GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2020

10612

MATHEMATICS

PAPER 2

TIME: 3 hours

MARKS: 150

14 pages + 1 information sheet

P.T.O.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs etc. that you have used to determine the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An INFORMATION SHEET with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

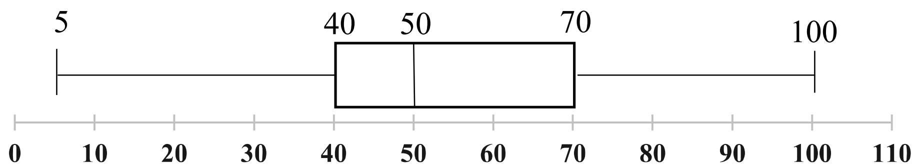
The A-Rithmetic High School decided to compare the results of 31 Grade 12 learners in Mathematics and Physical Sciences in the 2019 Preparatory Examination.

- The Mathematics results are recorded in the table below.
- The box and whisker plot below illustrates the results of Physical Sciences.
- Marks are recorded as percentages.

Mathematics Results

7	11	15	19	19	23	28	28	31	38	39
40	41	48	48	52	53	55	57	59	59	64
67	72	76	83	85	87	89	92	96	-	-

Physical Sciences Results



1.1 Calculate the mean mark of the Mathematics learners. (2)

1.2 Comment on the skewness of the Mathematics data. (1)

1.3 Determine which subject performed better in the 2019 Preparatory Examination. Give a reason for your conclusion. (2)

1.4 Write down a possible mark for a learner who achieved the tenth lowest mark in Physical Sciences. (2)

1.5 A learner scored the fourth highest in both subjects. The learner obtained the GREATEST possible difference between both subjects. Calculate the learner's mark in Physical Sciences. (2)

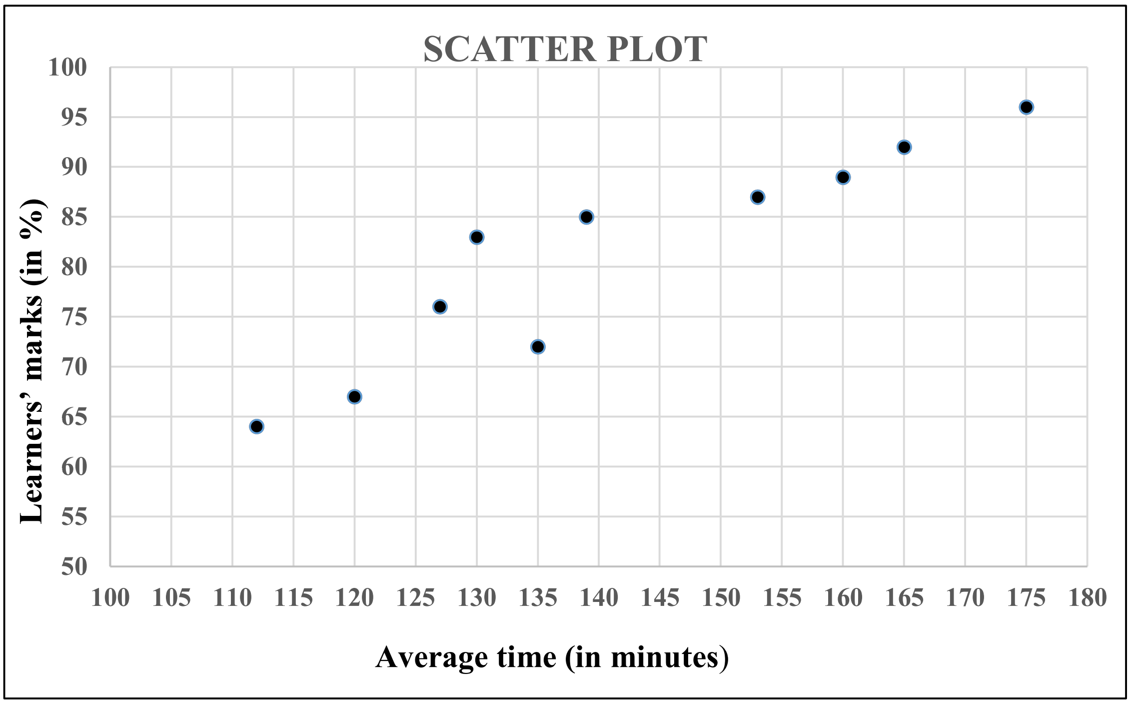
[9]

QUESTION 2

A question raised by many educators is whether the results that a learner achieves in an examination is dependent on the time that the learner takes to complete the examination.

