MATHEMATICS P2 COMPLETE REVISION & PRACTICE SSIP: NSC EXAM KIT 2020

MGSLG

CONTENTS

MARKS IN THE PAPER ACCORDING TO TOPICS

TOPICS	**MARKS**
Statistics	20 ± 3
Analytical Geometry	40 ± 3
Statistics	40 ± 3
Euclidean Geometry	50 ± 3

TAXONOMY LEVELS EXPECTED TO BE IN THE PAPER

Cognitive levels	Description of skills to be demonstrated	Marks Out of 150
Knowledge 20%	• Straight recall • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary	30
Routine procedures 35%	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formulae the information sheet (No changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class.	52,5
Complex procedures 30%	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real-world context • Could involve making significant connections between different representations • Require conceptual understanding	45
Problem solving 15%	• Unseen, non-routine problems (which are not necessarily difficult) • Higher order understanding and processes are involved • Might require the ability to break the problem down into its constituent parts	22,5

STATISTICS

NOTES TO REVISE WITH

1 **MEAN:** the average of the set of data. Sum of all data items ÷ number of items.

2 **MEDIAN:** of…. there are as many items above the median as below it.

Odd number of data items:

2; 4; **6**; 7 ; 10 Median is **6** – the value in the **centre**

OR: 2; 4; 6; 7 ; 10 ; 12 ; 18

Even number of data items:

$2 ; 4 ; \boxed{6 ; 9} ; 12 ; 15$ $\text{Median} = \frac{6 + 9}{2} = \frac{15}{2} = 7{,}5$

3 **MODE** of… (Value that occurs most often…remember M**O**ST)

2 ; 4 ; **6 ; 6** ; 7 ; 12 Mode = 6

MODE of …

2 ; 4 ; **6 ; 6** ; 7 ; **12 ; 12** ; 15 Modes = 6 and 12

This set of data is bimodal.

4 **RANGE** = highest value – lowest value

2 ; 4 ; 6 ; 7 ; 10 Range = 10 – 2 = 8

5 **QUARTILE 1 / Lower quartile of an odd number of data items**

$\underbrace{7 ; \mathbf{9} ; 10}$; **10** ; 11 ; 12 ; 15

Median = 10

Lower Quartile = 9 It is the median of the data items to the left of the median

OR the Lower Q of 9 data items: 2 ; 4 ↓ 6 ; 6 ; **7** ; 8 ; 9 ; 12 ; 15

6 **Quartile 3 / Upper quartile of an odd number of data items**

7 ; 9 ; 10 ; **10** ; $\underbrace{11 ; \mathbf{12} ; 15}$

Median = 10

Upper Quartile = 12 It is the median of the data items to the right of the median

OR the Upper Q of 9 data items: 2 ; 4 ; 6 ; 6 ; **7** ; 8 ; 9 ↓ 12 ; 15

7.1 **Quartile 1 and 3 of an even number of data items**

7 ; 9 ↑ 10 ; 10 ↑ 11 ; 12 ↑ 15 ; 22 The median of this set is $\frac{10 + 11}{2} = 10{,}5$

Quartile 1 = $\frac{9 + 10}{2}$ = 9,5 Quartile 3 = $\frac{12 + 15}{2}$ = 13,5

OR: **Quartile 1 and 3 of an even number of data items**

$2 ; 4 ; \overset{\downarrow}{6} ; 6 ; 7 ; 8 ; 9 ; \overset{\downarrow}{12} ; 15 ; 20$

7.2 **Quartile 1 and Quartile 3 of a large set of data (Position)**

Suppose there are 8527 data items

$$Quartile\ 1 = \frac{1}{4}(8527 + 1) = 2132nd\ data\ item \qquad Quartile\ 3 = \frac{3}{4}(8527 + 1) = 6\ 396th\ data\ item$$

8 PERCENTILES

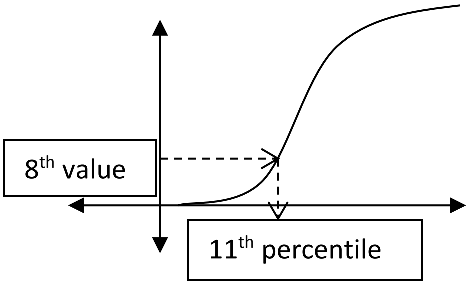
11th percentile of 70 values

11% of 70 = 7,7th value $\approx$ 8th value

On an ogive: get the 8th value on the vertical axis.

Draw a horizontal line to the graph and drop a perpendicular line to the horizontal axis.

This is the 11th percentile.



9 INTER QUARTILE RANGE: Quartile 3 - Quartile 1

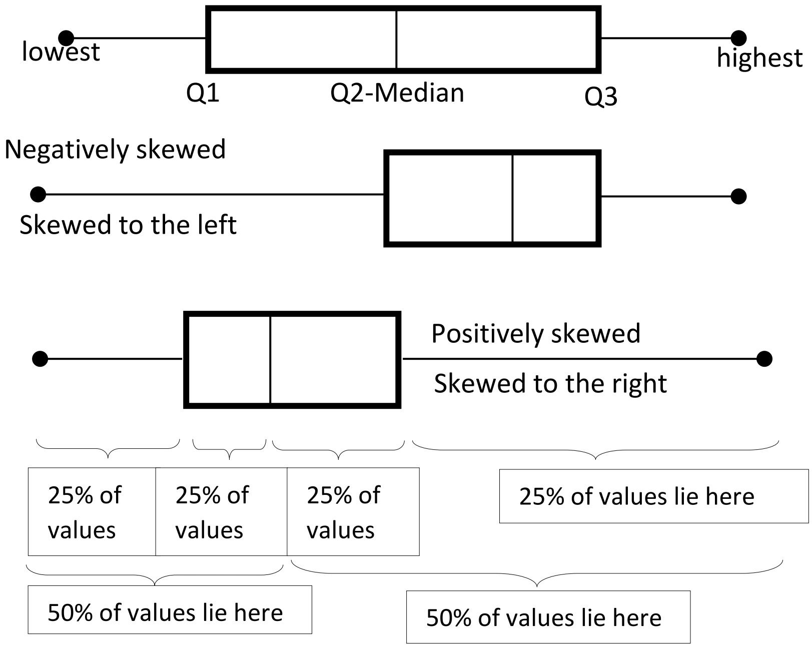
10 SEMI INTER QUARTILE RANGE: $\dfrac{Quartile\ 3\ -\ Quartile\ 1}{2}$

11 FIVE NUMBER SUMMARY OF A SET OF DATA

(1) Lowest value (2) Quartile 1 (3) Median (4) Quartile 3 (5) Highest value

12 BOX AND WHISKER DIAGRAM - USE THE FIVE NUMBER SUMMARY

Use the 5-number summary



13 IDENTIFICATION OF OUTLIERS

An outlier is a data value which is much larger or smaller than the rest of the values in the data set. This value will not follow the general trend of the data.

A value is defined to be an outlier if it fulfils one of the conditions below:

- **The value is less than** $Q_1 - (1,5 \times IQR)$
- **The value is greater than** $Q_3 + (1,5 \times IQR)$

14 BAR GRAPHS

In a bar graph the bars don't touch.

Data about anything that can be counted is displayed on a bar graph.

Note:

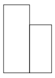
- The bar graph is drawn in a frame
- The graph must have a proper heading
- Both axes must be labelled

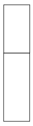
15 PIE CHART

- Can you draw a pie chart? Basically, it represents the same data than a bar chart. The frequency of each bar is calculated as a percentage of the total.
- The same percentage is calculated of 360°. THIS IS THE SIZE OF THE SECTOR ANGLE (the sector angle is the angle of the slice of e.g. a piece of pizza – the angle at the centre determines the size of the slice.)

16 COMPOUND BAR GRAPHS:

- boys/ girls distribution in a number of classes
- yes/no responses to a number of questions



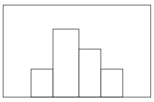


DIVIDED BAR CHART: same information but on top of each other

17 HISTOGRAM

- The intervals can be height, time, age, length, marks and so forth.
- The number of data items that fall within a certain interval is the frequency of the interval. The height of the bar reflects the frequency of the items falling within an interval.
- The **bars touch** because the number intervals are continuous. We say: numerical class intervals are displayed on the horizontal axis.
- From the histogram you can be asked to draw an ogive.

18 FREQUENCY POLYGON



The midpoints of the class intervals of a histogram are calculated. These midpoint values are indicated at the tops of the bars. Then these midpoints are connected. The polygon is anchored to the horizontal axis on the left-hand side of the first bar and the right-hand side of the last bar.

19 LINE GRAPH / BROKEN LINE GRAPH

Most often seen in the financial pages of newspapers (e.g. JSE).

- **Shows increasing or decreasing trends or changes over time**
- **Help to make predictions based on the trend observed in the graph.**

20 OGIVE OR CUMULATIVE FREQUENCY

You will be given a frequency table and from that you have to draw an ogive. This is a very cryptic example but shows the principle.

DRAWING: Cumulative frequency on the y – axis and class intervals on the x - axis

Age intervals	**Frequency (amount of people falling within this age interval)**	**Cumulative frequency**	**Ordered number pairs that are "made" with the information**
$0 < x \leq$ **10**	2	**2**	**(10 ; 2)**
$10 < x \leq$ **20**	13	**15**	**(20 ; 15)**
$20 < x \leq$ **30**	18	**33**	**(30 ; 33)**
$30 < x \leq$ **40**	16	**49**	**(40 ; 49)**
$40 < x \leq$ **50**	6	**55**	**(50 ; 55)**

ORDERED PAIRS

NB: TO DETERMINE THE ORDERED PAIR USE THE VALUE OF THE UPPER LIMIT OF THE CLASS INTERVAL AND CUMULATIVE FREQUENCY

GROUNDING

Use the value of the lower limit of the first class interval and zero then ground the graph.

Sometimes in the class interval inequalities the inequalities change position e.g.

$0 < x \leq$ **10** will be $0 \leq x <$ **10**

20.1 QUARTILES (Q1, Q3 and the Median) FROM AN OGIVE

Lower Quartile $(Q_1)=\frac{1}{4}\times(n+1)=Position$

Median $(Q_2)=\frac{1}{2}\times(n+1)=Position$

Upper Quartile $(Q_3)=\frac{3}{4}\times(n+1)=Position$

Once all the values have been found draw a line from the y – axis to the graph and then read the x – values at this point.

20.2 BOX AND WHISKER FROM AN OGIVE

20.3 FREQUENCY TABLE FROM AN OGIVE.

This means you must work backwards and determine the frequency of each interval that originally was in the frequency table. **Practise this.**

20.4 From the ogive, you can be asked to draw a histogram.

21 AVERAGE e.g. AGE WHEN GIVEN A FREQUENCY TABLE:

Age intervals	**Midpoint (average) of each age interval** $\bar{x}$	**Frequency (amount of people falling within this age interval)** f	$\bar{x}.f$
0 < x **≤ 10**	$\frac{0+10}{2}=5$	**2**	$2\times5=10$
10 < x **≤ 20**	$\frac{10+20}{2}=15$	**13**	$13\times15=195$
20 < x **≤ 30**	**25**	**18**	$18\times25=450$
30 < x **≤ 40**	**35**	**16**	$16\times35=560$
40 < x **≤ 50**	**45**	**6**	$6\times45=270$
		55 people	**10+195+450+560+270=1 485**

Approximate average of all the ages = $\frac{1485}{55}$ = 27

ONE LOSES THE DETAIL OF THE DATA IN A TABLE LIKE THIS.

EG. The 6 ages data in the last interval **40 <** x **≤ 50** could have been: 41 47 49 50 46 42
If we take the midpoint (average) of the interval, then we say the ages are: 45 45 45 45 45 45

21.1 THE MODAL CLASS INTERVAL: the class interval with most data items.

21.2 THE CLASS INTERVAL WHERE THE MEDIAN OF THE DATA LIES

In this case: **20 <** x **≤ 30 THE 27**TH **DATA ITEM LIES HERE.**

22 VARIANCE: it is a measure of the dispersion of the data.

[(standard deviation)^2 = variance]

x	$x - \overline{x}$		$(x - \overline{x})^2$	**Answer**
3	3-5	-2	$(-2)^2$	4
4	4-5	-1	$(-1)^2$	1
8	8-5	3	$(3)^2$	9
5	5-5	0	$(0)^2$	0
Total: 20				Total: 14 14 = $\sum (x - \overline{x})^2$
Mean: $\frac{20}{4} = 5$				Mean of these squared values = $\frac{14}{4} = 3,5$ THE VARIANCE IS 3,5

22.1 STANDARD DEVIATION (δ) = $\sqrt{3,5}$ = 1,87 : it is a measure of the dispersion of the data about the mean.

22.2 AN INTERVAL THAT IS ONE STANDARD DEVIATION from the mean:

[5 - δ ; 5 + δ] (5= mean) [5 - 1,87 ; 5 + 1,87]

which gives [3,13; 6,87]

How many of the data items in the table lie in this interval?

22.3 AN INTERVAL THAT IS TWO STANDARD DEVIATIONS from the mean:

[5 – 2. δ ; 5 + 2. δ] [5 – 1,87 – 1,87 ; 5 + 1,87 + 1,87]

which gives: [1,26 ; 8,74] How many data items lie in this interval?

23 SCATTER PLOT

- The data are recorded as ordered number pairs. CAN YOU PLOT POINTS?
- The ordered number pairs are plotted on a Cartesian plane.
- The data investigate a possible relationship between things like:
  (1) Years/Numbers (2) Wrist size/shoe size (3) Mathematics marks/ English marks
  (4) Shelf space in a shop/Weekly sale of an item
- The line of best fit is the trend line drawn on a scatter plot or scatter diagram. It shows whether the points are "scattered" in a straight line, parabola, hyperbola or an exponential graph shape.
- Being able to extrapolate (reading off information from extended parts of the line of best fit).
- To interpolate means to estimate by "observing" the known data around a point (reading off information based on the line of best fit of the known data)
- The correlation refers to how strong the relationship seems to be. It could be non-existent, weak, strong or very strong.

23.1 LEAST SQUARES REGRESSION LINE

- It is a straight line in the form y = mx + c is represented as y = A + Bx where B is the gradient and A is the y – intercept.
- The line will always pass through the mean point of the data set$(\bar{x};\bar{y})$
- The mean point is determined by determining the mean of the x – values $(\bar{x})$ and the mean point of the y - values$(\bar{y})$.

CASIO CALCULATOR

USING A CALCULATOR TO DETERMINE REGRESSION LINES AND THE MEAN POINT

STEP	BUTTON TP PRESS/METHOD	CALCULATOR DISPLAY
1	**Mode**	A list of various modes
2	**2**(STAT)	A list of options in STAT mode
3	**2**(A +BX)	A table to input x and y values
4	Input each data value for (x) and (y)one at a time , pressing the = button after each entry	
5	Once all the data is entered press AC	0
6	**Shift 1** (STAT)	A list of options to choose
7	**5** (Reg)	1: **A** 2: **B** 3: **r** 4:$\hat{x}$ 5:$\hat{y}$
8	Pressing **1** will display the value of **A**. Pressing **2** will display the value of **B**. Pressing **3** will display correlation coefficient **r**.	
9	**1** (**A**) and then =	**24,8121547**
10	To determine the value of B repeat 6, 7 and 8 pressing 2 (B) instead of 1	
11	**1**(**B**) and then =	**0,2779005525**
12	To determine the mean $(\bar{x};\bar{y})$ point of the data set repeat step 6	
13	**4** (Var)	1: **n** 2:$\bar{x}$ 3:δx 4: sx 5:$\bar{y}$ 6:δy 7:**sy**
14	Pressing **2** will display the x – value of the mean point (82) and pressing **5** will display the y – value of the mean point(47,6)	

e.g.

Time (Minutes)	**10**	**40**	**80**	**120**	**160**
Temperature (℃)	**30**	**35**	**43**	**60**	**70**

The equation of the line best fit: y = 24,81 + 0,28x

Mean point (82 ; 47,6)

Correlation coefficient (r) = 0,9881730865

NOTES ON REGRESSION LINES

- **It is important to understand that a linear regression line can be fitted to any data set.**
- **The linear line is not always the ideal line to use.**
- **Different association can be shown on bivariate data which are:**
  - ✓ **Quadratic association**
  - ✓ **Inverse association**
  - ✓ **Exponential association**

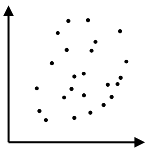
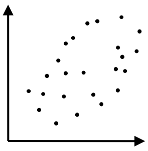
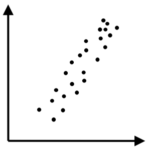
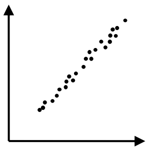
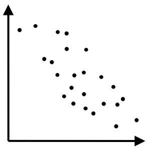
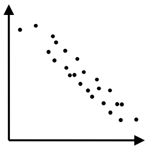
23.2 CORRELATION COEFFICIENT

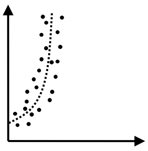
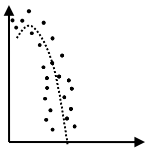
- Correlation coefficient is represented by r
- Correlation coefficient tell us about the strength of the relationship between the variables.
- It tells us well the data fits the line of best fit.
- Correlation coefficient always lie between and including 1 and -1

NB: DO NOR ROUND OF THE CORRELATION COEFFICIENT TO AN INTERGER

$r \approx$	Correlation
1	**Perfect positive association**
0,9	**Strong positive association**
0,5	**Moderate positive association**
0,2	**Weak positive association**
0	**No correlation**
-0,2	**Weak negative association**
-0,5	**Moderate negative association**
-0,9	**Strong negative association**
-1	**Perfect negative association**

e.g.

					
$r \approx 0{,}2$	$r \approx 0{,}4$	$r \approx 0{,}7$	$r \approx 0{,}9$	$r \approx -0{,}5$	$r \approx -0{,}8$

	
EXPONENTIAL	**PARABOLIC**

24 THE STEM AND LEAF PLOT OR DIAGRAM

1 Row 1: 2, 3
Row
Row
Row

Leading digits (TENS)	Trailing digits (UNITS)
0	2 3
1	0 2 4 5
2	3 4 6
3	2 5 7 7

2: **1**0, **1**2, **1**4, **1**5
3: **2**0, **2**3, **2**4, **2**5
4: **3**2, **3**5, **3**7, **3**7

The bold 1, 2 and 3 come from the first column in the table and represent tens.

NB Calculator usage is an Important skill in this topic

TECHNICAL REPORT FINDINGS

COMMON ERRORS AND MISCONCEPTION

Learners entering incorrect values on the calculator.
Identification of independent variable and dependent variable is challenging on determine the equation of a least square regression equation.
Learners still swapping the value of A and B on the equation of the least square regression line
Drawing an Ogive is still a challenge.

