GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PREPARATORY EXAMINATION

2020

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

11 pages and 1 information sheet

P.T.O.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. Answer ALL the questions.
2. This question paper consists of 11 questions.
3. Present your answers according to the instructions of each question.
4. Clearly show ALL calculations, diagrams, graphs et cetera which were used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.
6. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet is included on Page 12 of the question paper.
10. Number the questions correctly according to the numbering system used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 + 5x = 7$ (correct to TWO decimal places) (4)

1.1.2 $2x^2 = 9x + 5$ (3)

1.1.3 $x^2 - 5x > -4$ (4)

1.1.4 $x - 3x^{\frac{1}{2}} = 4$ (6)

1.2 Show that the equation $2^{2x+1} + 7.2^x - 4 = 0$ has only ONE solution. (4)

1.3 Solve for x and y simultaneously:
$x = y - 13$ and $\sqrt{2 - x} = y - 3$ (6)

[27]

QUESTION 2

2.1 Given the following quadratic sequence:

$1 ; 7 ; 15 ; 25 ; x ; \ldots$

2.1.1 Write down the value of x in the sequence. (1)

2.1.2 Determine the expression for the n^{th} term of this sequence. (4)

2.1.3 W_n represents the general term of a sequence for the first differences.

Determine the value of the n^{th} term of the quadratic sequence if $W_n = 50$. (5)

2.2 Consider the following:

$$0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 \ldots$$

Assume that the pattern continues consistently.

2.2.1 Write down the value of the 191st term of this sequence. (1)

2.2.2 Determine the sum of the first 500 terms of this sequence. (4)

2.3 Consider the following geometric series:

$$4\left(\frac{1-k}{5}\right)+8\left(\frac{1-k}{5}\right)^{2}+16\left(\frac{1-k}{5}\right)^{3}\ldots$$

Determine the values of k. (4)

[19]

QUESTION 3

The first THREE terms of an infinite geometric sequence are 16, 8 and 4 respectively.

3.1 Determine ALL possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

3.2 Calculate the sum to infinity of this sequence. (2)

[5]

QUESTION 4

Given: $f(x) = \dfrac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Calculate:

4.2.1 the y-intercept of f. (2)

4.2.2 the x-intercept of f. (2)

4.3 Sketch the graph of f showing clearly the asymptotes and the intercepts of the axes. (3)

4.4 Determine the equation of the line of symmetry of f that has a negative gradient.
Leave your answer in the form $y = \ldots$ (3)

[12]

QUESTION 5

Given: $p(x) = \log_3 x$

5.1 Write down the equation of p^{-1}, the inverse of p, in the form $y = \ldots$ (2)

5.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axis. Show clearly the intercepts of the axes and ONE other point on each graph. (6)

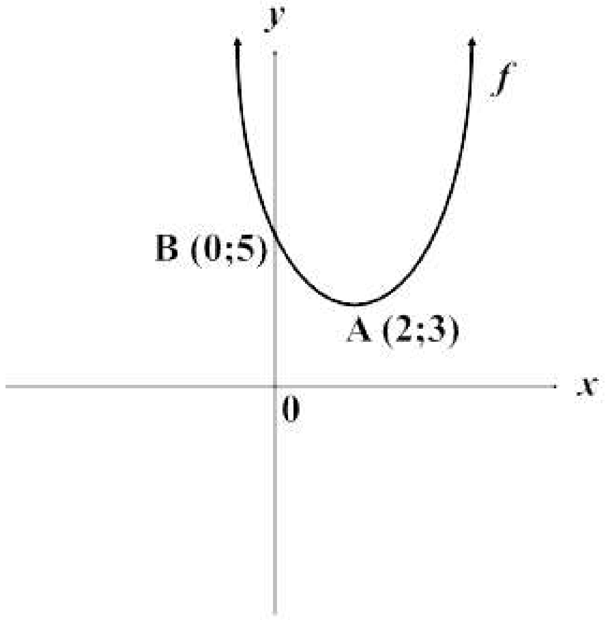
5.3 Determine the values of x for which $p(x) \leq 2$. (3)

5.4 Write down the x-intercept of h if $h(x) = p(-x)$. (2)

[13]

QUESTION 6

The sketch below shows the parabola f with turning point A(2 ; 3) and y-intercept B(0 ; 5).



