



PREPARATORY EXAMINATION

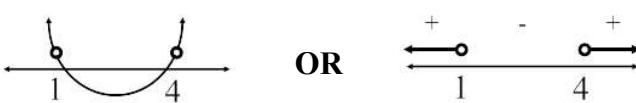
2020

MARKING GUIDELINES

MATHEMATICS P1 (10611)

17 pages

QUESTION 1

1.1.1	$3x^2 + 5x = 7$ $3x^2 + 5x - 7 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-7)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{109}}{6}$ $x = 0,91 \text{ or } x = -2,57$ <p>PENALIZE 1 MARK FOR INCORRECT ROUNDING IN THIS QUESTION ONLY.</p>	<ul style="list-style-type: none"> ✓ standard form ✓ subst. into correct formula ✓ ✓ answers <p style="text-align: right;">(4)</p>
1.1.2	$2x^2 = 9x + 5$ $2x^2 - 9x - 5 = 0$ $(2x+1)(x-5) = 0$ $x = -\frac{1}{2} \text{ or } x = 5$ <p>ANY OTHER VALID METHOD</p>	<ul style="list-style-type: none"> ✓ standard form ✓ factors ✓ both answers correct <p style="text-align: right;">(3)</p>
1.1.3	$x^2 - 5x > -4$ $x^2 - 5x + 4 > 0$ $(x-4)(x-1) > 0$ <div style="text-align: center;">  </div> $x < 1 \text{ or } x > 4$	<ul style="list-style-type: none"> ✓ standard form ✓ factors ✓ critical values ✓ both correct answers <p style="text-align: right;">(4)</p>

1.1.4	$x - 3x^{\frac{1}{2}} = 4$ $x - 4 = 3\sqrt{x}$ $x^2 - 8x + 16 = 9x$ $x^2 - 17x + 16 = 0$ $(x - 16)(x - 1) = 0$ $x = 16 \text{ OR } x = 1$ <p style="text-align: center;">N/A</p>	<ul style="list-style-type: none"> ✓ isolating $3\sqrt{x}$ ✓ squaring both sides ✓ standard form ✓ factors ✓ critical values ✓ selection/rejection <p style="text-align: right;">(6)</p>
1.2	$2^{2x+1} + 7 \cdot 2^x - 4 = 0$ <p>let $2^x = k$</p> $\therefore 2k^2 + 7k - 4 = 0$ $(2k - 1)(k + 4) = 0$ $k = \frac{1}{2} \text{ OR } k = -4$ $\therefore 2^x = 2^{-1} \quad 2^x \neq -4$ $x = -1 \quad \text{Not a solution}$	<ul style="list-style-type: none"> ✓ standard form ✓ factors ✓ critical values ✓ rejection of 1 answer <p style="text-align: right;">(4)</p>
1.3	$x = y - 13 \dots (1) \text{ and } \sqrt{2 - x} = y - 3 \dots (2)$ <p style="text-align: center;">sub (1) into (2)</p> $\sqrt{2 - (y - 13)} = y - 3$ $\sqrt{15 - y} = y - 3$ $15 - y = y^2 - 6y + 9$ $0 = y^2 - 5y - 6$ $0 = (y - 6)(y + 1)$ $y = 6 \text{ OR } y = -1$ $x = -7$ <p style="text-align: center;">OR</p>	<ul style="list-style-type: none"> ✓ substitution ✓ square both sides ✓ standard form ✓ factors ✓ y-values ✓ x-value <p style="text-align: center;">OR</p>

	$x = y - 13 \text{ and } \sqrt{2-x} = y - 3$ $\therefore y = x + 13 \dots (1) \quad \sqrt{2-x} = y - 3 \dots (2)$ <p>Sub for y into equation 2</p> $\sqrt{2-x} = x + 13 - 3$ $\sqrt{2-x} = x + 10$ $2 - x = x^2 + 20x + 100$ $0 = x^2 + 21x + 98$ $0 = (x + 14)(x + 7)$ $x = -14 \text{ OR } x = -7$ $y \neq -1 \text{ OR } y = 6$	<ul style="list-style-type: none"> ✓ substitution ✓ square both sides ✓ standard form ✓ factors ✓ x-values ✓ y-values <p style="text-align: right;">(6)</p>
		[27]