- Grounding the graph
- Using of the correct values (Cumulative frequency or frequency)
- Using lower class limit and upper-class limit.

Rounding off to the required decimal is still a challenge.
Word such as “Predict” challenge learners as such learners just guess the answer.
Using the information to prove as if it is the given information.
Calculator usage is still a problem

SUGGESTION FOR IMPROVEMENT

Learners to be given multiple opportunities to practice calculator skills
Learners should be made aware that the operation procedure varies from one brand of the calculator to another.
It is advisable to use the same brand that has been used during the year even in the examination.
Teachers to always emphasise correct rounding off.
Teachers should explain each definition or concept in detail.
It is important for teachers to **discuss the independent and dependent variables**.
Learners should be taught to **use the regression to make predictions**.
Teachers should stress to learners that it is not permissible to use information that they must prove as if it is given information.
Graphs are an integral part of Statistics, learners should be given more practice on drawing graphs, reading off from the graphs and interpreting the graphs.
Grounding of an ogive should be emphasised.

Drawing an ogive
SEE NUMBER 20 on the NOTES

EXAM TIPS WHEN ANSWERING STATISTICS QUESTIONS

- Read the information more than once.
- Analyse and understand the data(Information) given
- It is advisable to Use colour to analyse information given.
- Make sure your calculator is in good working condition.

WARM UP QUESTIONS OR PRACTICE QUESTIONS

NOVEMBER GRADE 11 2018

QUESTION 1

A school held a sports day. One of the items on the programme was an obstacle race. Teams of 10 parents and learners participated in this race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

4	12	13	16	17	18	20	22	22	25

1.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)

1.2 Determine the mean time taken by this team. (2)

1.3 Calculate the standard deviation for the data. (1)

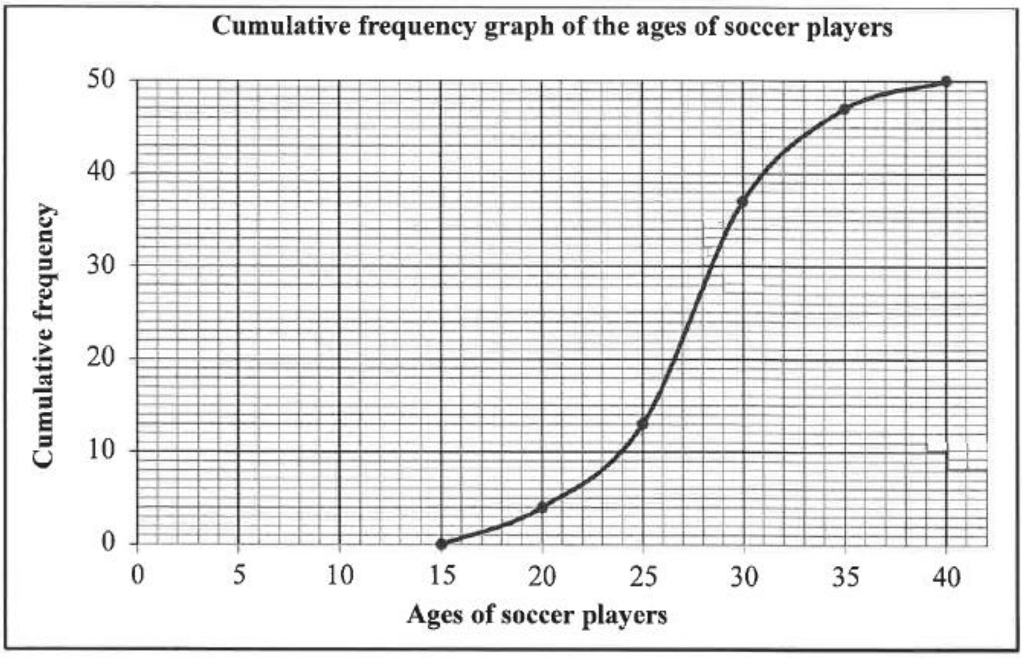
1.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)

1.5 It took another team a total time of $x+5$ minutes to complete the race. Calculate the value of x if the overall mean of the two teams combined was 18 minutes. (3)

[10]

QUESTION 2

2.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



2.1.1 How many players took part in the soccer tournament? (1)

2.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)

2.1.3 Complete the frequency column of the table below in the ANSWER BOOK.

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

(3)

2.1.4 Use the grid provided in the ANSWER BOOK to draw a frequency polygon for the data. (4)

2.2 Two Grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

CLASS A $(30\ ;\ 48\ ;\ 65\ ;\ 82\ ;\ 90)$

CLASS B $(50\ ;\ 58\ ;\ 65\ ;\ 75\ ;\ 90)$

The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes.

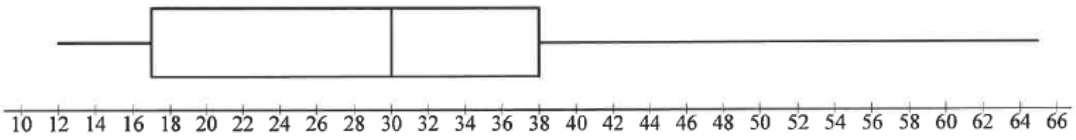
Do you agree with this claim? Use at least TWO different arguments to justify your answer. (3)

[13]

NOVEMBER GRADE 11 2017

QUESTION 1

1.1 Mr Brown conducted a survey on the amount of airtime (in rands) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below.



1.1.1 Write down the five-number summary of the data. (2)

1.1.2 Determine the interquartile range. (1)

1.1.3 Comment on the skewness of the data. (1)

1.2 A group of 13 students indicated how long it took (in hours) before their cellphone batteries required recharging. The information is given in the table below.

5	8	10	17	20	29	32	48	50	50	63	y	107

1.2.1 Calculate the value of y if the mean for this data set is 41. (2)

1.2.2 If $y = 94$, calculate the standard deviation of the data. (1)

1.2.3 The mean time before another group of 6 students needed to recharge the batteries of their cellphones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cellphones. (3)

[10]

QUESTION 2

A student conducted a survey among his friends and relatives to determine the relationship between the age of a person and the number of marketing phone calls he or she received within one month. The information is given in the table below.

AGE OF PERSON IN SURVEY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

2.1 Complete the frequency and cumulative frequency columns in the table given in the ANSWER BOOK. (4)

2.2 How many people participated in this survey? (1)

2.3 Write down the modal class. (1)

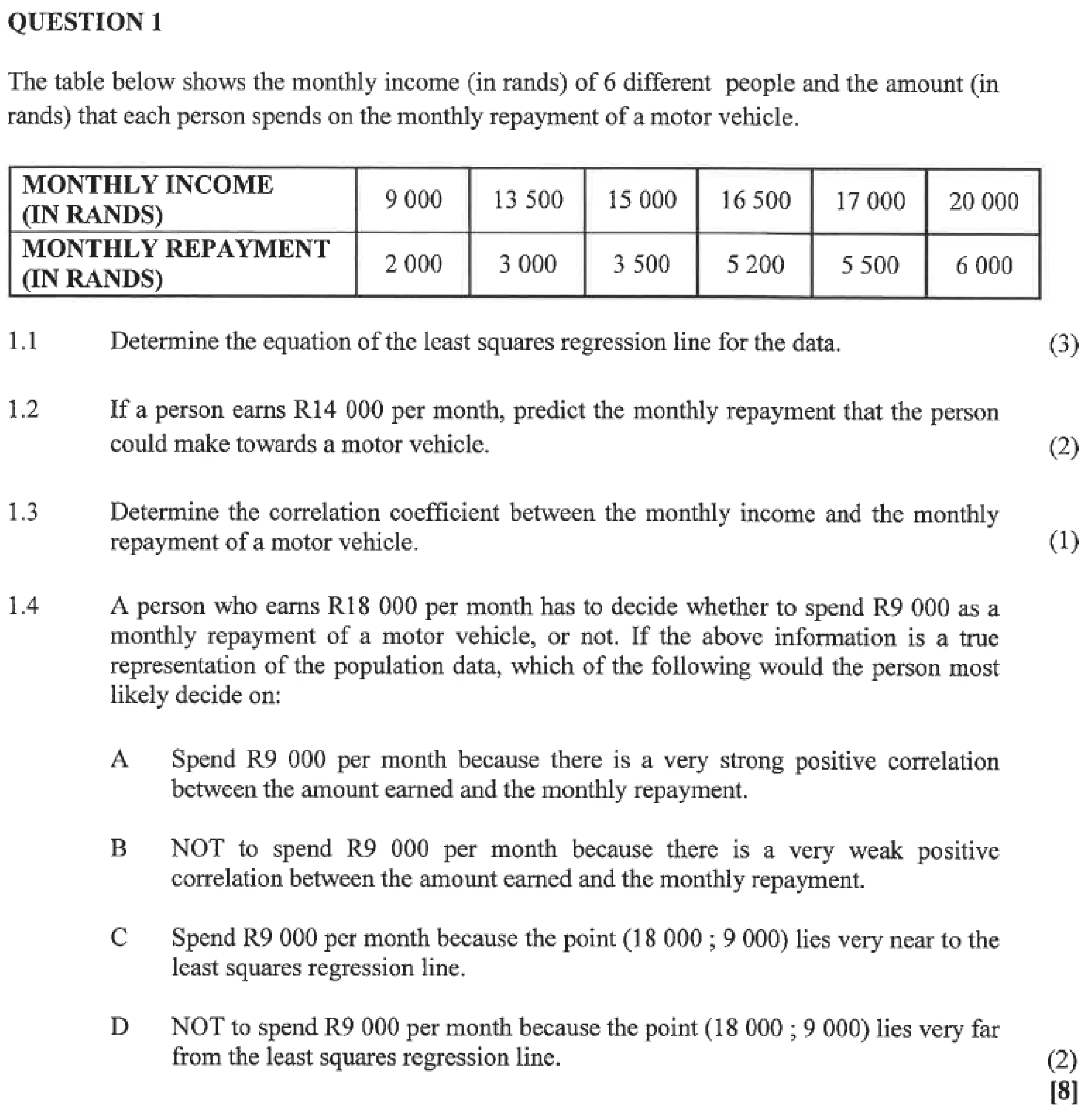
2.4 Draw an ogive (cumulative frequency graph) to represent the data on the grid given in the ANSWER BOOK. (3)

2.5 Determine the percentage of marketing calls received by people older than 54 years. (3)

[12]

TYPICAL EXAM QUESTIONS

NOVEMBER 2019



QUESTION 1

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

MONTHLY INCOME (IN RANDS)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS)	2 000	3 000	3 500	5 200	5 500	6 000

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)

1.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)

1.4 A person who earns R18 000 per month has to decide whether to spend R9 000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:

A Spend R9 000 per month because there is a very strong positive correlation between the amount earned and the monthly repayment.

B NOT to spend R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.

C Spend R9 000 per month because the point (18 000 ; 9 000) lies very near to the least squares regression line.

D NOT to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. (2)

[8]

QUESTION 2

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

2.1 How many people paid R200 or less on their monthly cellphone contracts? (1)

2.2 Use the information above to show that $a = 24$ and $b = 16$. (5)

2.3 Write down the modal class for the data. (1)

2.4 On the grid provided in the ANSWER BOOK, draw an ogive (cumulative frequency graph) to represent the data. (4)

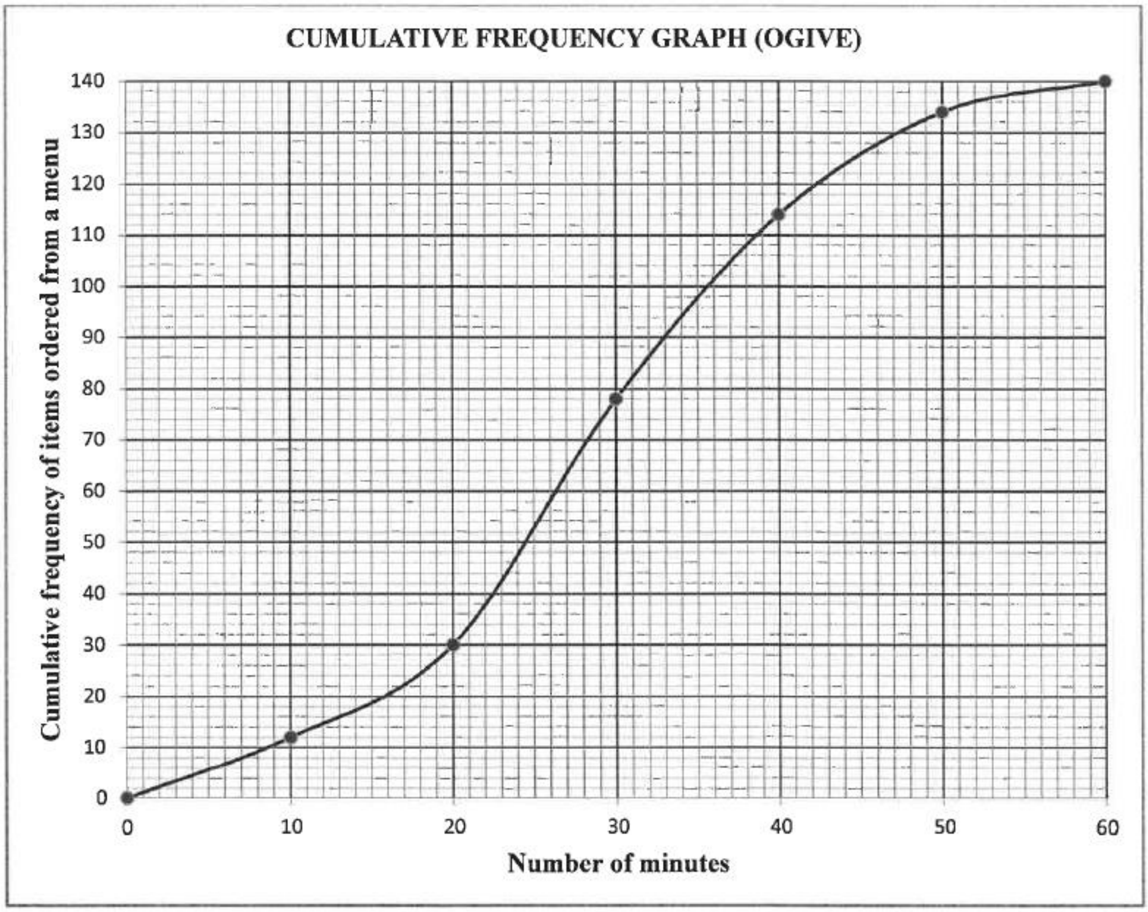
2.5 Determine how many people paid more than R420 per month for their cellphone contracts. (2)

[13]

NOVEMBER 2018

QUESTION 1

1.1 The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from a menu over a period of 1 hour.



1.1.1 Write down the total number of food items ordered from the menu during this hour. (1)

1.1.2 Write down the modal class of the data. (1)

1.1.3 How long did it take to order the first 30 food items? (1)

1.1.4 How many food items were ordered in the last 15 minutes? (2)

1.1.5 Determine the 75th percentile for the data. (2)

1.1.6 Calculate the interquartile range of the data. (2)

1.2 Reggie works part-time as a waiter at a local restaurant. The amount of money (in rands) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1 Calculate:

(a) The mean of the data (2)

(b) The standard deviation of the data (2)

1.2.2 Mary also works part-time as a waitress at the same restaurant. Over the same 15-day period Mary collected the same mean amount in tips as Reggie, but her standard deviation was R14.

Using the available information, comment on the:

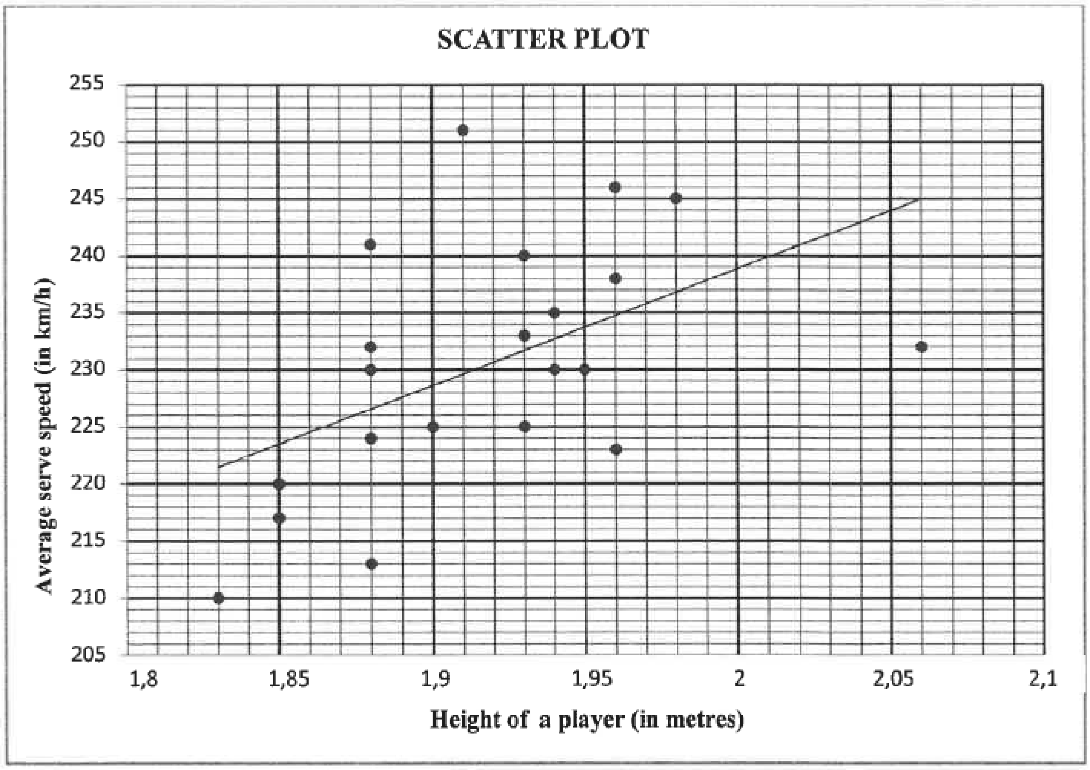
(a) Total amount in tips that they EACH collected over the 15-day period (1)

(b) Variation that EACH of them received in daily tips over this period (1)

[15]

QUESTION 2

A familiar question among professional tennis players is whether the speed of a tennis serve (in km/h) depends on the height of a player (in metres). The heights of 21 tennis players and the average speed of their serves were recorded during a tournament. The data is represented in the scatter plot below. The least squares regression line is also drawn.