The average time taken by each of the top 10 Mathematics learners was recorded. The data is represented in the table and scatter plot below.

Average time (in minutes)	175	165	160	153	139	130	127	135	120	112
Learners' marks (in %)	96	92	89	87	85	83	76	72	67	64



2.1 Calculate the equation of the least squares regression line for the data. (3)

2.2 A learner completed the exam in 2,5 hours. Predict the mark that the learner achieved. (2)

2.3 Explain within the context why the regression line is not reliable. (1)

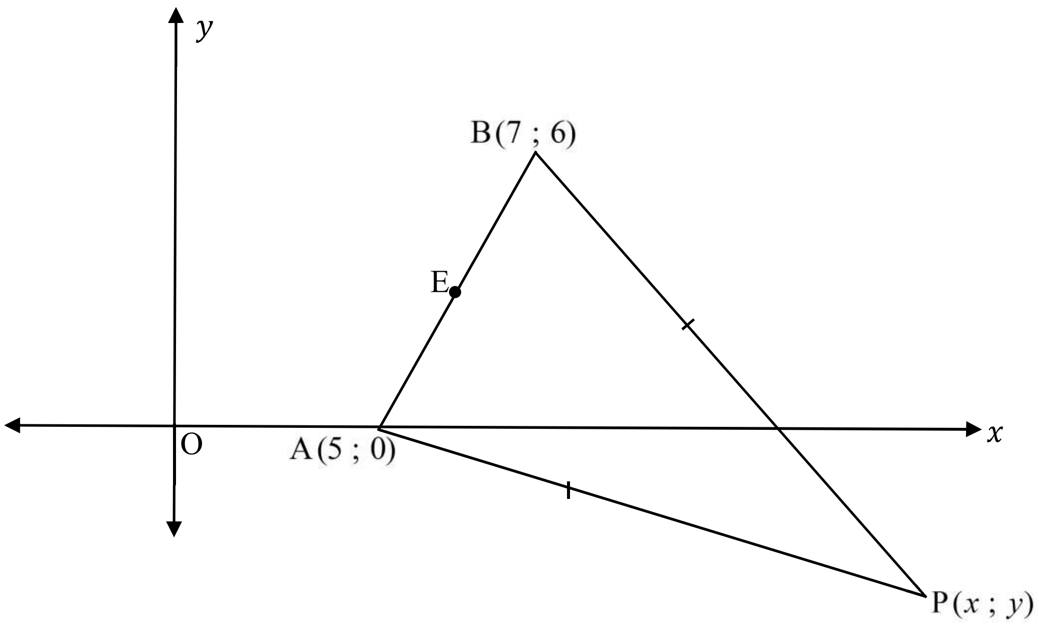
2.4 Calculate the standard deviation of the top 10 Mathematics learners. (2)

2.5 It is further given that $(p\,;103{,}59)$ is the interval of 15 random learners' marks within ONE standard deviation of the mean. If $\bar{x} = 63{,}96$, calculate the value of p. (3)

[11]

QUESTION 3

In the diagram below, points $A(5\ ;\ 0)$, $B(7\ ;\ 6)$ and $P(x\ ;\ y)$ form a triangle.
BP = AP and E is the midpoint of AB.