QUESTION 2

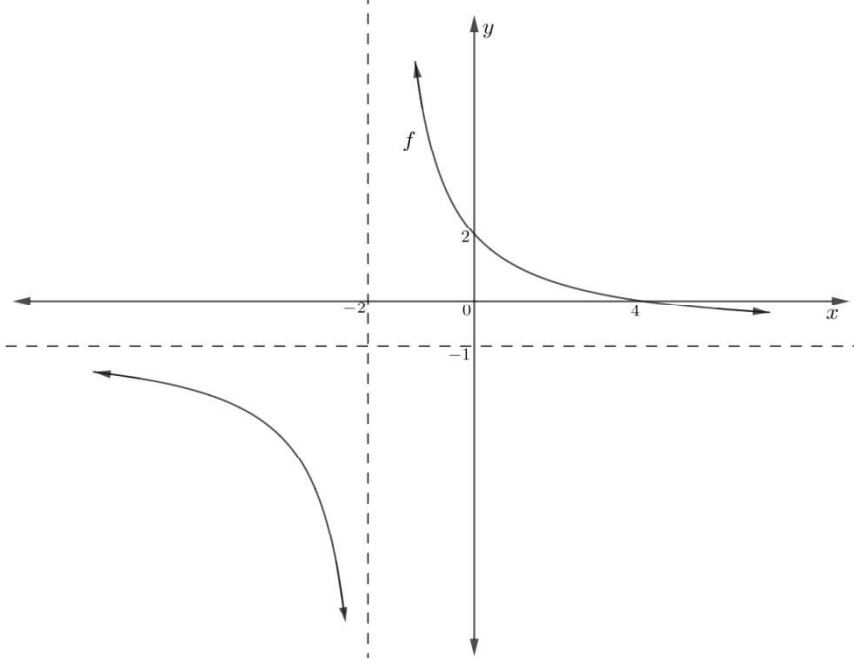
2.1.1	37	✓ answer (1)
2.1.2	$ \begin{array}{c} 1 \quad ; \quad 7 \quad ; \quad 15 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 6 \quad \quad \quad 8 \\ \swarrow \quad \searrow \\ 2 \end{array} $ $2a = 2$ $a = 1$ $3a + b = 6$ $3(1) + b = 6$ $b = 3$ $a + b + c = 1$ $1 + 3 + c = 1$ $c = -3$ $T_n = n^2 + 3n - 3$	✓ second difference ✓ $a = 1$ ✓ $b = 3$ ✓ $c = -3$ (4)
2.1.3	$W_n = 2n + 4$ $50 = 2n + 4$ $46 = 2n$ $n = 23$ $T_{23} = 23^2 + 3(23) - 3$ $= 595$	✓ $W_n = 2n + 4$ ✓ equating to 50 ✓ $n = 23$ ✓ substitution ✓ answer (5)
2.2.1	$T_{191} = 0$	✓ answer (1)
2.2.2	$-\frac{1}{2} ; \frac{1}{2} ; \frac{3}{2} \dots 250 \text{ terms}$ $d = 1$ $S_{250} = \frac{250}{2} \left[2 \left(-\frac{1}{2} \right) + (250 - 1)(1) \right]$ $S_{250} = 31000$ $\therefore 0 + -\frac{1}{2} + 0 + \frac{1}{2} + 0 + \frac{3}{2} \dots \text{to } 500 \text{ terms}$ $S_{500} = 31000$	✓ value of d ✓ correct sub. into correct formula ✓ $S_{250} = 31\ 000$ ✓ answer (4)

2.3	$r = 2\left(\frac{1-k}{5}\right)$ $\therefore -1 < 2\left(\frac{1-k}{5}\right) < 1$ $-\frac{1}{2} < \left(\frac{1-k}{5}\right) < \frac{1}{2}$ $-\frac{5}{2} < 1-k < \frac{5}{2}$ $-\frac{7}{2} < -k < \frac{3}{2}$ $-\frac{3}{2} < k < \frac{7}{2}$	<p>✓ r</p> <p>✓ $-1 < r < 1$</p> <p>✓ correct critical values</p> <p>✓ answer</p> <p>(4)</p>
		[19]