6.1 Show that the equation of the parabola f can be written as $y = \frac{1}{2}x^2 - 2x + 5$. (5)

6.2 WITHOUT calculating the discriminant, give an appropriate reason whether the discriminant of f is positive, negative or zero. (2)

6.3 Use the graph to determine the value(s) of k for which the equation $\frac{1}{2}x^2 - 2x + 5 = k$ has real and un-equal roots. (2)

6.4 The parabola is shifted vertically until the new y-intercept is the origin. Determine the equation of the NEW parabola. (1)

[10]

QUESTION 7

7.1 Sarah's investment earns interest at 11% p.a. compounded semi-annually.
Mary's investment earns an effective interest of 11,42% *p.a.*
Whose investment, Sarah's or Mary's, earns a higher rate of interest per annum. (3)

7.2 Buhle decided to start saving before retirement. She makes payments of R10 000 monthly into an account yielding 7,72% *p.a.* compounded monthly, starting on 1 November 2016 with a final payment on 1 April 2026.

7.2.1 Calculate how much will be in the savings account immediately after the last deposit is made. (4)

7.2.2 At the end of the investment period Buhle re-invested the full amount in order for her to be able to draw a monthly pension from the fund.

She re-invested the money at an interest rate of 10% p.a. compounded monthly. If she draws an amount of R30 000 per month from this investment, for how many full months will she be able to receive R30 000? (4)

7.2.3 After withdrawing R30 000 for 20 months Buhle requires R1 500 000. Determine whether she can access this amount of money from this annuity. (4)
[15]

QUESTION 8

8.1 Determine $f'(x)$ from FIRST principles if

$f(x) = -2x^2 + 6x$ (4)

8.2 Evaluate:

8.2.1 $\frac{dy}{dx}$ if $y = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.2.2 $f'(x)$ if $f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4}$ (4)

8.3 The tangent to the curve of $y = 2x^2 - 3x - 5$ is drawn at the point (2 ; –3). A straight line parallel to the tangent passing through the y-intercept of the curve is then drawn. Determine the equation of the straight line. (4)

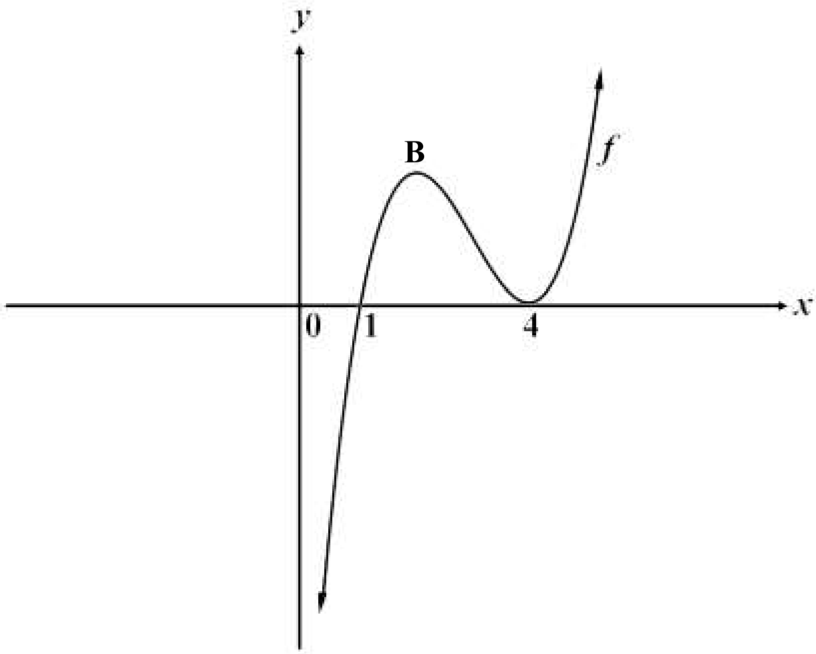
[14]

QUESTION 9

9.1 Determine the points on the curve of $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1. (5)

9.2 The graph of the cubic function with equation $y = x^3 + ax^2 + bx + c$ is drawn below.

- $f(1) = f(4) = 0$
- f has a local maximum at point B and a local minimum at $x = 4$.