2.1 Write down the fastest average serve speed (in km/h) achieved in this tournament. (1)

2.2 Consider the following correlation coefficients:

A. $r = 0{,}93$ B. $r = -0{,}42$ C. $r = 0{,}52$

2.2.1 Which ONE of the given correlation coefficients best fits the plotted data? (1)

2.2.2 Use the scatter plot and least squares regression line to motivate your answer to QUESTION 2.2.1. (1)

2.3 What does the data suggest about the speed of a tennis serve (in km/h) and the height of a player (in metres)? (1)

2.4 The equation of the regression line is given as $\hat{y} = 27{,}07 + bx$.
Explain why, in this context, the least squares regression line CANNOT intersect the y-axis at $(0\,;27{,}07)$. (1)

[5]

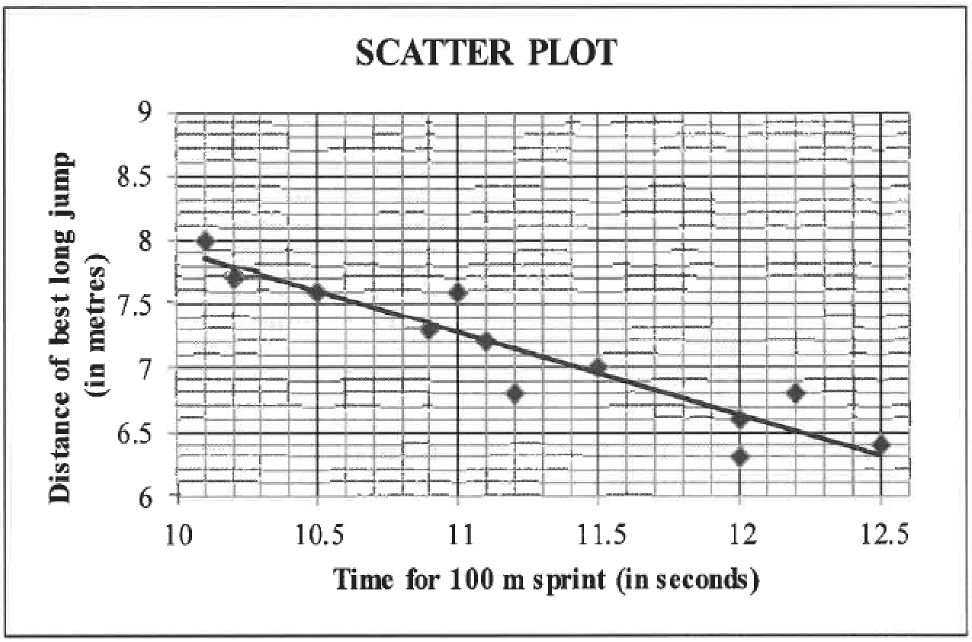
NOVEMBER 2017

QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)

[7]

QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)

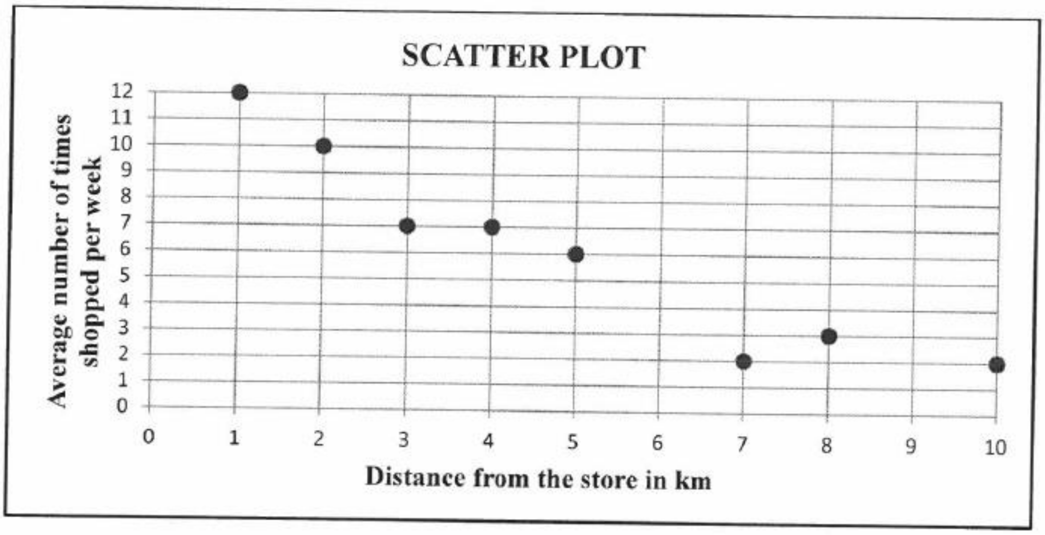
[13]

NOVEMBER 2016

QUESTION 1

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



1.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

1.2 Calculate the correlation coefficient of the data. (1)

1.3 Calculate the equation of the least squares regression line of the data. (3)

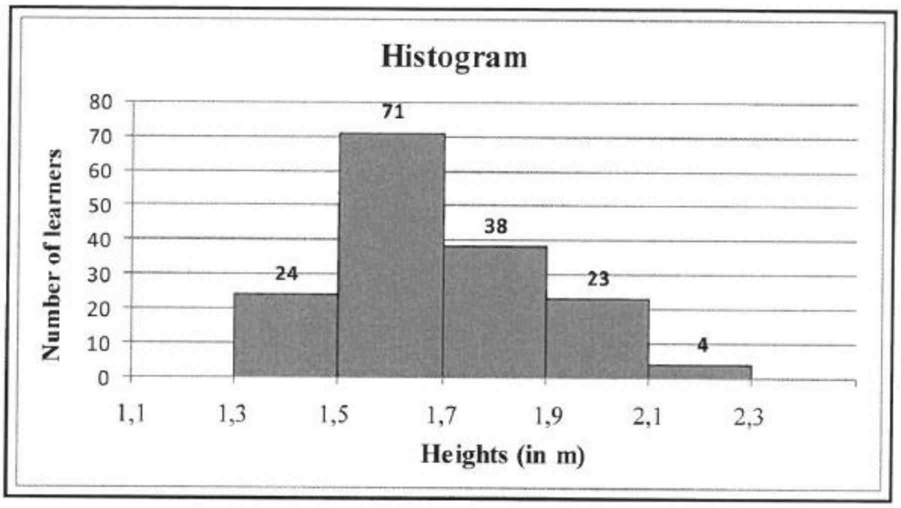
1.4 Use your answer at QUESTION 1.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)

1.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

[9]

QUESTION 2

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

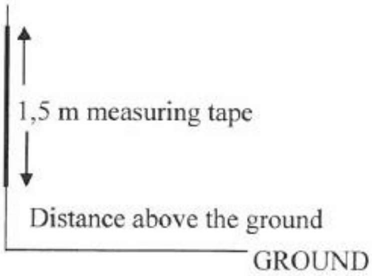
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

2.6.1 Mean of the data set (1)

2.6.2 Standard deviation of the data set (1)

[13]

NOVEMBER 2015

QUESTION 1

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)

1.2 The equation of the least squares regression line is $\hat{y} = 154{,}60 + 77{,}13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)

1.3 Identify an outlier in the data set. (1)

1.4 Calculate the value of the correlation coefficient. (2)

1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)

[11]

QUESTION 2

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	**Frequency**
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)

[9]

ANALYTICAL GEOMETRY

NOTES TO REVISE WITH

1. The distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2. What to do if the distance of a line is given and either a y or an x is missing.

Suppose the distance equals 5 and one variable is missing

$$\textit{Then}: \quad 5 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Square both sides and solve.

3. The midpoint formula.

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$$

4. What to do if the ordered number pair at the midpoint is given and they want a y or an x at one of the end points of the line.

$x = \frac{x_1 + x_2}{2}$ *and / en* $y = \frac{y_1 + y_2}{2}$ where x and y are the values at the midpoint that are known, and the unknown value(s) is/are one or two of the other coordinates in the end point ordered number pairs.

5. How to calculate the gradient (steepness, slope) of a straight line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

6. The gradients of parallel lines are equal.

$$IF\ AB \parallel CD\ then\ m_{AB} = m_{CD}$$

7. The product of the gradients of perpendicular lines is equal to -1.

$$If \quad AB \perp CD \quad then \quad m_{AB} \times m_{CD} = -1$$

8. How to use the gradients of two straight lines to calculate a missing *x* or *y.*
Set up an equation.

- **If the lines are parallel**
  Make your own equation. The two m values (gradients) are equal.

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_3 - y_4}{x_3 - x_4}$$

(Gradient of the one line = Gradient of the other line)

Then solve for the unknown

- **If the lines are perpendicular (1)** $m_{AB} \cdot m_{CD} = -1$
  Again, make your own equation.

$$\frac{y_1 - y_2}{x_1 - x_2} \times \frac{y_3 - y_4}{x_3 - x_4} = -1$$

(Gradient of the one line × Gradient of the other line = -1)

Then solve for the unknown

- **If the lines are perpendicular (2)**
  Apply the theorem of Pythagoras:
  $AB^2 + BC^2 = AC^2$

Then solve for the unknown

9. How to determine the equation of a straight line:

- **When an *m*-value (Gradient) and a point are given.**
  Substitute the point into x_1 *and* y_1, and the gradient into *m*.

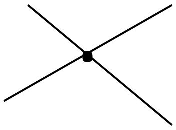
$$y - y_1 = m(x - x_1)$$

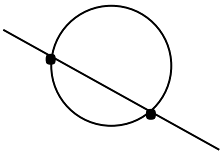
You can also use:

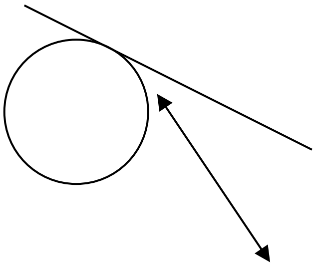
$$y = mx + c$$

- **When two points are given,**
  you first calculate the gradient, then use one of the above equations.
- **You can be given the c - value and a point:**
  Substitute into $y = mx + c$. **Solve for *m.* Write down the equation.**
- **You can be given the c – value and the *m* – value:**
  Substitute into $y = mx + c$

10. If you need the point(s) of an intersection you **ALWAYS** look for a ***SIMULTANEOUS EQUATION.***

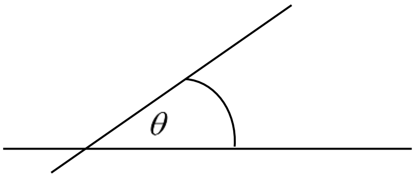


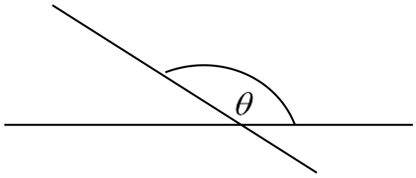




A line will TOUCH a circle if a simultaneous equation produces two equal *x*- and hence two equal *y*-values or vice versa then it is a tangent.

11. The angle of **inclination** is the angle on the RHS of the line where it cuts the ***x* axis.** **It is formed between the line and the positive x - axis**





Questions like these appear:

12. You calculate the angle of inclination by going:

2nd fn tan (+value) = on the calculator.

If the angle is obtuse, then after this you go: 180° **- angle.**

Examples:

1) If the gradient equals 4, then you go:

2nd fn tan 4 = ...this inclination equals: 57,96º

This <u>57,96º</u> is the angle of inclination.

2) If the gradient equals -4, then you go:

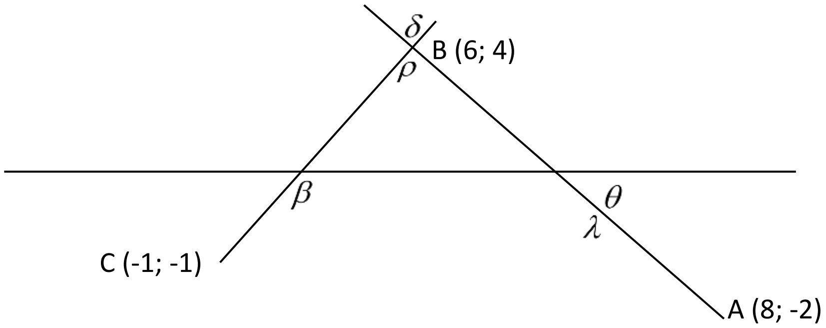
2nd fn tan +4 = ...this equal: 57,96º

But the negative indicates an <u>obtuse angle of inclination,</u>

so you go: 180º - 57,96º = 122,04º

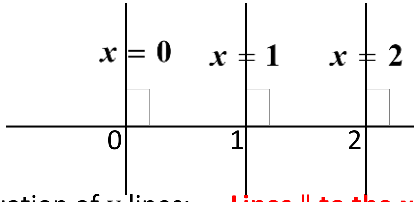
This 122,<u>04º</u> is the angle of inclination.

Calculate all the angles by making use of the above.



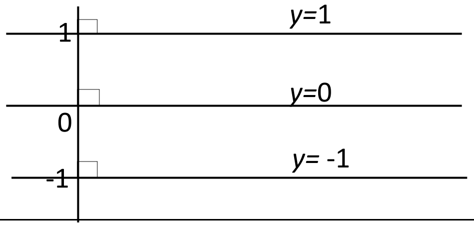
13. The equation of x lines: **Lines || to the *y* axis.**

Equation:



14. The equation of y lines: **Lines || to the *x* axis.**

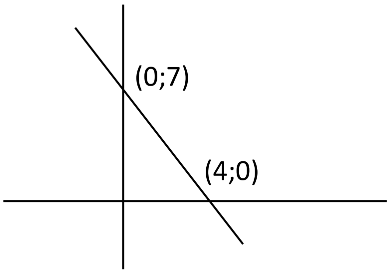
Equation:



15. If the x-intercept is (4; 0) and the y-intercept is (0 ; 7) then the equation of the line is:

$$\frac{x}{4}+\frac{y}{7}=1$$

x 28: $\therefore 7x+4y=28$



The standard form is:

$$\frac{x}{a}+\frac{y}{b}=1 \quad \textit{where } a \textit{ and } b \textit{ are the } x \textit{ and } y \textit{ intercepts respectively.}$$

THE ONE IS ALWAYS 1. It is part of the standard form.

Or read the gradient and the y-intercept off the graph and substitute into

$y = mx + c$

16. How to prove an angle is a right angle?

It the gradient of one line, times the gradient of the other line equals -1, then the lines are **perpendicular (cut at 90º)**

Example:

You must prove that two lines, AB and CD are perpendicular.

$m_{AB} = \frac{7}{23}$ and $m_{CD} = -\frac{23}{7}$ then $m_{AB} \cdot m_{CD} = -1$

since $\frac{7}{23} \times -\frac{23}{7} = -1$, $AB \perp CD$

OR: IN Δ ABC calculate $AB^2 + BC^2$ and then AC^2.

IF $AB^2 + BC^2 = AC^2$ then $AB \perp BC$. (Converse of Pythagoras)

17. What does collinear mean?

It means that points lie in a straight line.

18. How to prove that points are collinear?

The question will be something like: PROVE THAT A (x; y), B (a; b) and C (s; t) are collinear.

<u>If they are collinear, then:</u>

$m_{AB} = m_{AC}$ *OR* *length* $(AB + BC) =$ *lenght* AC

STRATEGIES :

Calculate m_{AB} *separately from* m_{AC}.

If they are equal then the three points are collinear.

OR

Calculate length $(AB + BC)$ *separately from lenght* AC

If they are equal then the three points are collinear.

OR:

$m_{AB} = m_{BC}$ CALCULATE THE GRADIENTS SEPERATELY.

Hope they are equal!!!!!!!

19 **Still about collinear points**.

Three points A, B and C are said to be collinear. One of the x or y values is missing. ***How*** does one **find one missing x or y value?**

Use two gradients that are equal.

Build your own equation: $m_{AB} = m_{AC}$

Multiply by the LCD and solve.

19. Standard form (equation) of the circle with centre ***on*** the origin and radius r.

$(x - 0)^2 + (y - 0)^2 = r^2$ which becomes $x^2 + y^2 = r^2$

20. Standard form (equation) of the circle with centre $(a; b)$ *away from* **the** origin and radius r.

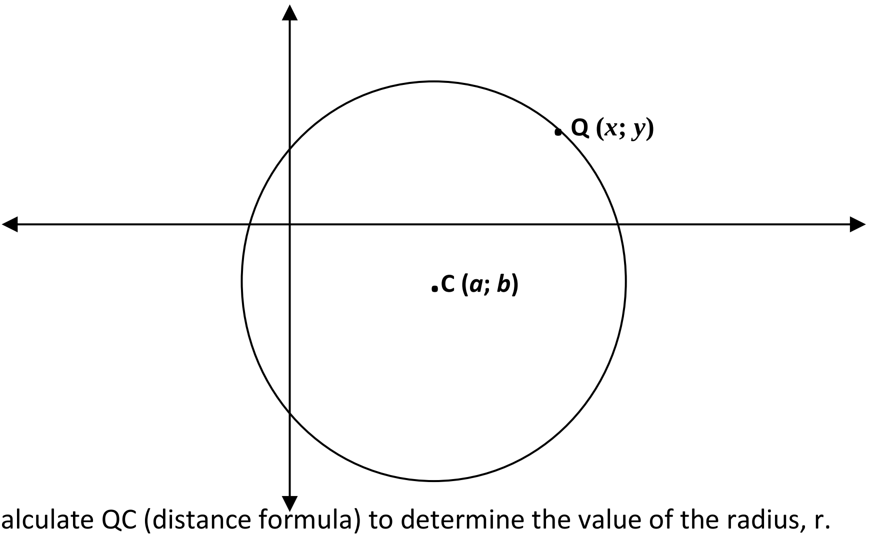
$$(x - a)^2 + (y - b)^2 = r^2$$

21. How to determine the equation of a circle with centre away from the origin:

Case 1: Given a circle with centre C $(a; b)$ and radius r, then....

$$(x - a)^2 + (y - b)^2 = r^2$$

Case 2: Given a circle with centre C and a point on the circle, say Q.



First calculate QC (distance formula) to determine the value of the radius, r.
Then substitute centre C and the value of r^2 correctly into the circle equation,

$$(x - a)^2 + (y - b)^2 = r^2$$

22. How to find the centre if the equation of a circle is given.

Shuffle the terms. *x* terms next to each other, y terms next to each other (if not given like this).

Complete the square for the *x* **terms** and complete the square for the *y* terms.

Factorise. Remember to add the **values that are added** on the L H Side to complete the square, to the R H Side.
The additive inverses of the constants in the brackets are the *x and the y* values of the centre.