3.1 Determine the coordinates of E. (2)

3.2 Determine the equation of line BA. (3)

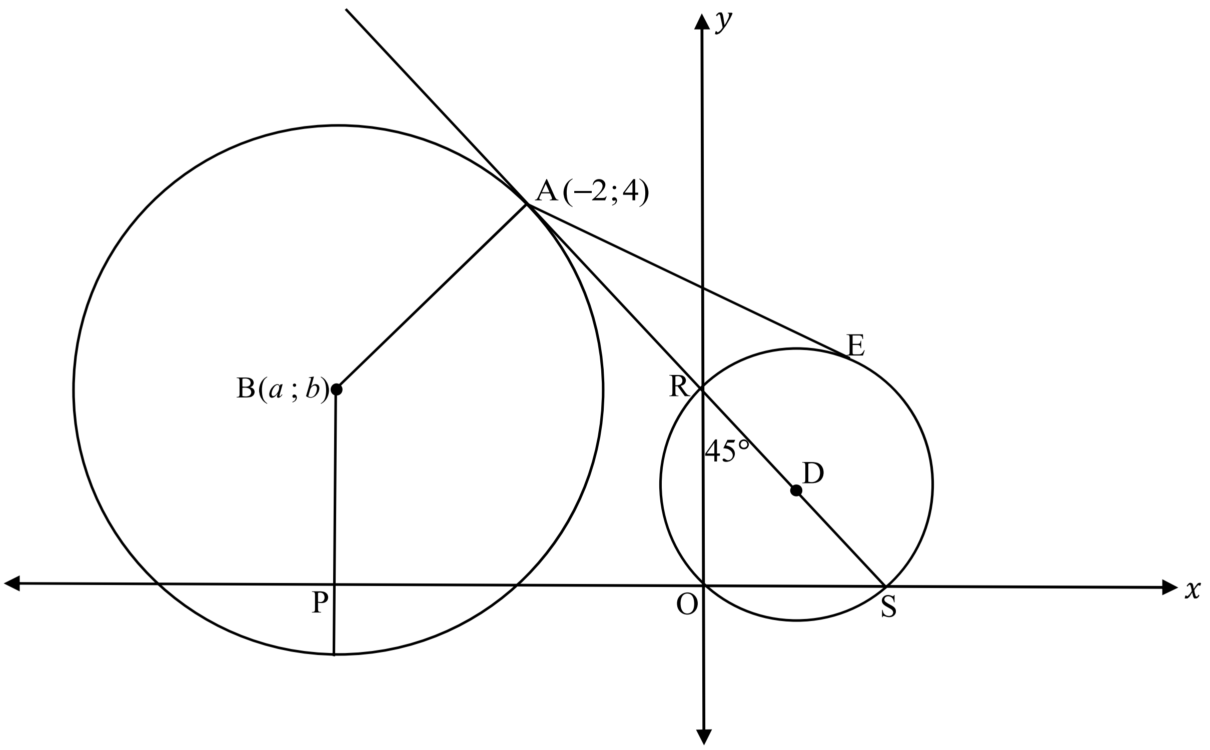
3.3 Line BA is parallel to the straight line with equation $rx - 3y + 5 = 0$.
Calculate the value of r. (3)

3.4 If the area of $\Delta AOP = 10$ units^2 and $y < 0$, calculate the coordinates of P. (7)

[15]

QUESTION 4

The diagram below shows a circle with centre $B(a ; b)$. BP is parallel to the y-axis with P on the x-axis. AS is a tangent to circle B at $A(-2 ; 4)$ and intersects the x-axis at S and the y-axis at R. AE is a tangent to the smaller circle with centre D and touches the circle at E. $O\hat{R}S = 45°$.



4.1 Determine the equation of tangent AS. (4)

4.2 If OP = 4 units, determine the values of a and b, the centre of the larger circle. (4)

4.3 Determine the equation of the circle with centre B. (3)

4.4 The equation of the smaller circle with centre D is $x^2 - 2x + y^2 - 2y = 0$.
Write this equation in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

4.5 Write down the coordinates of D, the centre of the smaller circle. (1)

4.6 Calculate the length of AE, the tangent to circle D at E. (6)
[21]

QUESTION 5

5.1 Calculate the value of $1-4\sin^2 15°$ without the use of a calculator. (5)

5.2 Simplify without the use of a calculator:

$$\frac{\sqrt{3}\sin x.\sin^2 72° + \sin^2 198°.\sqrt{3}\cos(x-90°)}{\tan 120°.\ \sin x}$$ (6)