9.2.1 Show that $a = -9$, $b = 24$ and $c = -16$. (2)

9.2.2 Calculate the coordinates of point B. (4)

9.2.3 Determine the value(s) of k for which $f(x) = k$ has ONLY negative roots. (2)

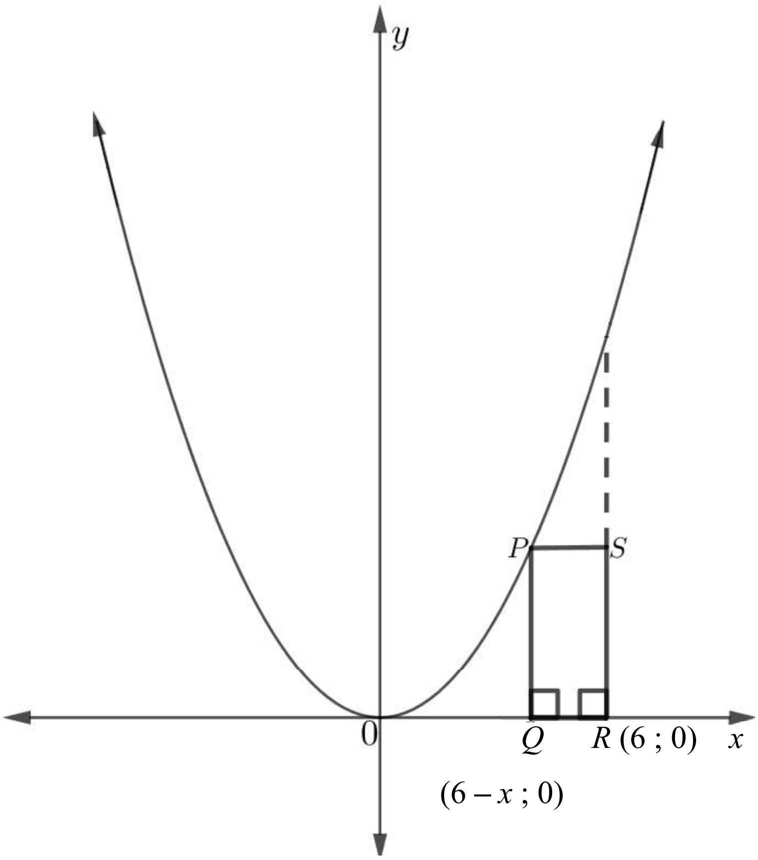
9.2.4 Determine the value(s) of x for which f is concave up. (2)

[15]

QUESTION 10

The diagram below represents the graph of $y = x^2$ and rectangle PQRS.

- Point P is a common point on both the graph and the rectangle.
- The graphs of $x = 6$ and $y = 0$ represent TWO adjacent infinite lines of rectangle PQRS.
- The coordinates of point R(6 ; 0) and point Q($6 - x$; 0) are given.



10.1	Write down the coordinates of point P in terms of x.	(1)
10.2	Determine the maximum area of rectangle PQRS.	(5) **[6]**

QUESTION 11

11.1 Given that A and B are independent events.

- P(B only) = 0,3
- P(A and B) = 0,2
- P(A only) = x
- P(not A or B) = y

Determine the values of x and y. (4)

11.2 Six players of a volleyball team stand at random positions in a row before the game begins. X and Y are two players in this team.
Determine the probability that X and Y will NOT stand next to each other. (3)

11.3 Determine how many 4-digit numbers can be formed from 10 digits, 0 to 9, if:

11.3.1 repetition of digits is allowed (2)

11.3.2 repetition of digits is NOT allowed (3)

11.3.3 the last digit must be 0 and repetition of digits is allowed (2)

[14]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$