EG :

Determine the centre and radius of $x^2 + 4x + y^2 - \frac{3}{5}y = 4\frac{91}{100}$

SOLUTION :

$$x^2 + 4x + y^2 - \frac{3}{5}y = 4\frac{91}{100}$$

$$x^2 + 4x + (2)^2 + y^2 - \frac{3}{5}y + (-\frac{3}{10})^2 = 4\frac{91}{100} + (2)^2 + (-\frac{3}{10})^2$$

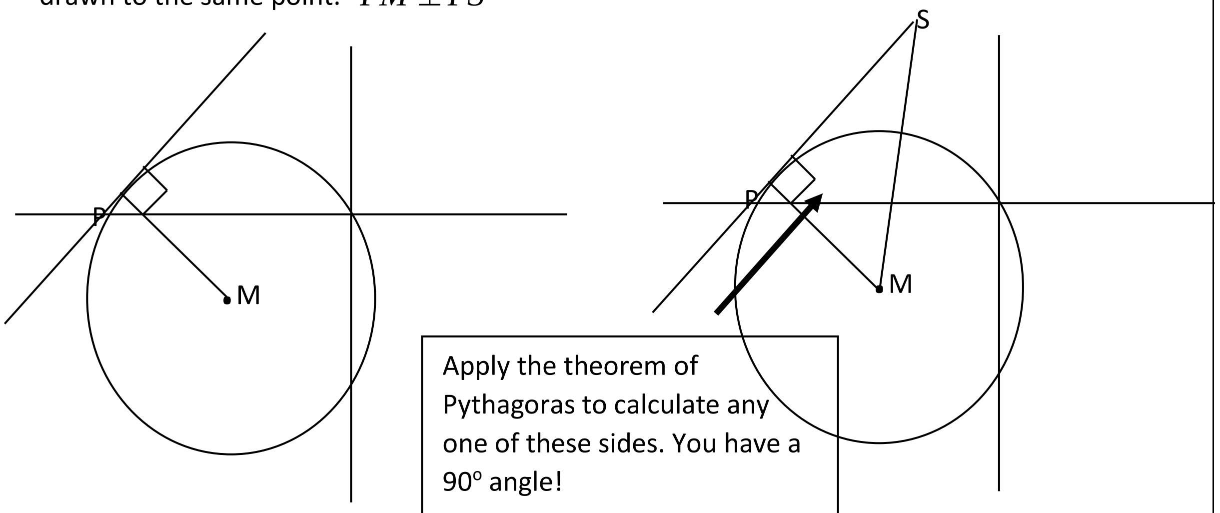
$$(x+2)^2 + \left(y - \frac{3}{10}\right)^2 = 4\frac{91}{100} + 4 + \frac{9}{100}$$

$$(x+2)^2 + \left(y - \frac{3}{10}\right)^2 = 8\frac{100}{100}$$

$$(x+2)^2 + \left(y - \frac{3}{10}\right)^2 = 9$$

Centre : $\left(-2\,;\, +\frac{3}{10}\right)$ $\qquad r^2 = 9, \ \therefore r = \sqrt{9} = 3$

23. **The tangent drawn to a circle at a specific point is perpendicular to the radius** of the circle drawn to the same point. $PM \perp PS$



Apply the theorem of Pythagoras to calculate any one of these sides. You have a 90° angle!

24. You need the gradient of the radius to get the gradient of the tangent.

This is a bargain! You get two for the price of one. If you have the one gradient, and the two lines are perpendicular, then you HAVE the other gradient as well.

Since the radius is perpendicular to the tangent, the product of the gradients will equal −1.

This implies that you can only determine the gradient of the tangent through the gradient of the radius (and vice versa).

$$m_{radius} \cdot m_{tangent} = -1$$

$$\text{So if } m_{radius} = \frac{3}{4}, \text{ then } m_{tangent} = -\frac{4}{3} \quad \text{since } \frac{3}{4} \times -\frac{4}{3} = -1$$

$$\text{or if } m_{radius} = -\frac{7}{6}, \text{ then } m_{tangent} = +\frac{6}{7} \quad \text{since } -\frac{7}{6} \times \frac{6}{7} = -1$$

25. Intercepts on the axes???

This is old news:

- if you want the *x* intercept(s): let *y* = 0 and solve for *x*.
- if you want the *y* intercept(s): let *x* = 0 and solve for *y*.

26. Questions about the diameter.

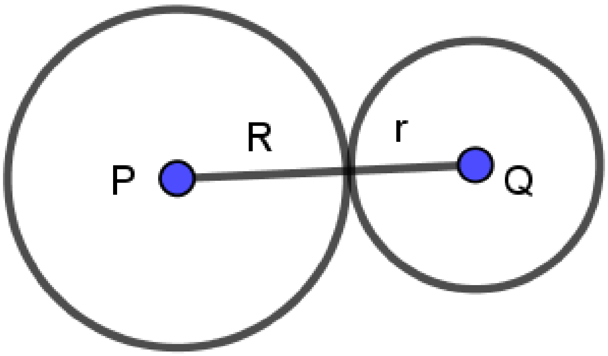
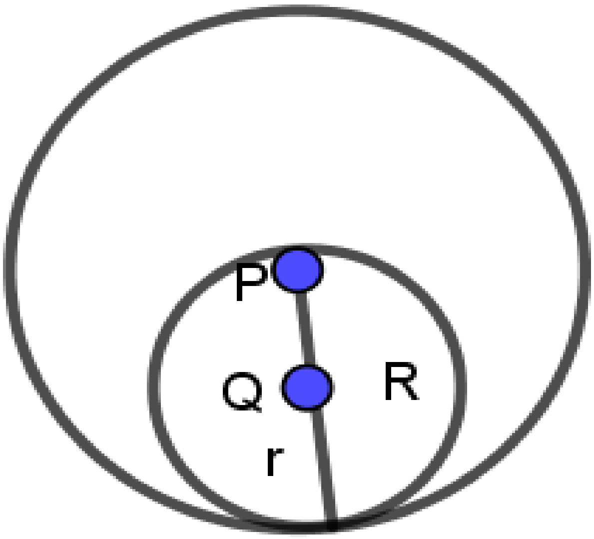
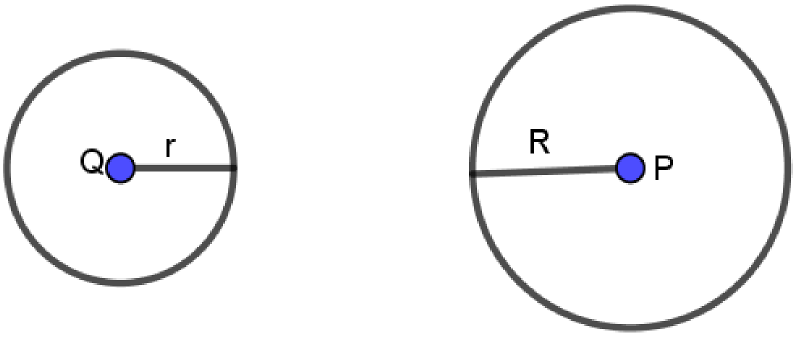
The circle has a centre, remember, and on the diameter **the radius = radius.**

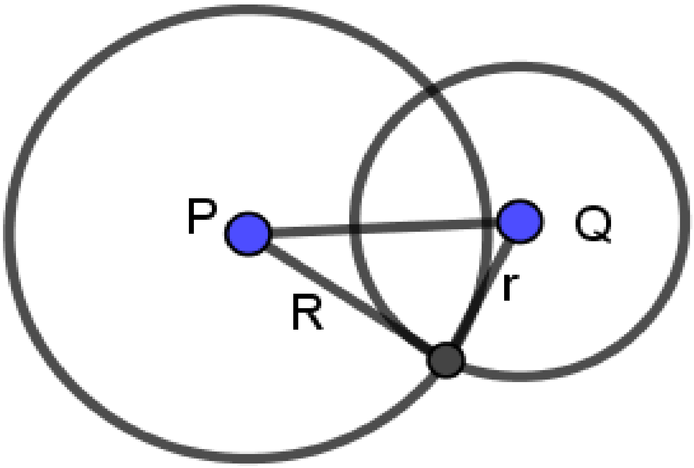
Watch out for questions involving the midpoint formula:

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$$

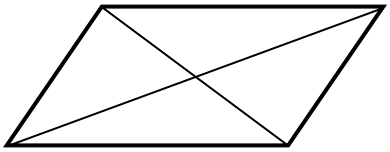
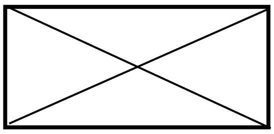
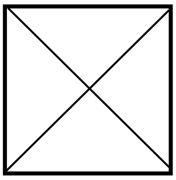
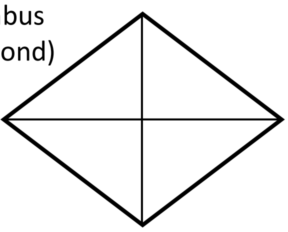
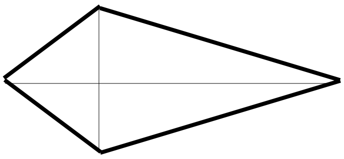
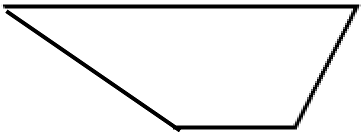
OR: they give the centre and one point on the circle. You go like in number 4.

27.CIRCLES

	Circle centre P and circle centre Q and the circles touch each other externally. ∴ PQ=R + r
	Circle centre P and circle centre Circles touch each other internally ∴ PQ=R – r
	Circle centre P and circle centre Q. Circles do not touch each other ∴ PQ > R + r

	**Circle centre P and circle centre P intersect in 2 points** **∴ $PQ < R + r$**

28. PROPERTIES (CHARACTERISTICS) OF A RECTANGLE, SQUARE, PARALLELOGRAM, RHOMBUS, KITE AND TRAPEZIUM.

Quadrilateral	**Side lengths**	**Side gradients (pairs of parallel sides?)**	**Angles at vertices**	**Diagonals**
Parallelogram 	two pairs of opposite sides are equal	two pairs of opposite sides are parallel	two pairs of opposite angles are equal	bisect each other
Rectangle 	two pairs of opposite sides are equal	two pairs of opposite sides are parallel	all 4 equals to 90º	are equal
Square 	all sides are equal	two pairs of opposite sides are parallel	all 4 equals to 90º	are equal, bisect each other and intersect at 90º
Rhombus (diamond) 	all sides are equal	two pairs of opposite sides are parallel	two pairs of opposite angles are equal	bisect each other at 90º
Kite 	two pairs of **adjacent** sides are equal	no parallel sides	one pair of opposite angles are equal	intersect at 90º but are unequal
Isosceles trapezium 	Non-parallel sides are equal	one pair of opposite sides are parallel	Two pairs of equal angles	equal
Trapezium 		one pair of opposite sides are parallel		

29.It is important that you revise the properties of these quadrilaterals, because a knowledge of them often is necessary in Analytical Geometry. HOW WOULD YOU NEED THIS?

PROVE THAT A QUADRILATERAL IS A….		**FORMULAE**
PARALLELOGRAM	• **2 pairs of opposite sides parallel**	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Or	• **2 pairs of opposite sides equal**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Or	• **One pair of opposite sides equal and parallel**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$
RECTANGLE	• **2 pairs of opposite sides parallel AND** **1 ANGLE = 90^{O}**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $m_{AB} \cdot m_{CD} = -1$
Or	• **2 pairs of opposite sides equal AND** **1 ANGLE = 90^{O}**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $m_{AB} \cdot m_{CD} = -1$
Or	• **The 2 diagonals equal and bisect each other**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ The two midpoints must be equal
SQUARE	• **4 sides equal AND** **1 ANGLE = 90^{O}**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $m_{AB} \cdot m_{CD} = -1$
Or	• **The two diagonals equal and bisect (halve) each other at 90°**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $m_{AB} \cdot m_{CD} = -1$ $\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ The two midpoints must be equal.
RHOMBUS	• **4 sides equal**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
RHOMBUS	• **Diagonals bisect (halve) each other at 90°**	$m_{AB} \cdot m_{CD} = -1$ $\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ The two midpoints must be equal.
KITE	• **2 pairs of adjacent sides equal**	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Or	• **One diagonal is bisected** • **The 2 diagonals intersect at 90°**	$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ $m_{AB} \cdot m_{CD} = -1$
TRAPEZIUM	• **One pair of opposite sides is parallel**	m_{AB} = ... m_{CD} =.......... $m_{AB} = m_{CD}$

30.AREA FORMULAE

Parallelogram	base. height
Rectangle	length. breadth
Square	side. side
Rhombus	base. height
Kite	½ (diagonal 1. diagonal 2) = ½ (product of the diagonals)
Trapezium	½ (sum of the parallel sides). height
Circle	π . r.r = π . r^2
Triangle ABC	½. base. height Or ½ *a.b*.sin C (SAS – trig formula may be used in Analytical Geometry)

TECHNICAL REPORT FINDINGS

COMMON ERRORS AND MISCONCEPTIONS

Gradient formula written incorrectly as e.g. $\frac{x_2-x_1}{y_2-y_1}$

Distance formula written incorrectly e.g.: $d=\sqrt{(x_2+x_1)^2-(y_2+y_1)^2}$

Learners are inconsistent in substituting values.

Learners substitution values that do not lie on the line of circle.

Lack of **interpretation of diagrams.**

Learners not indicate the figure they are working in.

Integration of topics is still a challenge.

SUGGESTION FOR IMPROVEMENT

Emphasise to learners to copy the formulae from the formula sheet.

Learners to label their coordinates as $(x_1;y_1)$ and $(x_2;y_2)$.

It should be emphasised that only points that lie on the graph can be used.

Analyse the diagram before attempting to answer question.

In the analysis of the diagram use colour and mark off information on the diagram.

Indicate the figure you are working in. e.g. In $\triangle ABC$.

Different topics in Mathematics should be integrated. Learners must be able to establish the connection between Euclidean Geometry and Analytical Geometry

EXAM TIPS WHEN ANSWERING STATISTICS QUESTIONS

- *Learners must learn which formula is to be used to prove the most basic aspects of Analytical Geometry*.
  E.g. Bisect is 2 mid-points
  Perpendicular is the product of 2 gradients = - 1
- *Learners should then follow the method laid out below:*
  - Select the correct formula from the data sheet
  - Label the ordered pairs using the correct two points, e.g. A and C.
  - Substitute correctly and accurately into your chosen formula
  - Perform the arithmetic, preferably *without* a calculator
- Often Analytical Geometry questions *follow on*, (scaffolding). Look out for that, as you might have already proven an aspect above, that you will require for the next sub-question
- *Use the diagram more effectively.*

E.g. Highlight the sides you are going to use for proving perpendicular, so you can see clearly which points you are going to use for the substitution.

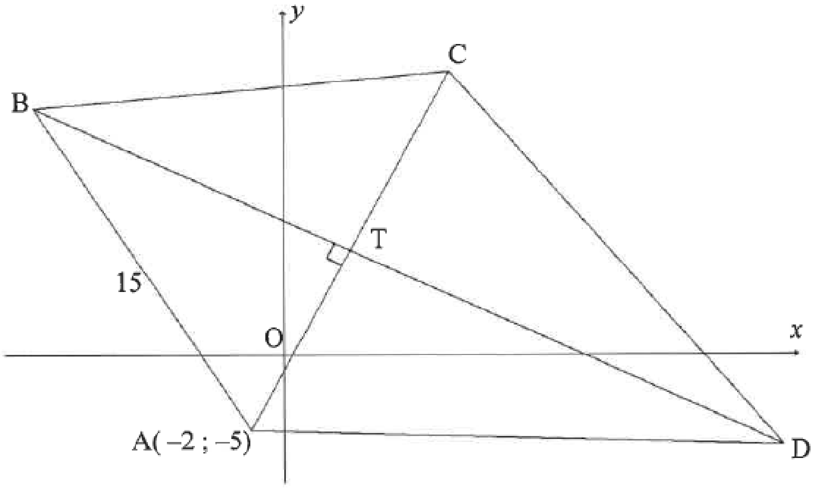
- You must answer the question, and remember to conclude, exactly what you were asked to show / prove / conclude. Use wording to do this.

WARM UP OR PRACTICE QUESTIONS

NOV 2017 GRADE 11

QUESTION 3

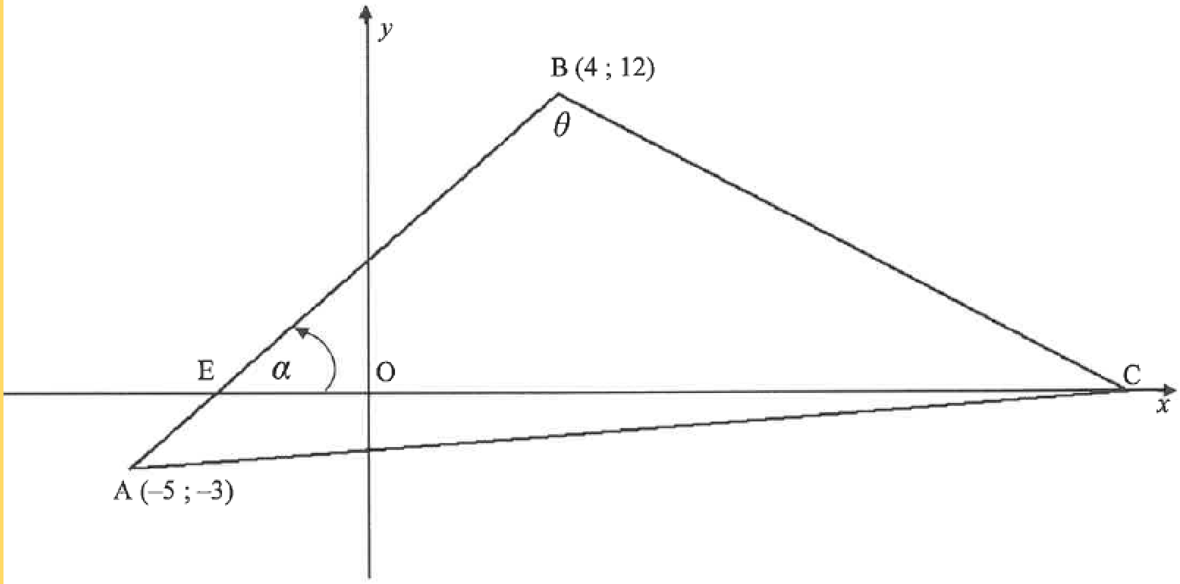
A(–2 ; –5), B, C and D are the vertices of quadrilateral ABCD such that diagonal AC is perpendicular to diagonal BD at T.
The equation of BTD is given by $2y+x=18$ and AB = 15 units.



3.1	Determine the gradient of line AC.	(2)
3.2	Determine the equation of AC in the form $y=mx+c$.	(2)
3.3	If the equation of AC is $y=2x-1$, calculate the coordinates of T.	(3)
3.4	If ABCD is a kite with AB = BC:	
3.4.1	Determine the coordinates of C	(2)
3.4.2	Calculate the length of BT	(4)
3.4.3	Write down the length of the radius of the circle passing through points B, C and T	(2)
		[15]

QUESTION 4

C, a point on the x-axis, $A(-5\,;\,-3)$ and $B(4\,;\,12)$ are the vertices of a triangle.
AB intersects the x-axis at E.
$A\hat{B}C=\theta$ and $B\hat{E}C=\alpha$.