5.3 Determine the general solution of the following:

$$6\sin x.\cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0$$ (7)

5.4 Prove that:

$$(1-\tan \mathrm{A})\left(\frac{\cos \mathrm{A}}{\cos 2\mathrm{A}}\right) = \frac{1}{\cos \mathrm{A} + \sin \mathrm{A}}$$ (4)

5.5 If $\sin 2\theta = k$ and $0° < 2\theta < 90°$, determine in terms of k:

5.5.1 $\cos 2\theta$ (2)

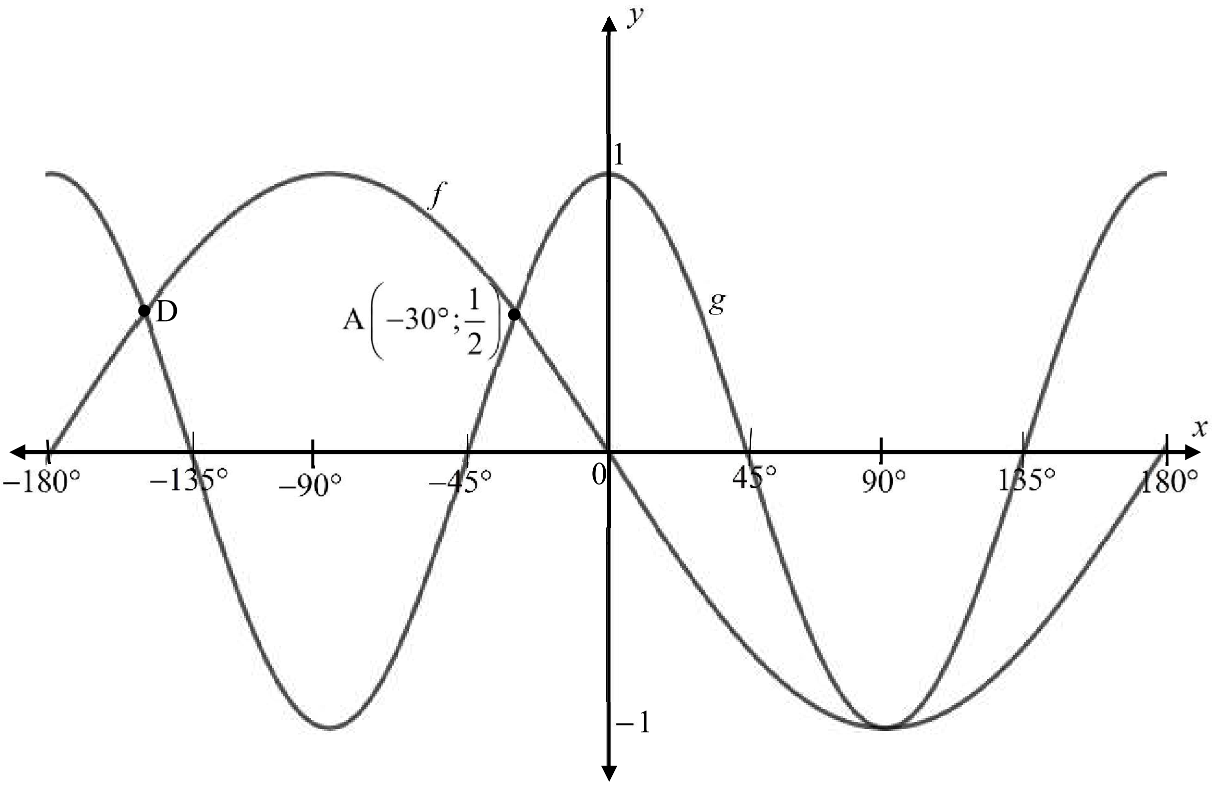
5.5.2 $\dfrac{\sin 2\theta}{\tan \theta}$ (5)

[29]

QUESTION 6

The sketch below shows the graphs of $f(x) = a\sin x$ and $g(x) = \cos dx$ for $x \in [-180° ; 180°]$.