QUESTION 3

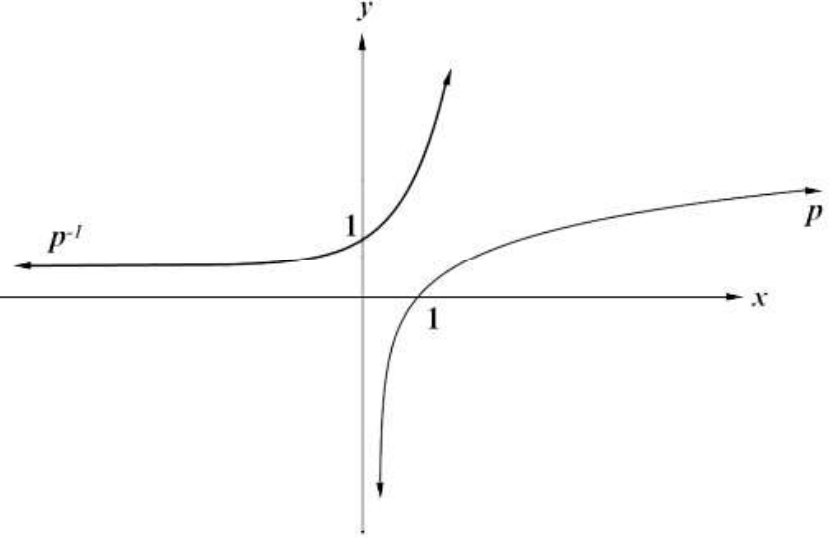
3.1	$S_n > 31$ $\therefore \frac{16 \left(1 - \left(\frac{1}{2} \right)^n \right)}{1 - \left(\frac{1}{2} \right)} > 31$ $\left(1 - \left(\frac{1}{2} \right)^n \right) > \frac{31}{32}$ $-\left(\frac{1}{2} \right)^n > -\frac{1}{32}$ $-\left(\frac{1}{2} \right)^n > -\left(\frac{1}{2} \right)^5$ $\therefore n > 5 \text{ or } n \geq 6$ <p>ANSWER ONLY: FULL MARKS</p>	<p>✓ correct substitution into correct formula</p> <p>✓ simplification $\left(-\frac{1}{32}\right)$</p> <p>✓ correct answers</p> <p style="text-align: right;">(3)</p>
3.2	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{16}{1 - \frac{1}{2}}$ $S_\infty = 32$	<p>✓ correct substitution into correct formula</p> <p>✓ answer</p> <p style="text-align: right;">(2)</p>
		[5]

QUESTION 4

4.1	$x = -2$ $y = -1$	✓ answer ✓ answer (2)
4.2.1	$y = \frac{6}{0+2} - 1$ $y = 2$	✓ $x = 0$ ✓ answer (2)
4.2.2	$0 = \frac{6}{x+2} - 1$ $1 = \frac{6}{x+2}$ $x+2 = 6$ $x = 4$	✓ $y = 0$ ✓ answer (2)
4.3		✓ shape ✓ asymptotes ✓ x -and y- intercepts (3)

4.4	$y = -(x + 2) - 1$ $y = -x - 3$ <p>OR</p> $y = mx + c$ $-1 = -1(-2) + c$ $-3 = c$ $\therefore y = -x - 3$	✓ $m = -1$ ✓ correct subst. of point $(-2; -1)$ ✓ answer OR ✓ $m = -1$ ✓ correct subst. of point $(-2; -1)$ ✓ answer (3)
		[12]

QUESTION 5

5.1	$p(x) = \log_a x$ $x = \log_a y$ $\therefore p^{-1}: y = 3^x$	✓ ✓ answers (2)
5.2		✓ ✓ shape p ✓ point on p ✓ ✓ shape p^{-1} ✓ point on p^{-1} (6)
5.3	$2 = \log_a x$ $x = 9$ $0 < x \leq 9$	✓ value of x ✓ ✓ answer (3)
5.4	x -intercept of p is $(1; 0)$ x -intercept of h is $(-1; 0)$	✓ ✓ answer (2)
		[13]

QUESTION 6

6.1	$y = a(x - p)^2 + q$ $5 = a(0 - 2)^2 + 3$ $5 = 4a + 3$ $2 = 4a$ $\therefore a = \frac{1}{2}$ $y = \frac{1}{2}(x - 2)^2 + 3$ $y = \frac{1}{2}(x^2 - 4x + 4) + 3$ $y = \frac{1}{2}x^2 - 2x + 2 + 3$ $y = \frac{1}{2}x^2 - 2x + 5$	<p>✓ ✓ sub. point A and B correctly</p> <p>✓ value of a</p> <p>✓ sub. a, p and q</p> <p>✓ simplification</p> <p style="text-align: right;">(5)</p>
6.2	<p>The graph does NOT cut the x-axis.</p> <p>\therefore No real roots</p> <p>$\therefore \Delta < 0$</p>	<p>✓ explanation</p> <p>✓ $\Delta < 0$</p> <p style="text-align: right;">(2)</p>
6.3	$\frac{1}{2}x^2 - 2x + 5 = k$ <p>$\therefore k > 3$</p>	<p>✓ ✓ answer</p> <p style="text-align: right;">(2)</p>
6.4	$y = \frac{1}{2}x^2 - 2x + 5 - 5$ $y = \frac{1}{2}x^2 - 2x$	<p>✓ answer</p> <p style="text-align: right;">(1)</p>
		[10]