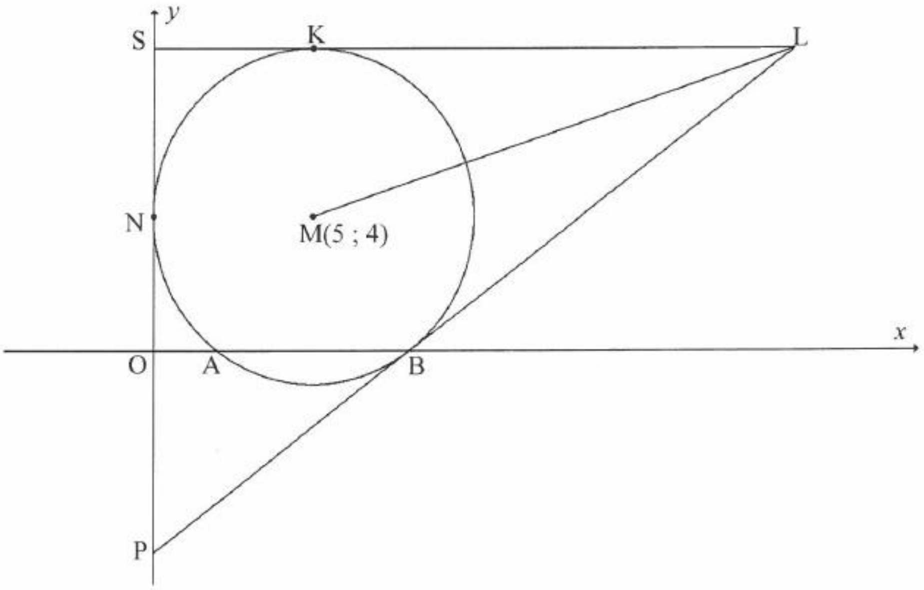
4.1	Calculate the gradient of AB.	(2)
4.2	Determine the coordinates of point E.	(3)
4.3	Determine the size of α. Round off to the nearest whole number.	(2)
4.4	If $\theta=76^\circ$, determine the equation of the line through A parallel to BC.	(5)
		[12]

TYPICAL GRADE 12 EXAM TYPE QUESTIONS

NOVEMBER 2014

QUESTION 3

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y=mx+c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

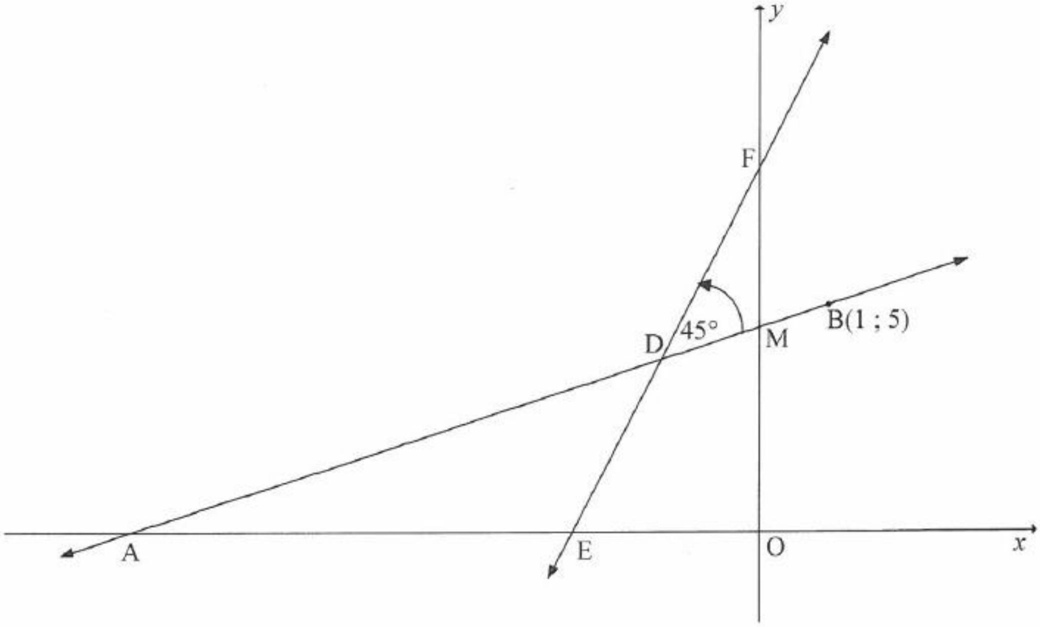
3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

QUESTION 4

In the diagram below, E and F respectively are the x- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has x- and y-intercepts A and M respectively.



4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

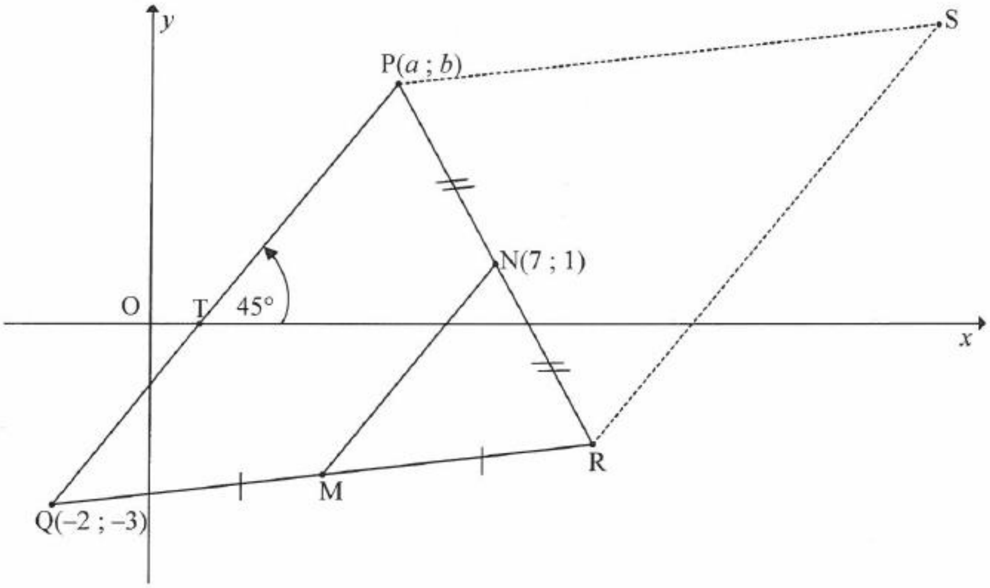
4.5 Calculate the area of quadrilateral DMOE. (6)

[19]

NOVEMBER 2015

QUESTION 3

In the diagram below, the line joining $Q(-2\,;-3)$ and $P(a\,;b)$, a and $b>0$, makes an angle of 45° with the positive x-axis. $QP = 7\sqrt{2}$ units. $N(7\,;1)$ is the midpoint of PR and M is the midpoint of QR.

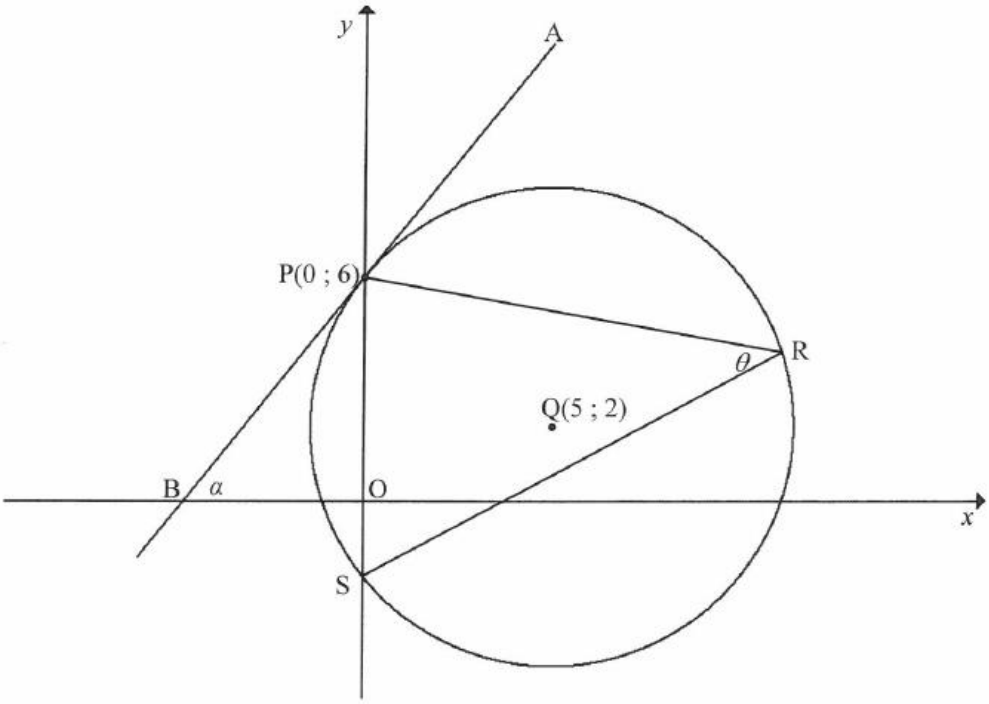


Determine:

3.1	The gradient of PQ	(2)
3.2	The equation of MN in the form $y = mx + c$ and give reasons	(4)
3.3	The length of MN	(2)
3.4	The length of RS	(1)
3.5	The coordinates of S such that PQRS, in this order, is a parallelogram	(3)
3.6	The coordinates of P	(6)
		[18]

QUESTION 4

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.

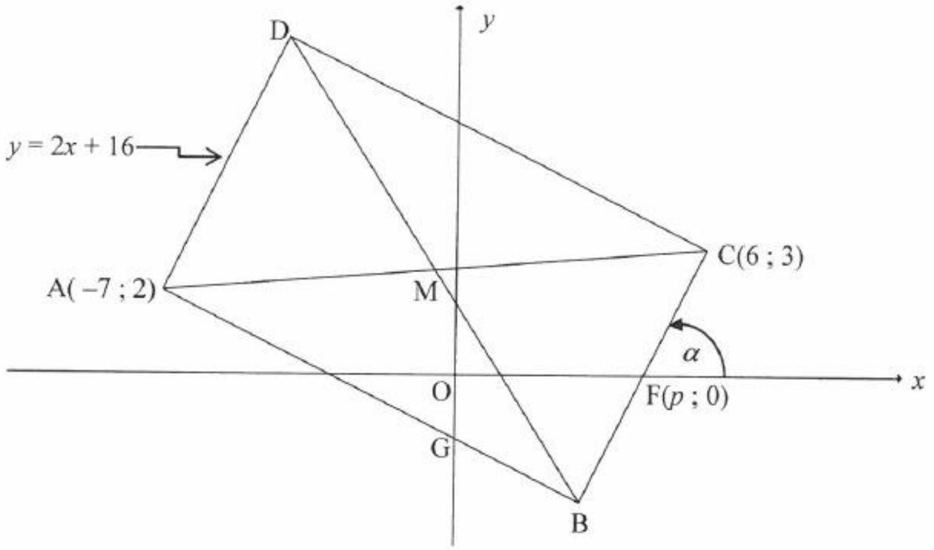


4.1	Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$.	(3)
4.2	Calculate the coordinates of S.	(3)
4.3	Determine the equation of the tangent APB in the form $y = mx + c$.	(4)
4.4	Calculate the size of α.	(2)
4.5	Calculate, with reasons, the size of θ.	(4)
4.6	Calculate the area of ΔPQS.	(4) **[20]**

NOVEMBER 2016

QUESTION 3

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

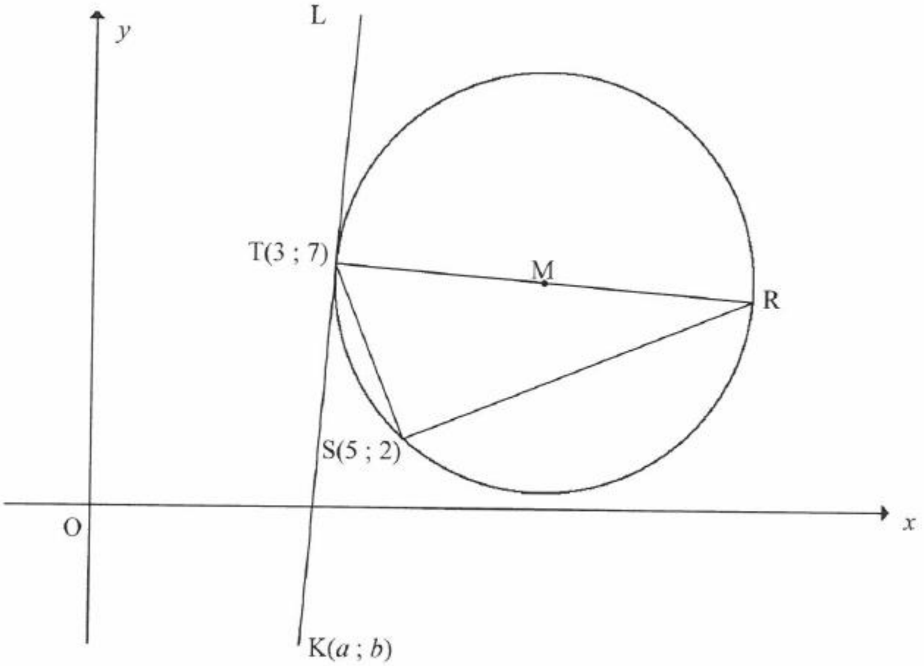
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)

[19]

QUESTION 4

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4th quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

4.4.4 Show that $b = 12a - 29$. (3)

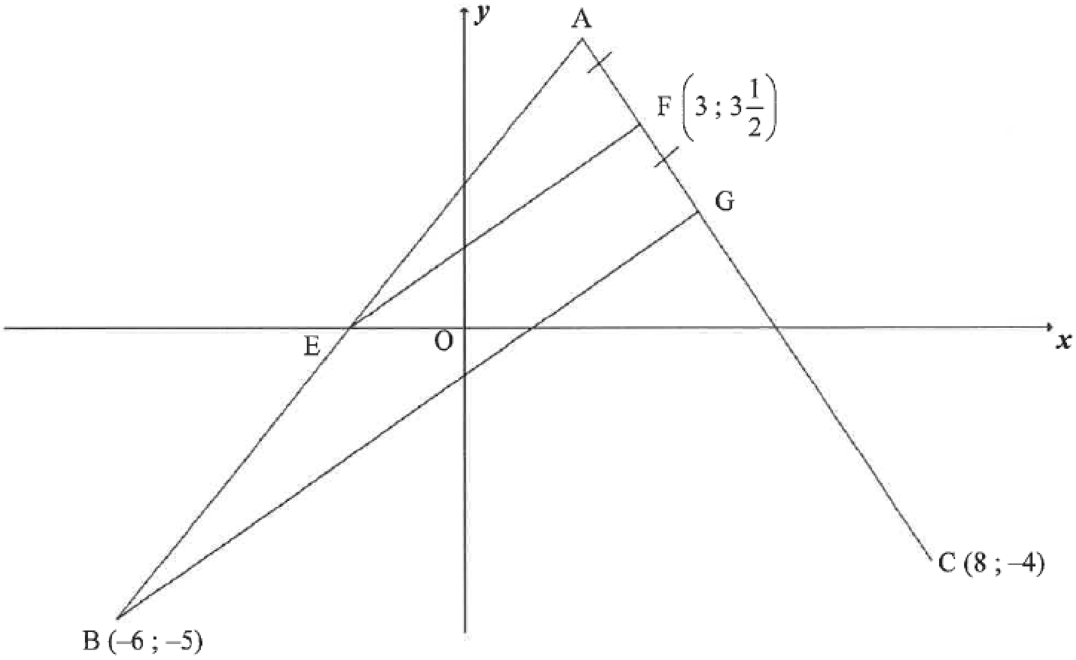
4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

NOVEMBER 2017

QUESTION 3

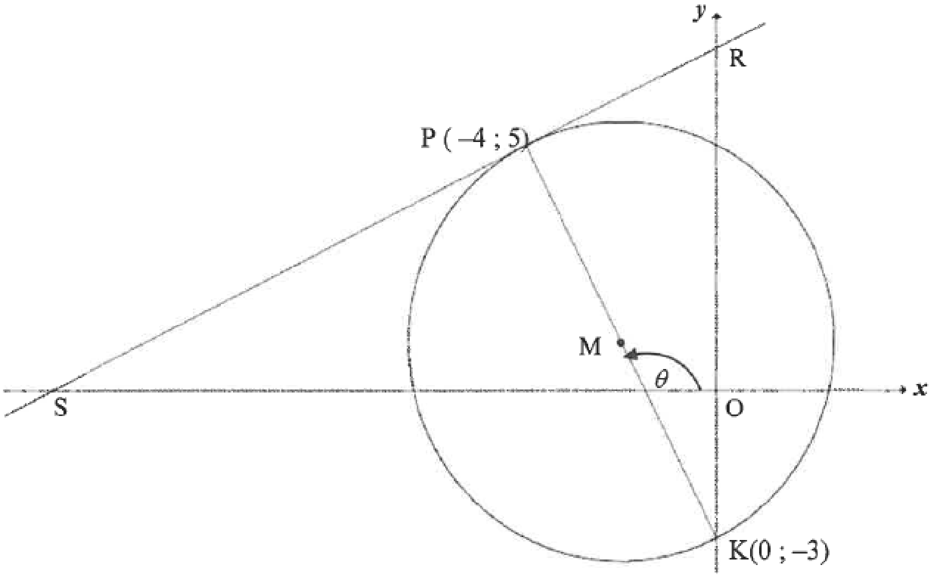
In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3;\,3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



3.1	Calculate:		
	3.1.1	The equation of AC in the form $y = mx + c$	(4)
	3.1.2	The coordinates of G if the equation of BG is $7x - 10y = 8$	(3)
3.2	Show by calculation that the coordinates of A is (2 ; 5).		(2)
3.3	Prove that EF || BG.		(4)
3.4	ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D.		(4) **[17]**

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

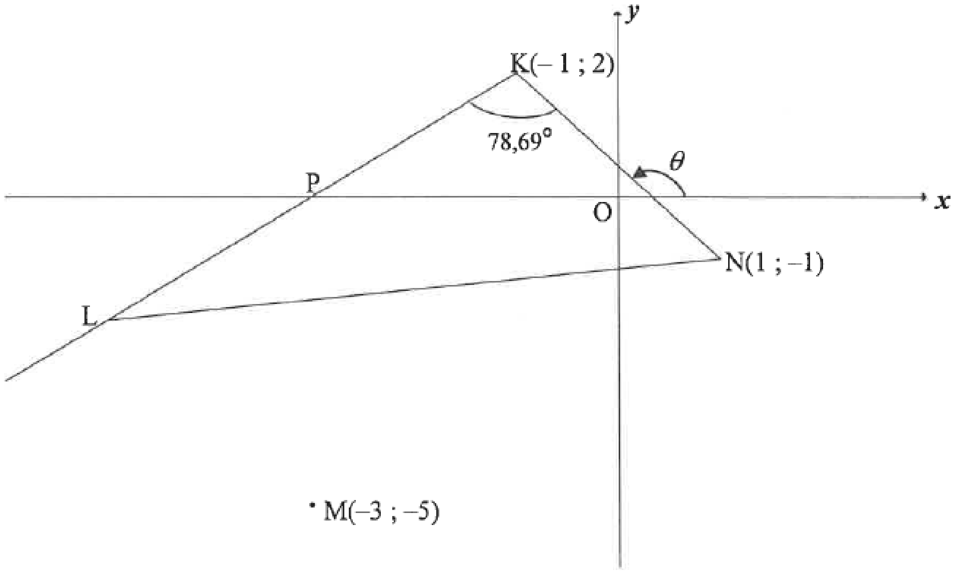
4.3 Calculate the area of ΔSMK. (5)