$A\left(-30° ; \frac{1}{2}\right)$ is a point of intersection of f and g.



6.1 Write down the values of a and d. (2)

6.2 Determine the coordinates of D. (1)

6.3 For which value(s) of x is:

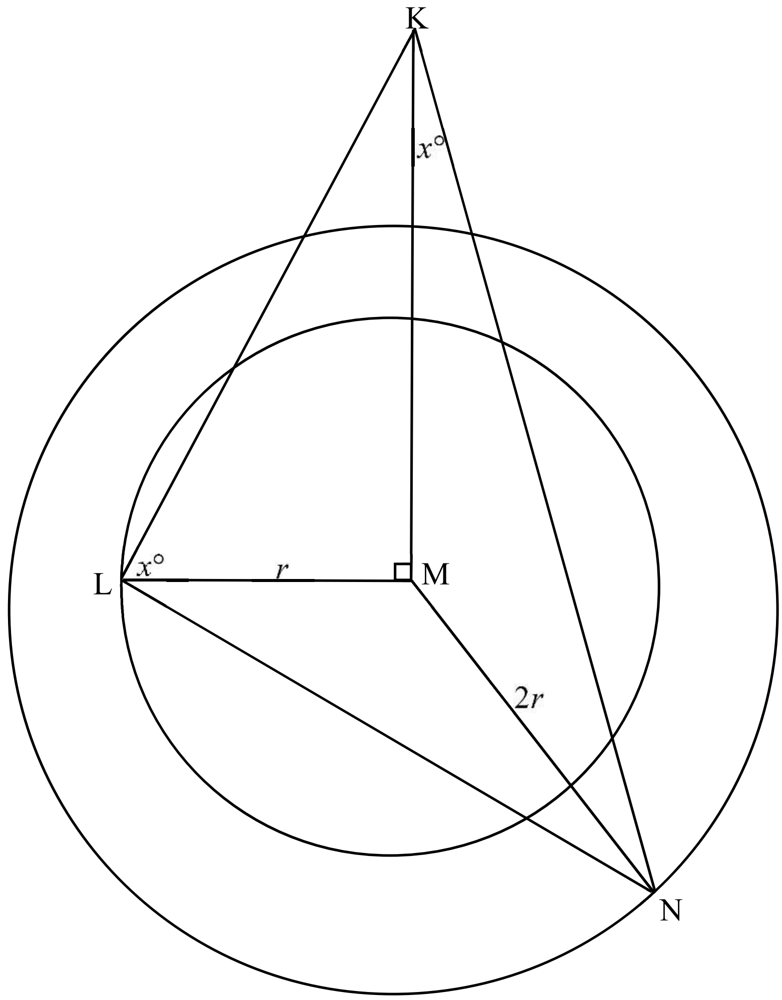
6.3.1 f decreasing for $x \in [-180° ; 180°]$? (2)

6.3.2 $f(x) \cdot g(x) < 0$ for $x \in [-180° ; 0°]$? (2)

[7]

QUESTION 7

In the figure below, KM is a vertical flag post set in the centre of two circles which lie on the same horizontal plane. $M\hat{K}N = M\hat{L}K = x°$. The radius of the inner circle $ML = r$ units and the radius of the outer circle $MN = 2r$ units.