QUESTION 7

7.1	$1 + i_{eff} = \left(1 + \frac{0,11}{2}\right)^2$ $i_{eff} = \left(1 + \frac{0,11}{2}\right)^2 - 1$ $i_{eff} = 11,30\%$ <p>\therefore Mary has secured the better rate.</p>	<ul style="list-style-type: none"> ✓ correct sub. into correct formula ✓ answer ✓ conclusion <p style="text-align: right;">(3)</p>
7.2.1	$Fv = \frac{10\,000 \left[\left(1 + \frac{0,0772}{12}\right)^{114} - 1 \right]}{\frac{0,0772}{12}}$ $= R1\,674\,501,44$	<ul style="list-style-type: none"> ✓ value of i ✓ value of n ✓ correct sub. into correct formula ✓ answer <p style="text-align: right;">(4)</p>
7.2.2	$R1\,674\,501,44 = \frac{30\,000 \left[1 - \left(1 + \frac{0,1}{12}\right)^{-n} \right]}{\frac{0,1}{12}}$ $0,46513\dots = \left[1 - \left(1 + \frac{0,1}{12}\right)^{-n} \right]$ $0,53486\dots = \left(1 + \frac{0,1}{12}\right)^{-n}$ $\log_{\left(1 + \frac{0,1}{12}\right)} 0,53486\dots = -n$ $n = 75,4$ <p>She will be able to receive the money in 75 full months.</p>	<ul style="list-style-type: none"> ✓ subst. of P, x and i into correct formula ✓ simplification ✓ correct use of logs ✓ answer <p style="text-align: right;">(4)</p>
7.2.3	$Pv = \frac{30\,000 \left[1 - \left(1 + \frac{0,1}{12}\right)^{-55} \right]}{\frac{0,1}{12}}$ $Pv = R\,1\,319\,260,60$ <p>\therefore No</p>	<ul style="list-style-type: none"> ✓ subst. x and i into correct formula ✓ correct value of n ✓ answer ✓ conclusion <p style="text-align: right;">(4)</p>
[15]		

QUESTION 8

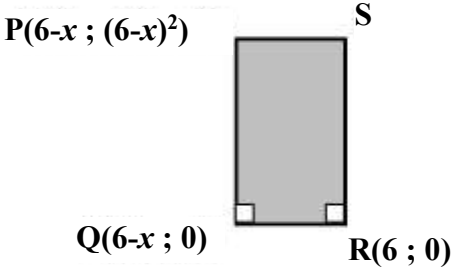
8.1	$f(x) = -2x^2 + 6x$ $f(x+h) = -2(x+h)^2 + 6(x+h)$ $f(x+h) = -2(x^2 + 2xh + h^2) + 6x + 6h$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 6x + 6h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 6x + 6h - (-2x^2 + 6x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 6x + 6h + 2x^2 - 6x}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2 + 6h}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h + 6)}{h}$ $f'(x) = \lim_{h \rightarrow 0} -4x - 2h + 6$ $f'(x) = -4x + 6$	<p>✓ value of $f(x+h)$</p> <p>✓ correct sub. into correct formula</p> <p>✓ factorise</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>
8.2.1	$f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ $f'(x) = 4x + 2x^3$	<p>✓ $4x$</p> <p>✓ $2x^3$</p> <p style="text-align: right;">(2)</p>
8.2.2	$f(x) = \frac{x^3 - 5x^2 + 4x}{x-4}$ $f(x) = \frac{x(x^2 - 5x + 4)}{x-4}$ $f(x) = \frac{x(x-4)(x-1)}{x-4}$ $f(x) = x^2 - x$ $f'(x) = 2x - 1$	<p>✓ factorising</p> <p>✓ factors</p> <p>✓ simplification of f</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>