[23]

NOVEMBER 2018

QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

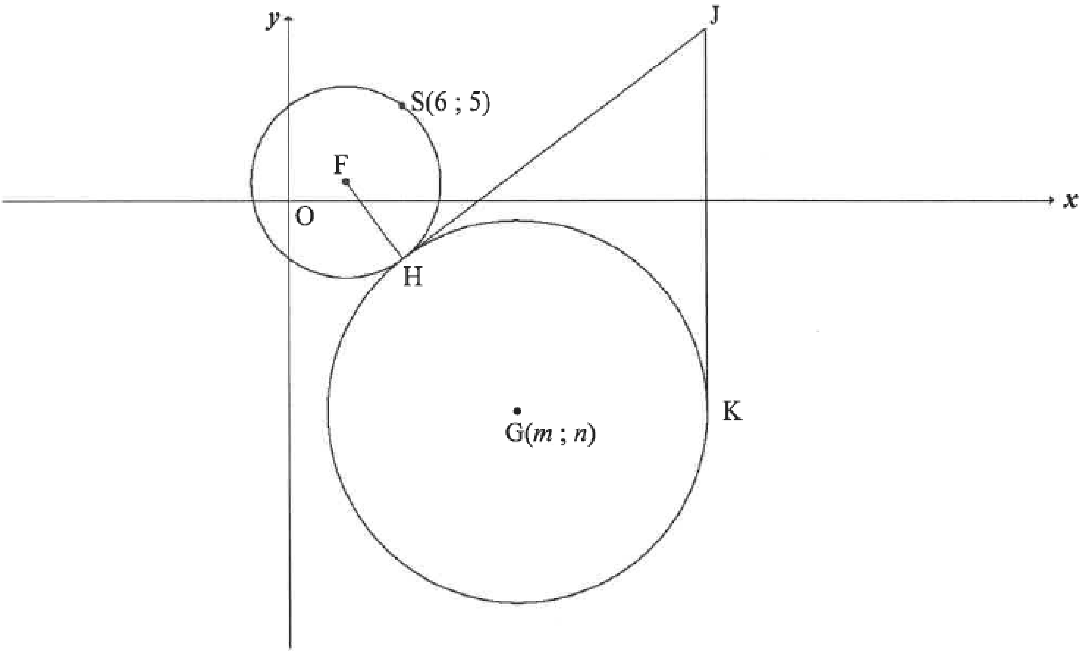
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)

[22]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G$(m\,;n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

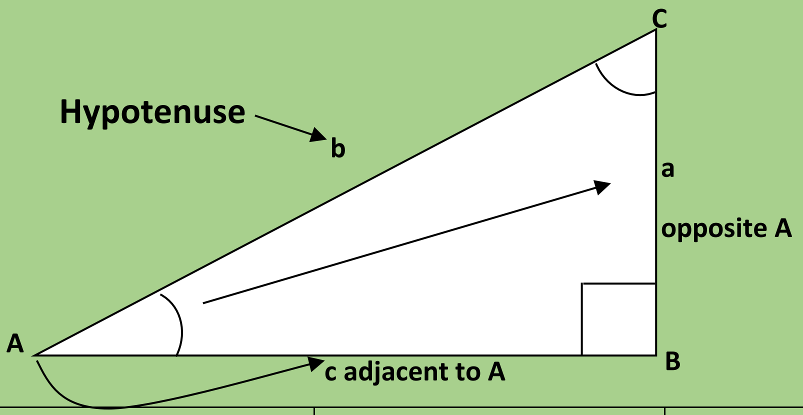
4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

TRIGONOMETRY

GRADE 11 TRIGONOMETRY

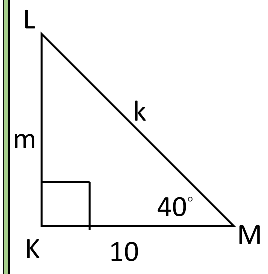
1 THE TRIGONOMETRIC RATIOS (FRACTIONS) IN A RIGHT-ANGLED TRIANGLE:



sin A = $\frac{a}{b}$	sin C = $\frac{c}{b}$	sin A= $\frac{opposite}{hypotenuse}$
cos A = $\frac{c}{b}$	cos C = $\frac{a}{b}$	cos A= $\frac{adjacent}{hypotenuse}$
tan A = $\frac{a}{c}$	tan C= $\frac{c}{a}$	tan A= $\frac{opposite}{adjacent}$

2 APPLICATION OF THESE RATIOS (mostly in 2 and 3 dimensional problems)

Solve Δ KLM fully. EXAMPLE



$$\frac{10}{k} = \cos 40^\circ$$

$$\frac{10}{k} \cdot \frac{k}{1} = \cos 40^\circ.k$$

$$10 = \cos 40^\circ.k$$

$$\frac{10}{\cos 40^\circ} = k$$

$$13{,}1 = k$$

For m there are 2 strategies:

$$\frac{m}{10} = \tan 40^\circ$$

$$\frac{m}{10} . \frac{10}{1} = \tan 40^\circ.10$$

$$m = 8{,}4$$

OR:

By Pythagoras:

$$m^2 + 10^2 = 13{,}1^2$$

$$m^2 + 100 = 171{,}61$$

$$m^2 = 171{,}61 - 100$$

$$m^2 = 71{,}61$$

$$m = 8{,}5$$

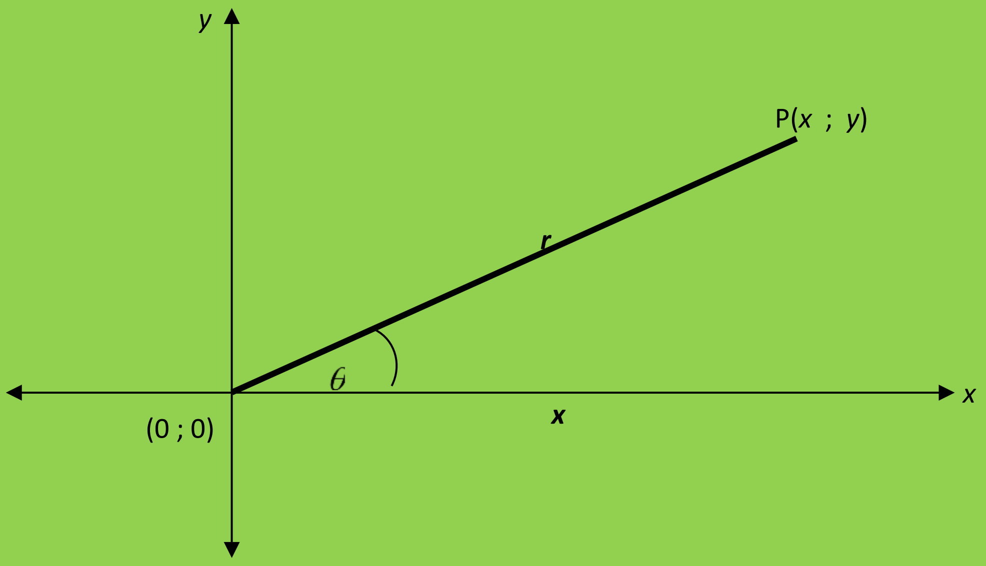
For L one will simply say:

L = $180^\circ - 90^\circ - 40^\circ$

L = 50° (3 $\angle$'s of **Δ**=180°)

Different answers due to rounding off!!

3 THE TRIGONOMETRIC RATIOS ON THE CARTESIAN PLANE.



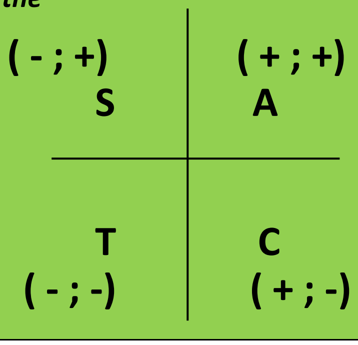
The 3 fractions on the Cartesian plane for the angle θ in <u>standard position</u> are:

$\sin\theta = \dfrac{y}{r}$	$\cos\theta = \dfrac{x}{r}$	$\tan\theta = \dfrac{y}{x}$

<u>In general:</u>

- Learn your definitions well! (HOW the fractions are formed.)
- ***<u>r is always positive</u>***, since *r* represents the distance from the origin to the ordered number pair on the terminal arm of the angle.
- Since *x* and *y* are coordinates and represent the position of a point, they can either be negative or positive.
- **The signs of *x* and *y* depend on the position of the angle: does it lie in quadrant I, II, III or IV?**
- While the angles will always be measured in degrees, it is important to remember that the trigonometric ratios (the values of the sin, cos and tan fractions) **<u>are always numbers!!!</u>**
- THE SIGNS OF THE TRIG RATIOS DEPEND ON THE SIGNS OF *x and y. **Since r is always positive it has no influence on the the sign of the trig ratio.***
- Maybe your teacher has taught you the CAST diagram to help you remember the signs of the trig ratios in different quadrants

The diagram is a diagram of positives: it indicates which trig ratio is positive in which quadrant. The other ones in that quadrant will be negative.



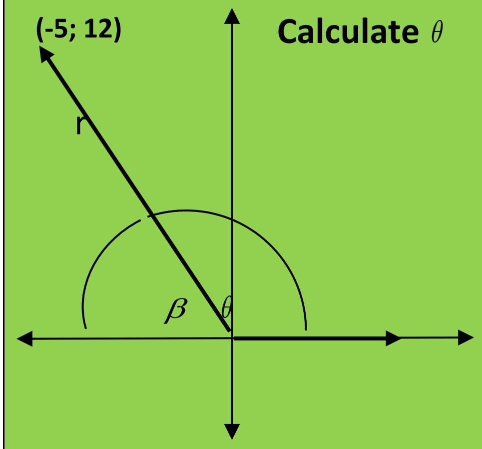
Quadrant I	**Quadrant II**	**Quadrant III**	**Quadrant IV**
$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$ $\cos\theta = \frac{x}{r} = \frac{+}{+} = +$ $\tan\theta = \frac{y}{x} = \frac{+}{+} = +$	$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$ $\cos\theta = \frac{x}{r} = \frac{-}{+} = -$ $\tan\theta = \frac{y}{x} = \frac{+}{-} = -$	$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$ $\cos\theta = \frac{x}{r} = \frac{-}{+} = -$ $\tan\theta = \frac{y}{x} = \frac{-}{-} = +$	$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$ $\cos\theta = \frac{x}{r} = \frac{+}{+} = +$ $\tan\theta = \frac{y}{x} = \frac{-}{+} = -$
All are positive (A)	**Only sin θ is positive (S)**	**Only tanθ is positive (T)**	**Only cos θ is positive (C)**

EXAMPLE 1

3.1 APPLICATION OF THESE RATIOS:

Calculate r

Calculate θ



By the circle formula:

$$x^2 + y^2 = r^2$$
$$(-5)^2 + 12^2 = r^2$$
$$25 + 144 = r^2$$
$$169 = r^2$$
$$13 = r$$

Now you can write down the value of any one of the trig ratios e.g.:

sin θ = $\frac{12}{13}$

tan θ = $\frac{12}{-5}$

One can also calculate the size of θ. How big a rotation did the **terminal arm** make? You go:

tanθ = $\frac{12}{-5} = -\frac{12}{5}$ (Now refer back to the diagram of Example 1)

On your calculator:
2nd fn/shift (+12÷5) = 67,4° **BUT THIS IS ACTUALLY THE SIZE OF** acute angle β

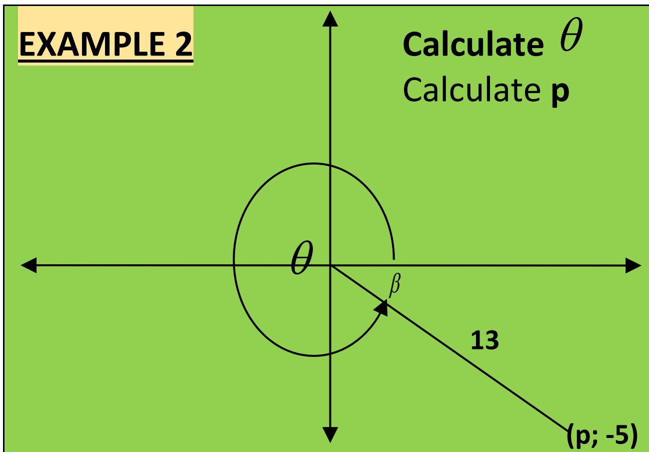
SO θ = 180° - 67,4°
θ = 112,6°

If you have x and y, calculate r.

If you have x and r, calculate y.

If you have y and r, calculate x,

EXAMPLE 2



Calculate θ

Calculate **p**

The rotation (angle) θ is quite big. We need the little acute angle β to calculate θ.

Since we have y and r, we can use the sin fraction to find θ . **You go:**

$$\sin\theta = \frac{-5}{13}$$

2^{nd} fn/shift (+5 ÷ 13) = 22,6°

BUT THIS IS β **(the reference angle) SO:**

θ = 360° - 22,6°

θ = 337.4°

For x:

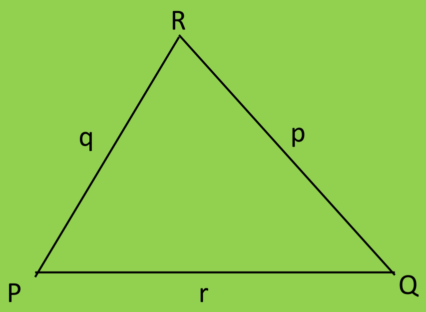
By the circle formula:

$$x^2 + y^2 = r^2$$
$$x^2 + (-5)^2 = 13^2$$
$$x^2 + 25 = 169$$
$$x^2 = 169 - 25$$
$$x^2 = 144$$
$$x = \pm\sqrt{144}$$
$$x = \pm 12$$
$$x = 12$$

One chooses +12 here, because the position of the ordered number pair is in the 4th quadrant; to the right of the y axis.

But x is the p value, ∴ p = 12

4 "Fractions" that are equal in any triangle: SINE RULE



IN ΔPQR IT IS ALWAYS TRUE THAT:

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} \quad or$$
$$\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$$

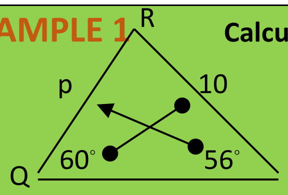
THIS IS KNOWN AS THE SINE RULE and given on the info sheet.

HOW DO WE USE THESE FRACTIONS?

- You write down a **proportion.**
- A **proportion** is 2 fractions that are equal.
- The proportion will include 3 values that you have, and one that you don't have.
- Solve the one you don't have.
- Your proportion will always look like this:

$$\frac{WANTED}{WHAT\ \ I\ \ HAVE} = \frac{WHAT\ \ I\ \ HAVE}{WHAT\ \ I\ \ HAVE}$$

EXAMPLE 1 **Calculate RQ (p)**



We want "baby p" so write down the proportion:

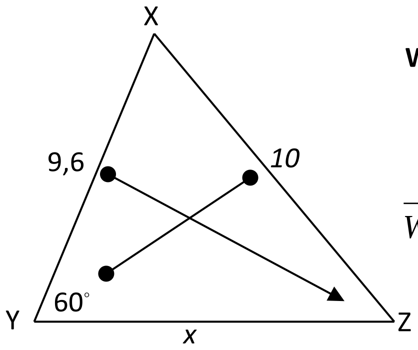
$$\frac{p}{\sin 56°} = \frac{10}{\sin\ 60°}$$

Now get p alone: × by

$$p = \frac{10\ .\ \sin 56°}{\sin\ 60°}$$

$$p =$$

EXAMPLE 2: **Calculate Z** **We calculate Z via sin Z in the sin rule.**



Write down the proportion

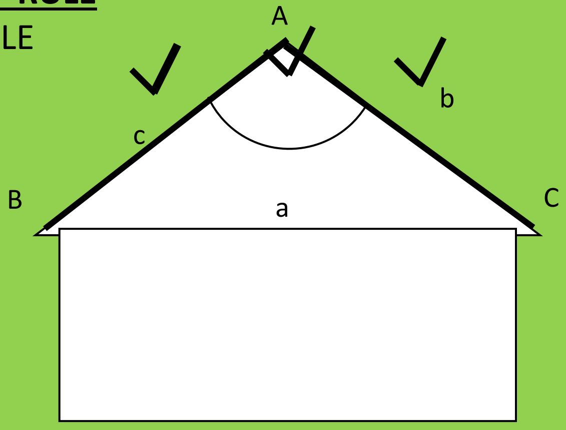
$$\frac{WANTED}{What\ I\ have} = \frac{What\ I\ have}{What\ I\ \ have}$$

$$\frac{\sin\ z}{9,6} = \frac{\sin\ 60°}{10}$$

$$\frac{\sin 60°.\ 9.6}{10}$$

5 THE COSINE RULE / COS RULE / "GABLE" RULE

Think about a little house. The COS RULE works like the **roof part** of the house. One applies the cos rule if one has 2 sides and the included angle of a triangle.



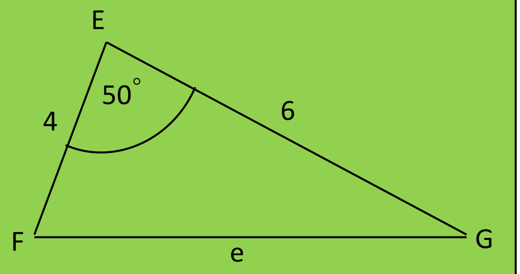
OR When one has three sides.