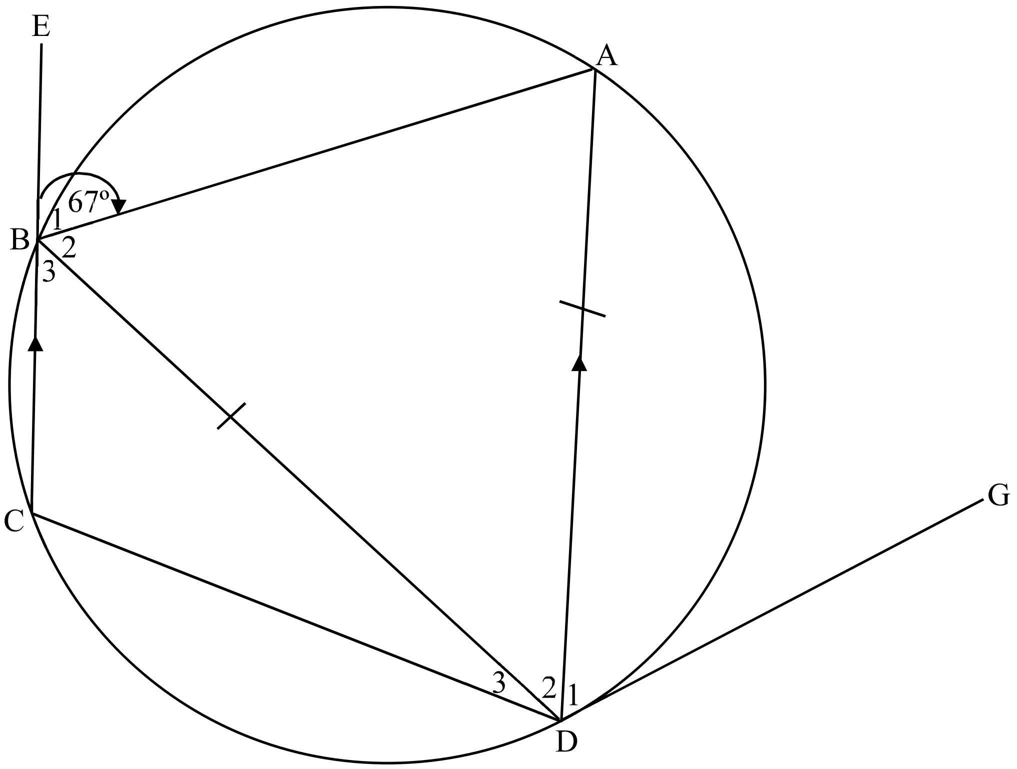
7.1 Calculate the value of x. (6)

7.2 If $r = 5m$ and $L\hat{M}N = 110°$, calculate the length of LN. (2)

[8]

QUESTION 8

In the diagram below, points A, B, C and D lie on the circumference of a circle with AD || EC. CB is produced to E. GD is a tangent to the circle at D and DB = AD.
$E\hat{B}A = 67°$.



8.1 Calculate, with reasons, the size of the following angles:

8.1.1 $A\hat{D}C$ (2)

8.1.2 $\hat{C}$ (1)

8.1.3 $\hat{A}$ (1)

8.1.4 $\hat{D}_2$ (3)

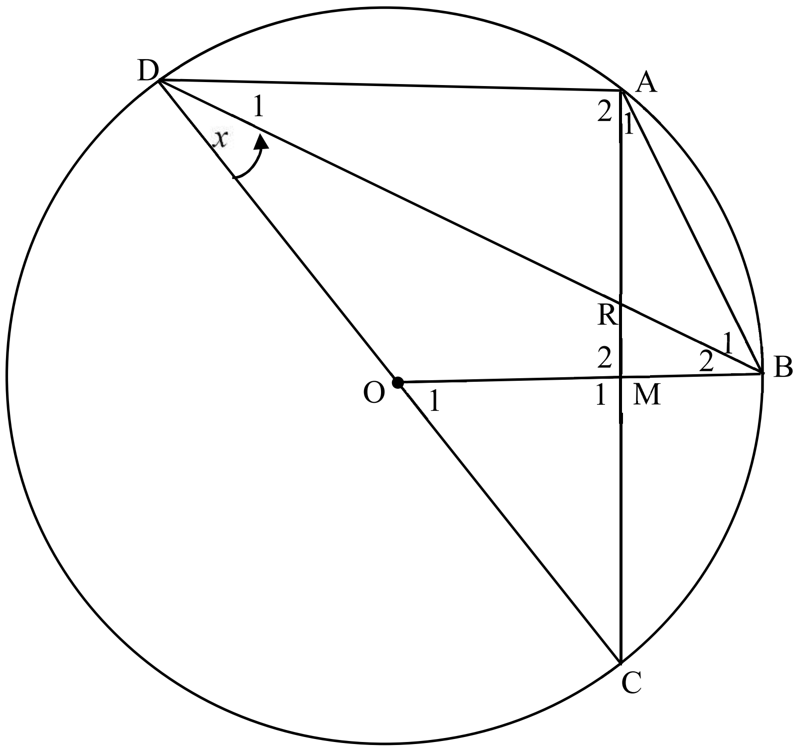
8.1.5 $B\hat{D}G$ (2)