8.3	$y = 2x^2 - 3x - 5$ $\frac{dy}{dx} = 4x - 3$ gradient at $x = 2$ $y = 2x^2 - 3x - 5$ $m = 4(2) - 3$ $m = 5$ $y = 5x - 5$	✓ derivative ✓ substitution ✓ value of m ✓ answer (4)
		[14]

QUESTION 9

9.1	<p>For $y = \frac{4}{x}$ the gradient of the tangent to the curve is –</p> <p>1.</p> $y = 4x^{-1}$ $\frac{dy}{dx} = -4x^{-2} = \frac{-4}{x^2}$ $\frac{-4}{x^2} = -1$ $4 = x^2$ <p>$\therefore x = -2$ OR $x = 2$</p> <p>$\therefore y = -2$ $y = 2$</p> <p>$(-2; -2)$ $(2; 2)$</p>	<p>✓ exponential form</p> <p>✓ derivative</p> <p>✓ derivative = -1</p> <p>✓ x-values</p> <p>✓ y-values</p> <p>(5)</p>
9.2.1	$y = (x-1)(x-4)^2$ $y = (x-1)(x^2 - 8x + 16)$ $y = x^3 - 9x^2 + 24x - 16$	<p>✓ $(x-1)(x-4)^2$</p> <p>✓ squaring binomial</p> <p>(2)</p>
9.2.2	$y = x^3 - 9x^2 + 24x - 16$ $\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$ $x^2 - 6x + 8 = 0$ $(x-2)(x-4) = 0$ <p>$x = 2$ OR $x = 4$</p> <p>$y = 4$ OR $y = 0$</p> <p>B(2 ; 4)</p>	<p>✓ derivative = 0</p> <p>✓ factors</p> <p>✓ y-values</p> <p>✓ coordinates of B</p> <p>(4)</p>
9.2.3	$k < -16$	<p>✓ ✓ answer</p> <p>(2)</p>
9.2.4	$f''(x) = 6x - 18$ $6x - 18 > 0$ $x > 3$	<p>✓ $6x - 18$</p> <p>✓ answer</p> <p>(2)</p>
		[15]

QUESTION 10

10.1	$P[6-x; (6-x)^2]$	✓ answer (1)
10.2	<div style="text-align: center;">  <p>$P(6-x; (6-x)^2)$ S</p> <p>$Q(6-x; 0)$ $R(6; 0)$</p> </div> <p>$A=L \times B$ $A = x[(6-x)^2]$ $A = x(36-12x+x^2)$ $A = x^3 - 12x^2 + 36x$</p> <p>$\frac{dA}{dx} = 3x^2 - 24x + 36$ $x^2 - 8x + 12 = 0$ $(x-6)(x-2)$ $x \neq 6$ OR $x = 2$ $y = 16$</p> <p>$A_{\max} = 16 \times 2$ $= 32$</p>	<p>✓ correct substitution into area formula</p> <p>✓ formula for area in terms of x</p> <p>✓ derivative = 0</p> <p>✓ choice of x-value to determine y-value</p> <p>✓ answer (5)</p>
		[6]

QUESTION 11

11.1	$(x+0,2) \times 0,5 = 0,2$ $x+0,2 = 0,4$ $x = 0,2$ $0,2+0,2+0,3+y = 1$ $y = 0,3$	✓ $P(A) \times P(B) = P(A \text{ and } B)$ ✓ x - value ✓ sum of probabilities = 1 ✓ y -value (4)
11.2	$P(X \text{ and } Y \text{ together}) = \frac{2!5!}{6!}$ $= \frac{1}{3}$ $P(X \text{ and } Y \text{ not together}) = 1 - \frac{1}{3}$ $= \frac{2}{3}$	✓ $\frac{2!5!}{6!}$ ✓ $\frac{1}{3}$ ✓ answer (3)
11.3.1	4 digit numbers (with repetition) $= 9 \times 10 \times 10 \times 10$ $= 9\ 000$	✓ product ✓ answer (2)
11.3.2	4 digit numbers (without repetition) $= 9 \times 9 \times 8 \times 7$ $= 4\ 536$	✓ 9×9 ✓ 8×7 ✓ answer (3)
11.3.3	4 digit numbers (with repetition and last digit = 0) $= 9 \times 8 \times 7 \times 1$ $= 4\ 536$	✓ product ✓ answer (2)
		[14]

TOTAL: 150