In any ΔABC,

$$a^2 = b^2 + c^2 - 2.b.c.\cos A$$

APPLICATION OF THE COS RULE:

EXAMPLE 1 **Calculate FG**



In ΔEFG:

$$e^2 = g^2 + f^2 - 2.g.f.\cos E$$

$$e^2 = 4^2 + 6^2 - 2.4.6.\cos 50^\circ$$

$$e^2 = 16 + 36 - 48.\cos 50^\circ$$

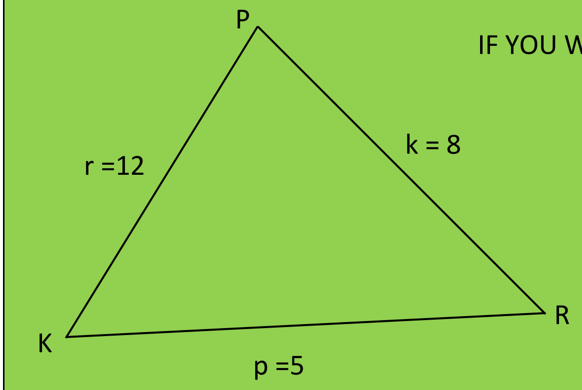
$$e^2 = 21{,}14619474$$

$$e = 4{,}6$$

This 2 is always a 2.

NB here: the side, included angle, side of the triangle were given.

EXAMPLE 2 **Three sides given:** **Calculate R**



IF YOU WANT ANGLE **R,** YOU START WITH **r²**

$$r^2 = p^2 + k^2 - 2.p.k.\cos R$$

$$12^2 = 5^2 + 8^2 - 2.5.8\cos R$$

$$144 = 25 + 64 - 80\cos R$$

$$144 = 89 - 80\cos R$$

$$144 - 89 = -80\cos R$$

$$55 = -80 \cos R$$

$$-\frac{55}{80} = \cos R$$

$$shift \cos(+55 \div 80) = 46{,}57^\circ$$

R is an obtuse angle, the negative shows it

$$\therefore R = 180^\circ - 46{,}57^\circ$$

$$R = 133{,}4^\circ$$

Remember if cos ***R*** is negative, then ***R*** is an obtuse angle. You go:

R = (180° – ref angle)

6 A NOTE ON REDUCTION FORMULAE

THE TRIG RATIO OF EVERY SINGLE OBTUSE OR REFLEX ANGLE (BIIIIG ANGLE), CAN BE REWRITTEN AS THE TRIG RATIO OF AN ACUTE ANGLE.

E.g.

sin 179° = + sin 1°
sin 181° = - sin 1°
sin 359° = - sin 1°
sin 361° = +sin 1°
sin (-1°) = - sin 1°

- How did I know it has to be + or – ? [check the quadrant in which the angle lies]
- How did I know the name of the trig ratio to go into the answer? [only with cos and sin of (90° +…) and(90° - …) the name changes, all the others stay the same]
- How did I know **that the reduced angle has to be 1°**? [I rewrite the "big' angle as compound angles:
  180° - … 180° + … or 360 - …

Remember:

$\sin(180° - \theta) = +\sin\theta$

$\sin(180° + \theta) = -\sin\theta$

$\sin(360° - \theta) = -\sin\theta$

$\sin(360° + \theta) = +\sin\theta$

$\sin(-\theta) = -\sin\theta$

$\sin(90° - \theta) = \cos\theta$

$\sin(90° + \theta) = \cos\theta$

$\cos(180° - \theta) = -\cos\theta$

$\cos(180° + \theta) = -\cos\theta$

$\cos(360° - \theta) = +\cos\theta$

$\cos(360° + \theta) = +\cos\theta$

$\cos(-\theta) = +\cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\cos(90° + \theta) = -\sin\theta$

$\tan(180° - \theta) = -\tan\theta$

$\tan(180° + \theta) = +\tan\theta$

$\tan(360° - \theta) = -\tan\theta$

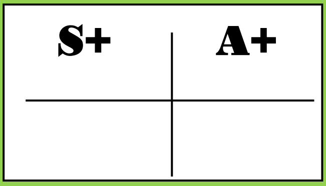
$\tan(360° + \theta) = +\tan\theta$

$\tan(-\theta) = -\tan\theta$

- How did I know it has to be + or – ?
- How did I know the name of the trig ratio to go into the answer?
- How did I know **that the reduced angle has to be** θ ?

THE SECRET IS: IN YOUR ANSWER THERE WILL ALWAYS BE

- **A SIGN**
- **A TRIG RATIO**
- **AN ACUTE ANGLE**



EXAMPLES IN THE QUADRANTS

sin 361° terminal arm in Q 1	**SIGN: +** **Because the angle is in Q1**	**TRIG RATIO:** stays sin	**ACUTE ANGLE:** 361° - 360° = 1°	**sin 361°** = + **sin 1°**
sin (-1°) terminal arm in Q 4	**SIGN: -** **Because the angle is in Q4**	**TRIG RATIO:** stays sin	**ACUTE ANGLE:** 1°	**sin (-1)** = -**sin 1°**

Trig ratio	**The terminal side lies in which quadrant?**	**What is the sign in this quadrant?**	**Angle as a compound angle in this quadrant.**	**Final answer**
tan 242°	Quadrant 3	tan ratio is positive in Q3	$(180^\circ + 62^\circ)$	tan 242° = + tan 62°
tan 340°	Quadrant 4	tan ratio is negative in Q4	$(360^\circ - 20^\circ)$	tan 340° = - tan 20°
cos 165°	Quadrant 2	cos ratio is negative in Q2	$(180^\circ - 15^\circ)$	cos 165° = - cos 15°
cos 395°	Quadrant 1	cos ratio is positive in Q1	Get the acute bit: $(395^\circ - 360^\circ)$	cos 395° = + cos 35°
tan (-60°)	Quadrant 4	tan ratio is negative in Q4	The acute bit is 60°	tan (-60°) = -tan 60°
cos (-71°)	Quadrant 4	cos ratio is positive in Q4	The acute bit is 71°	cos (-71°) = + cos 71°

THE ACUTE BIT IS SOMETIMES GIVEN AS A SYMBOL E.G.

θ *OR* β *OR* δ *OR* α

THE PRINCIPLE STAYS THE SAME:

- **SIGN**
- **TRIG RATIO**
- **ACUTE ANGLE**

	TERMINAL ARM?	SIGN?	ACUTE ANGLE?	ANSWER
sin $(180^\circ - \theta)$	Q 2	+	θ	sin $(180^\circ - \theta)$ = +sinθ
tan $(180^\circ + \theta)$	Q3	+	θ	tan $(180^\circ + \theta)$ = +tanθ
tan $(360^\circ - \theta)$	Q4	-	θ	tan $(360^\circ - \theta)$ = - tan θ
tan $(360^\circ + \theta)$	Q1	+	θ	tan $(360^\circ + \theta)$ = +tan θ
cos $(-\beta)$	Q4	+	β	cos $(-\beta)$ = + cos β
tan $(-\beta)$	Q4	-	β	tan $(-\beta)$ = -tan$(-\beta)$

WHEN DOES THE NAME OF THE TRIG RATIO CHANGE TO THE NAME OF THE CO-FUNCTION? ONLY WHEN WE HAVE:

sin $(90^\circ - \theta)$, sin $(90^\circ + \theta)$, cos $(90^\circ - \theta)$, cos $(90^\circ + \theta)$

sin $(90^\circ - \theta)$ = cos θ	cos $(90^\circ - \theta)$ = sinθ
sin $(90^\circ + \theta)$ = cos θ	cos $(90^\circ + \theta)$ = -sinθ

Huge positive and negative angles: TWO SPECIAL EXAMPLES:

(i) $\cos(-568)^\circ$: add full revolutions ...(**calculate the co-terminal angle)**

until you get the first positive angle

$(-568)^\circ + 360^\circ = (-208)^\circ$

$(-208)^\circ + 360^\circ = 152^\circ$

You may add as many 360° as you like. They are co terminal angles. The value of the trig ratios of co terminal angles are equal. Check: sin 1° sin361° sin 721° sin 1 081° and so forth.

$\cos 152^\circ$

$= \cos(180^\circ - 28^\circ)$

$= -\cos 28^\circ$

SO: $\cos(-568^\circ) = -\cos 28^\circ$

(ii) $\cos 1456^\circ$:

(subtract full revolutions until you get the first positive co-terminal angle)

$1456^\circ - 360^\circ = 1096^\circ$

$1096^\circ - 360^\circ = 736^\circ$

$736^\circ - 360^\circ = 376^\circ$

$376^\circ - 360^\circ = 16^\circ$

SO: $\cos 1456^\circ = \cos 16^\circ$

You may subtract as many 360° as you like. They are co terminal angles.

7 DIAGRAM QUESTIONS: EXAMPLES

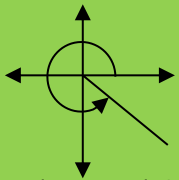
7.1 If sin A = $\frac{-\sqrt{7}}{4}$ and tan A < 0, use a sketch to determine the value of (i) cos A. (ii) A **(with a calculator)** (correct to one decimal place)

TIPS:

- Determine in which quadrant does angle A lie. How?
  THEY TELL YOU!

  Where is sinA ($\frac{y}{r}$) negative?

  Where is tan A ($\frac{y}{x}$) negative?



Now where is sin A and tan A both negative, simultaneously?

7.2 Don't use a calculator. Draw a diagram.

If 7cosA – 3 = 2, and sin A < 0, use a sketch to determine the value of (i) tan A. (ii) A (**with a calculator, correct to one dec. place**)

TIPS:

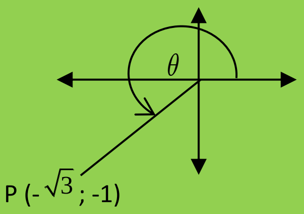
- Get cos A alone on the left hand side...isolate it [cosA= $\frac{5}{7} = \frac{x}{r}$]
- cos A is positive.
- Where is sin A negative?
- Where is cos A positive and sin A negative at the same time?
- This is the quadrant in which angle A lies.

7.3 If 40 tan B + 30 = 50, and cos B < 0, use a sketch to calculate the

value of: sinBcosB – cosBcosB [tan B= +½]

Cos-	**Tan+**
Cos- **Tan+**	

7.4



Without using a calculator, find the value of θ.

TIP:

- Do you perhaps remember the special angles?
  30° 45° 60°
- Check on the next page how to remember the trig ratio (fraction) values of the special angles.

7.5 If sin A = 2p, determine the value of $\cos^2 A$. [0° ; 90°]

TIPS:

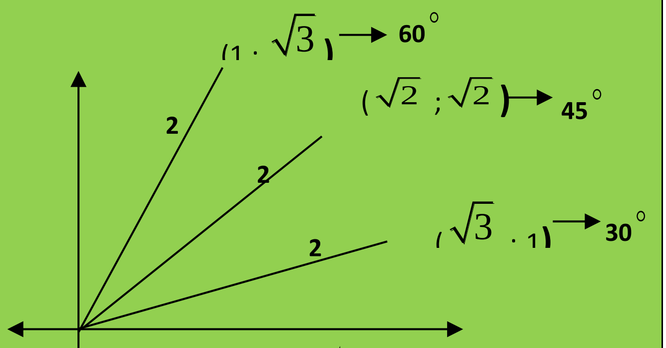
- Write sin A as $\frac{2p}{1}$
- This means you have ***y* and *r*** on the Cartesian plane,
- OR you have the **Opposite side and Hypotenuse** in a right-angled triangle.
- Make use of a **diagram** to **calculate *x*** or the adjacent side.

8 SPECIAL ANGLES: 30° 45° 60°

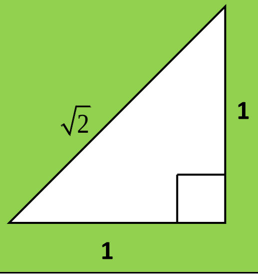
Table

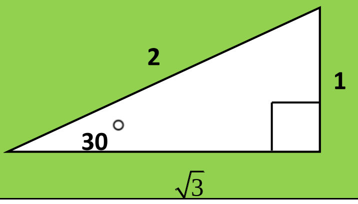
		30°	**45°**	**60°**
Adjacent	***x***	$\sqrt{3}$	**1**	**1**
Opposite	***y***	**1**	**1**	$\sqrt{3}$
Hypotenuse	***r***	**2**	$\sqrt{2}$	**2**

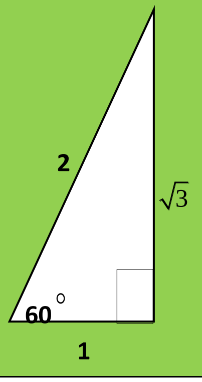
On the Cartesian plane



3 Triangles







9.**TRIGONOMETRIC FUNCTIONS**

$y = \sin x$	$y = \cos x$	$y = \tan x$
$y = a\sin x$	$y = a\cos x$	$y = a\tan x$
$y = \sin x + q$	$y = \cos x + q$	$y = \tan x + q$
$y = \sin kx$	$y = \cos kx$	$y = \tan kx$

The ‘***a***’ in each formula influences the **amplitude** of the graph.

If ‘***a***’ is negative, it causes the reflection of the graph in the x-axis.

The ‘*q*’ added to **e**ach ‘basic’ graph, shifts (translates) the whole graph just as it is, ***q* units**

units upwards if ***q*** is **positive** and ***q* units** downwards if ***q*** is **negative.**

The *k* changes the period of the graphs.

For the graphs of $y = \sin kx$ and $y = \cos kx$ it becomes $\frac{360°}{k}$ and for $y = \tan kx$ it becomes $\frac{180°}{k}$

$y = \sin(x + p)$ $y = \sin(x + 60°)$	$y = \cos(x + p)$ $y = \cos(x + 60°)$	$y = \tan(x + p)$ $y = \tan(x + 60°)$

If *p* is **positive** like in these examples, it moves (translates) the graph 60° **to the LEFT**

$y = \sin(x + p)$ $y = \sin(x - 60°)$	$y = \cos(x + p)$ $y = \cos(x - 60°)$	$y = \tan(x + p)$ $y = \tan(x - 60°)$

If *p* is **negative** like in these examples, it moves (translates) the graph 60° **to the RIGHT**

EXAMPLE

- Suppose you have to draw the graphs of $y = \sin 2x$ and $y = \cos(x - 60°)$

  Questions:

  - **Where do the two graphs intersect?** This is the simultaneous equation of $y = \sin 2x$ and $y = \cos(x - 60°)$

- It becomes the equation:

  $\sin 2x = \cos(x - 60°)$

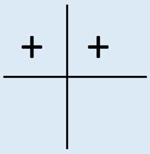
  Change the right-hand side to sin of an (angle).

  $\sin 2x = \sin[90° - (x - 60°)]$

  $\sin 2x = \sin[90° - x + 60°]$

  $\sin 2x = +\sin(150° - x)$

Quadrant where sin x>0:



Reference angle: (150° – *x*)

Solution: Q1 Q2

$2x = (150° - x) + k.360°$ or $2x = 180° - (150° - x) + k.360°$ $k \in$ □

$2x + x = 150° + k.360°$ or $2x = 180° - 150° + x + k.360°$ $k \in$ □

$3x = 150° + k.360°$ or $2x - x = 30° + k.360°$

$\div 3: \quad x = 50° + k.120°$ or $x = 30° + k.360°$

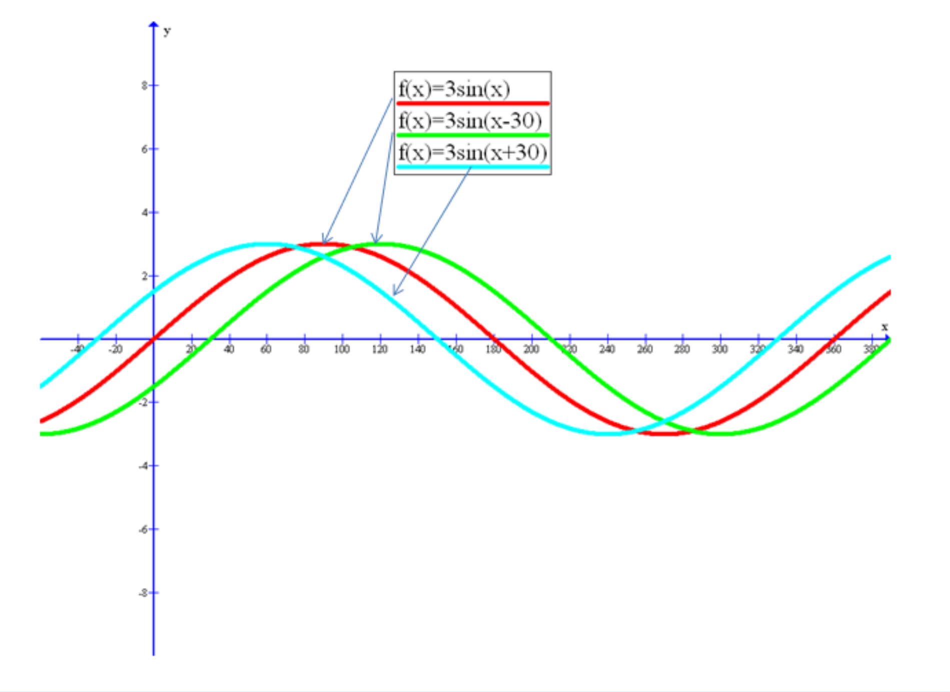
Substitute k = -3, -2, -1, 0, 1, 2, 3, etc. to get values in a specific interval.

- **What if they say**: show on the graph where the solutions are of
  $2\sin x\cos x = \cos x\cos 60° + \sin x\sin 60°$ or
  $\sin x\cos x = \frac{1}{2}[\cos x\cos 60° + \sin x\sin 60°]$
  Can you see that both these equations come from the graphs you drew?
  Manipulate equations like these until you get your original graphs.

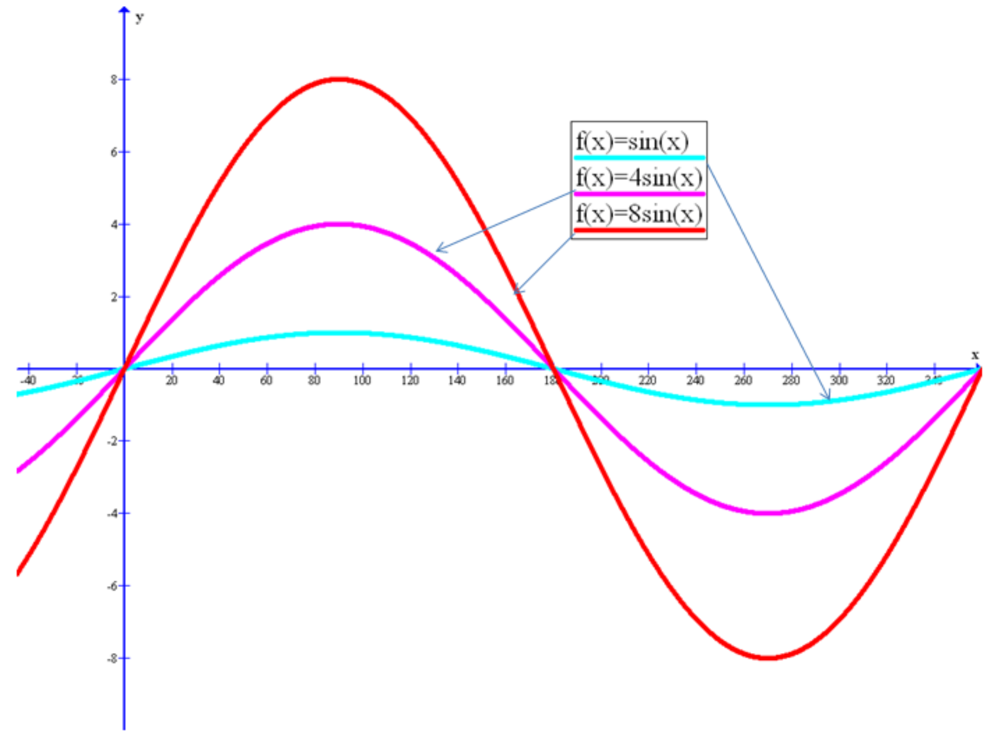
- **What if they say**: find the equation of *h*, where *h* is the function obtained (that you got) after $y = \sin 2x$ has been translated 4 units upwards. $h(x) = \sin 2x + 4$
  If one point on a graph is given, you should be able to get other points on the graph, e.g. if $(60° ; 0,87)$ lies on y = sin*x* then (240° ; -0,87) also lies on the graph.