8.2 Prove that AB = CD. (2)

[11]

QUESTION 9

9.1 In the diagram below, A, B, C and D are points on a circle with centre O. OB intersects AC at M, the midpoint of chord AC.
Let $B\hat{D}C = x$.



9.1.1 Determine, with reasons, in terms of x:

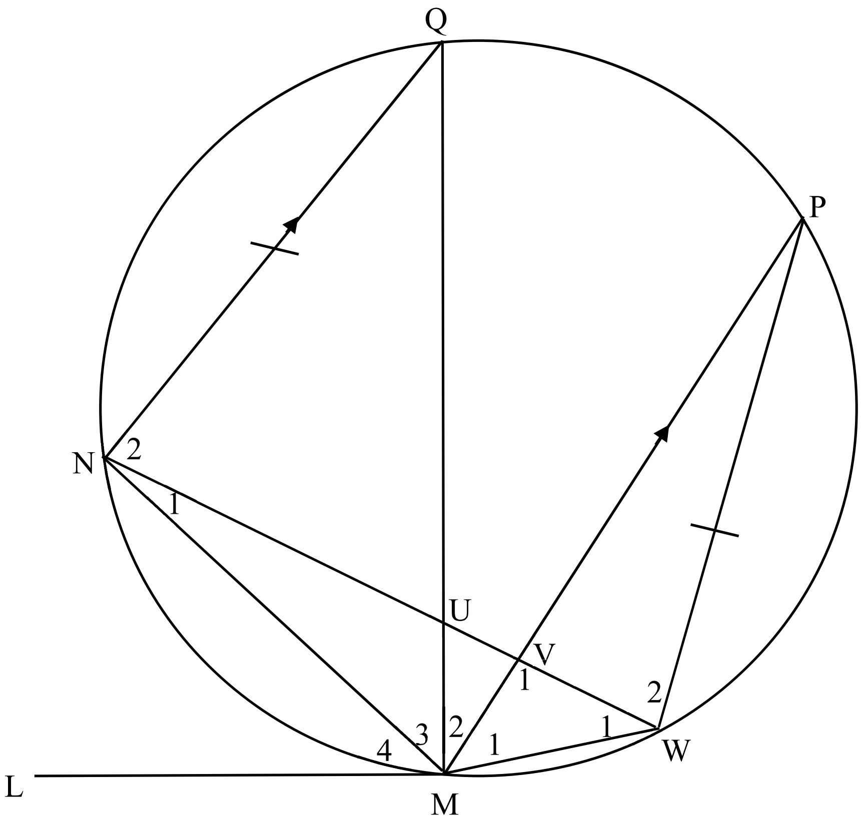
(a) $\hat{O}_1$ (1)

(b) $A\hat{B}O$ (4)

9.1.2 Prove that AB is a tangent to the circle that passes through points A, D and R. (6)

9.1.3 Prove that $AD^2 = 4DO^2 - 4AB^2 + 4MB^2$. (4)

9.2 In the diagram below, LM is a tangent to circle QNMWP at M. NW cuts QM and PM at U and V respectively.
NQ = WP and NQ || MP.



9.2.1 State, with reasons, THREE angles equal to $\hat{M}_2$. (3)

9.2.2 Prove that $\Delta WMV ||| \Delta QMN$. (3)

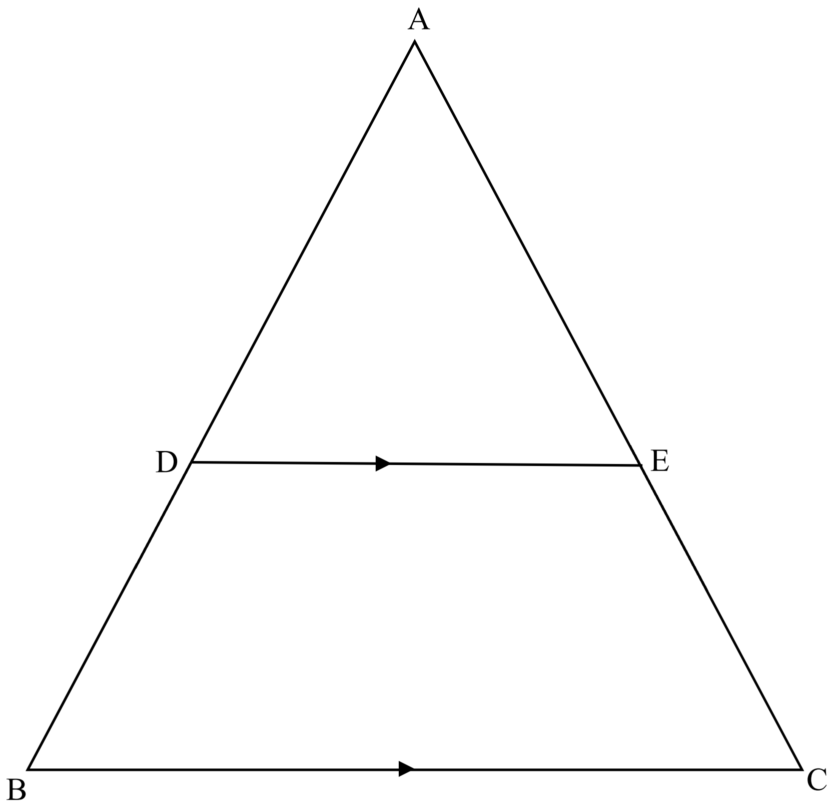
9.2.3 Prove that $\frac{MV}{WV} = \frac{MN}{PW}$. (3)

[24]

QUESTION 10

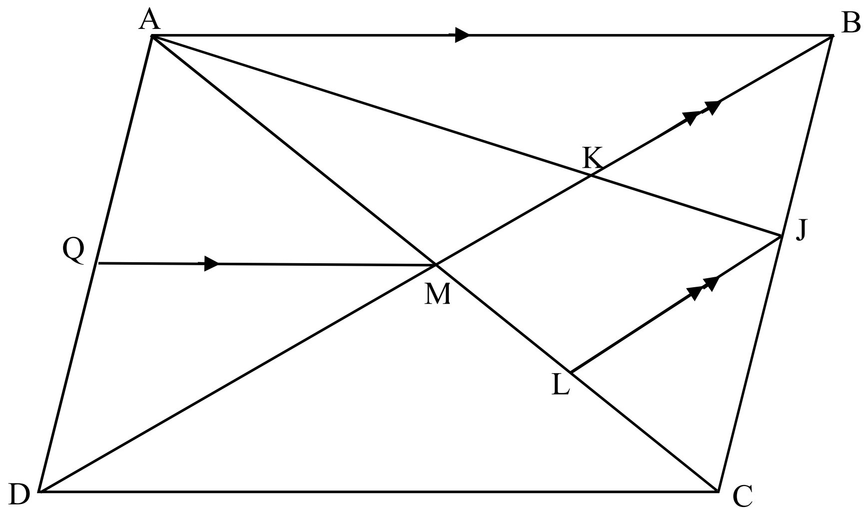
10.1 In ΔABC below, D and E are points on sides AB and AC respectively such that DE || BC.

Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 ABCD is a parallelogram with diagonals that intersect at M. J is a point on BC. BJ : JC is 2 : 3. AJ meets BD at K. BD || JL and JL meets AC at L. Q is a point on AD such that AB || QM.



10.2.1 Determine, with reasons, the following ratios:

(a) $\frac{ML}{LC}$ (2)

(b) $\frac{AK}{KJ}$ (3)

10.2.2 If $AB = \sqrt{10}$ units and $BC = \frac{2}{3}AB$.

Calculate the length of AQ. (4)

[15]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\quad$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}\ ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$