EXAMPLES OF TRIGONOMETRIC GRAPHS

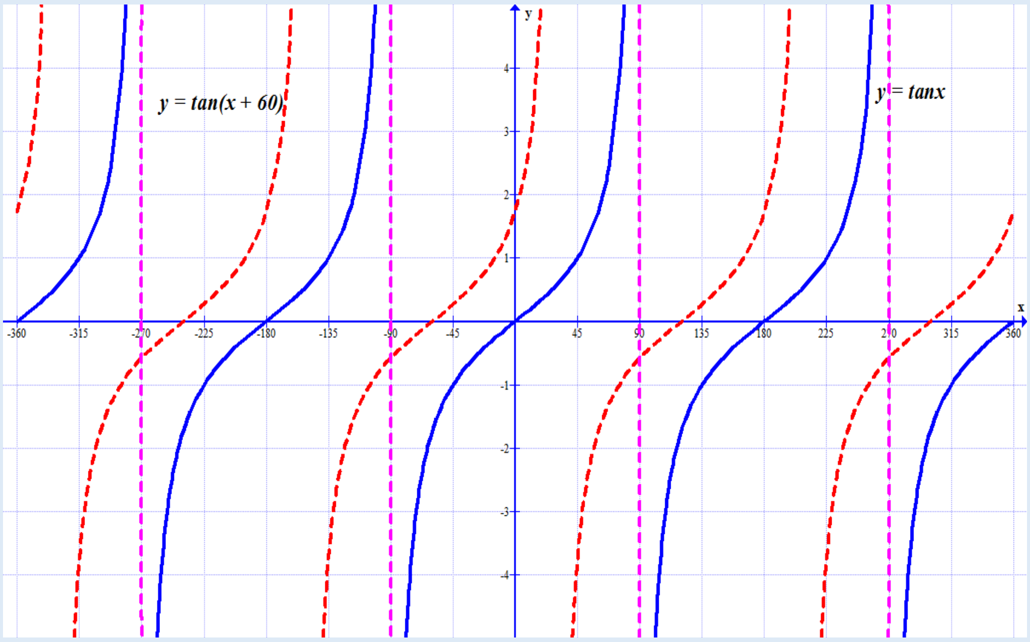
The following graph shows the effect of *p* in the equation $y = \sin(x + p)$



The following graph shows the effect of *r* in the equation $y = r\sin(x)$



The graph of $y = \tan(x + 60)$



WORKED EXAMPLE

1.1 Solve for $x \in [0°; 360°]$ for which $\cos 2x = \sin x$. (5)

1.2 Draw the graphs of $f(x) = \cos 2x$ and $g(x) = \sin x$ for the interval $x \in [-90°; 180°]$. Clearly indicate the intercepts with the axes as well as the coordinates of the turning points on the graphs. (7)

1.3 Use your answer on QUESTION 2.1 and also indicate the coordinates of the points of intersection of f and g on the graphs (2)

1.4 Hence use your graphs and determine the value(s) of $x \in [-90°; 180°]$ for which:

1.4.1 $g(x) \geq f(x)$ (3)

1.4.2 $f(x).g(x) \leq 0$ (3)

1.4.3 $g(x)$ increases while x increases (2)

SOLUTION

1.1 $\cos 2x = \sin x$

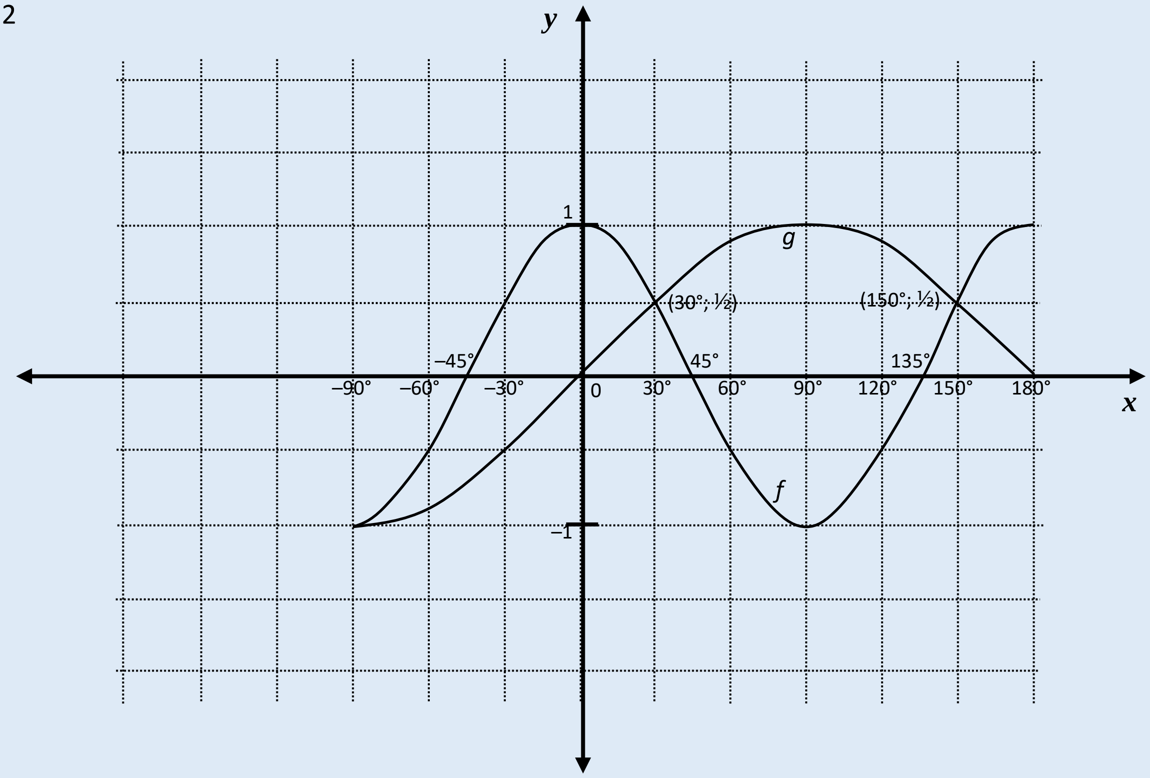
$= \cos(90° - x)$

$\therefore \quad 2x = 90° - x \quad$ or $\quad 2x = 360° - (90° - x)$

$\therefore \quad 3x = 90°$

$\therefore \quad x = 30° \qquad\qquad x = 270°$ (5)

1.2



(7)

1.3 $(30°;\frac{1}{2})$ *and* $(150°;\frac{1}{2})$

x coordinates correct

y coordinates correct (2)

1.4.1 $x = -90°$ or $30° \leq x \leq 150°$ (3)

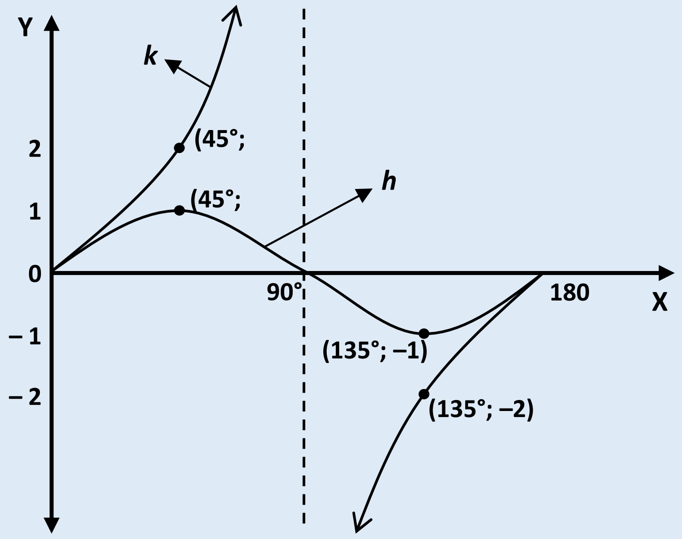
1.4.2 $-45° \leq x \leq 0°$ or $45° \leq x \leq 135°$ or $x = 180°$ (3)

1.4.3 $-90° < x < 90°$ (2)

EXAMPLE 2

The diagram alongside shows the graphs of $h(x) = \sin ax$ and $k(x) = b\tan x$ for $x \in [0°; 180°]$.

Use this diagram to answer the following questions.



2.1 Determine the values of *a* and *b*. (4)

2.2 For which values of $x \in [0°; 180°]$, will $h(x)$ decrease while *x* increases? (2)

2.3 Write down the range of the graph of *k*. (2)

2.4 What is the period of the function *h*? (2)

2.5 Write down the equation of the asymptote of *k*. (2)

SOLUTIONS

2.1 $a = 2$

$b = 2$ (4)

2.2 $45° < x < 135°$ **OR** $x \in (45° ; 135°)$ (2)

2.3 $-\infty < y < \infty$ **OR** $y \in (-\infty ; \infty)$; $y \in \mathbb{R}$ (2)

2.4 180° (2)

2.5 $x = 90°$ (2)

GRADE 12 TRIGONOMETRY

10 COMPOUND ANGLES AND DOUBLE ANGLES

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

DOUBLE ANGLES

$$\begin{aligned} cos2\theta &= cos^2\theta - sin^2\theta \\ &= 2cos^2\theta - 1 \\ &= 1 - 2sin^2\theta \end{aligned}$$

$sin2\theta = 2sin\theta cos\theta$

Found in:

10.1 Identities: substitute all possible double angle formulae, compound angle formulae and squared identities.

Quotient identity: tanA = $\frac{\sin A}{\cos A}$ or tanA.cosA **= sin A**

$\sin^2 A + \cos^2 A = 1$

Squared identities: $\sin^2 A = 1 - \cos^2 A$

$\cos^2 A = 1 - \sin^2 A$

Remember $\sin^2$A means (sin A)^2 = sin A **.** sin A

NB: in an identity you will NEVER multiply by the LCD. It is NOT an equation!

One often has to factorise in trigonometric identities.

$\sin^2$A - $\cos^2$A = (sin A - cosA) (sin A + cosA)

$\sin^2$A = 1 - $\cos^2$A = (1 - cosA) (1 + cosA)

$\cos^2$A = 1 - $\sin^2$A = (1 - sin A) (1 + sin A)

$\cos^2$A + 2cosAsinA + $\sin^2$A = (cosA + sin A) (cosA + sin A)

10.2 Trigonometric equations

Finding the General solution

Step 1: Write the equation in simplest form

Step 2: Q - Quadrants where the solution lies Use CAST diagram

R - Reference angle

S - Solution:

Step 3:

Reference angle + k.360° (sin A and cosA) or +k.180° (tanA) $k \in Z$

Step 4: Finalise the solution. Sometimes you have to divide by the coefficient of the angle.

Step 5: Finding the solution in a specific interval. Substitute integers into k: k = -3 k = -2 k = -1 k = 0 k = 1 k = 2 k = 3 Which ones fall within the given interval?

10.3 Simplify expressions and terms:

EXAMPLE 1: Evaluate without a calculator using a compound angle formula:

$\cos 95^o \sin 5^o - \sin 95^o \cos 5^o = ????$,

"Take out" a common factor namely -1:

$\cos 95^o \sin 5^o - \sin 95^o \cos 5^o$

$= -1(\sin 95^o \cos 5^o - \cos 95^o \sin 5^o)$

$= -1[\sin (95^o - 5^o)$

$= -1\sin 90^o$

$= -1$

EXAMPLE 2: Evaluate without a calculator.

$\sin 391° . \sin (-331°) - \cos 751° . \cos (-1051°)$

REMEMBER HUGE NEGATIVE OR POSITIVE (ADD 360° OR SUBTRACT 360°)

$\mathbf{391° - 360° = 31°}$

$\mathbf{-331° + 360° = 29°}$

$\mathbf{751° - 720° = 31°}$

$\mathbf{-1051 + 720° = 331°}$ **then** $\mathbf{360° - 29°}$

$\sin 391° . \sin (-331°) - \cos 751° . \cos (-1051°)$

$= \boldsymbol{sin31°.sin29° - cos31°.cos29°}$

$= \boldsymbol{-(cos31°.cos29° - sin31°.sin29°)}$

$= \boldsymbol{-cos(31+29)}$

$= \boldsymbol{-cos60°}$

$= \boldsymbol{-\frac{1}{2}}$

TECHNICAL REPORT FINDINGS

Common Errors and misconceptions

Leaners still lack knowledge of reduction formulae .Learners still reduce as follows $\cos(90° - x) = cosx$

Learners unable to relate double angles e.g. $sin^2 15° - cos^2 15°$ cannot relate to $cos^2 15° - sin^2 15°$

Identifying of compound expansion is still a challenge. e.g. $\sin(x + 15°)\, cos15° - \cos(x + 15°)\, sin15°$

Learners still cannot differentiate between domain and range.

Properties of quadrilateral still a challenge to learners.

Learners cannot differential and visualize shapes on different plans, vertical plane and horizontal plane.

Learners still not understanding writing something in terms of x. e.g. Writing AK in terms of x

Suggestion of improvement

Emphasis and differentiation to be explained between $90° \pm \theta$; $180° \pm \theta$ *and* $360 \pm \theta$'
Give more practise on reduction formulae.
Learners to always check the double angles in the formula sheet
Teachers to ensure that learners revise Grade 11 Trigonometry regularly in Grade 12 year.
Learners need exposure to the simplification of expressions containing double and compound angles.
Examples on double angles should include variables for angles as well as specific angle values.
Learners should also be required to write down the compound angles when given expansion.
Teachers should not confine the teaching of graphs to sketching .Learners should also be exposed to exercises in which they have to interpret graphs and read off solutions from the graphs.
Teachers to inform learners that Determine AK in terms of x means that AK must be the subject of an expression and that the expression must be in terms of x.
Initially expose learners to numerical values questions on solving 3-D problems This makes it easy for learners to develop strategies on how to solve such questions.

TIPS FOR ANSWERING EXAM QUESTIONS

Read the information given **more than once**.
Analyse and add more information regarding the information.
Break the question up and make connections
Learners to **always draw a cartesian plane** with ratios and reduction formulae in quadrants to indicate which ratios are positive in which quadrants. The cartesian plane will assist again in showing that the reduction formulae is in which quadrant.
Analyse the question or the diagram
When answering 3D questions **highlight the different triangles** using different colours.

TIPS ON SOLVING GENERAL SOLUTION

1. **Simplify** the equation using algebraic methods and trigonometric identities
2. Determine the reference angle.
3. Use **CAST** diagram to determine where the function is positive or negative (Depending on the given information/equation)
4. Restricted values: Determine the angles that lie within a specified interval by adding or subtracting of the multiples appropriate period.
5. **General Solution:** Determine the angles in the interval $[0°; 360°]$ that satisfy the equation and add multiples of the period to each answer
6. **Check** answers using a calculator

TIPS FOR PROVING IDENTITIES:

- Change all trigonometric ratios to sine and cosine.
- Choose one side of the equation to simplify and show that it is equal to the other side.
- Usually it is better to choose the more complicated side to simplify.
- Sometimes we need to simplify both sides of the equation to show that they are equal.
- A square root sign often indicates that we need to use the square identity.
- We can also add to the expression to make simplifying easier:
  - Replace 1 with $sin^2\theta + cos^2\theta$.
  - Multiply by 1 in the form of a suitable fraction, for example $\frac{1+sin\theta}{1+sin\theta}$

WARM UP QUESTIONS

1.SIMPLIFY WITHOUT A CALCULATOR: special angles mixed with other angles that eventually simplify

EXAMPLE

1.1 $$\frac{\sin 780°.\cos 135°.\tan(420°)}{\tan(-330°).\sin 315°.\cos(-150°)}$$

1.2 $$\frac{\sin 380°.\tan 721°.\cos 320°}{\cos 220°.\sin 160°.\tan 359°}$$

1.3 $$\frac{\sin 160°.\cos 380°}{\tan 200°} + \sin^2 340°$$

For 1.3 you will have to remember your trig identities.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

1.4 $$\frac{\cos 350°.\sin 190°.\sin 370°}{\sin 170°.\tan(-10°).\cos^2 10°}$$

NOTES ON NUMBER 1

1.1 **FOR 780° AND 420° YOU GO -360°, -360° ETC.**
FOR -330° AND -150° YOU GO +360°

1.2 FOR 721° YOU GO - 360°, - 360°
Try to reduce the angles to the same size acute angle.

1 **Prove that:**

$$\frac{1+\sin y}{1-\sin y} - \frac{1-\sin y}{1+\sin y} = \frac{4\tan y}{\cos y} \qquad (LCD!!)$$

State the restrictions on sin y and hence the values of y for which the identity is not valid. (This simply means indicate values of sin y and values of y that will make the denominators equal to 0.)

$$L = \frac{1+\sin y}{1-\sin y} - \frac{1-\sin y}{1+\sin y}$$

2 **Prove that / Bewys dat:**

$$\tan 2x + 2\sin x = \frac{2\sin x(2\cos x - 1)(\cos x + 1)}{\cos 2x}$$

3 **Prove that / Bewys dat:**

$$\frac{\sin 2x - \cos x}{\sin x - \cos 2x} = \frac{\cos x}{\sin x + 1}$$

5 ***Prove that / Bewys dat :***

$$\frac{1}{1-\cos(180^\circ - x)} + \frac{1}{1-\sin(90^\circ - x)} = \frac{2}{\sin^2 x}$$

a) <u>Simplify:</u>

1) $\dfrac{\cos(A+B)-\cos(A-B)}{\sin(A+B)-\sin(A-B)}$

2) $\cos(300^\circ + \beta) + \cos(300^\circ - \beta) - \cos\beta$

c) Evaluate:

1) cos 75º 2) 2 sin15 ºcos 15 º 3) 2 sin 22,5 ºcos 22,5 º

4) cos 165º 5) 2 sin 150ºcos 330º 6) sin 225º

7) 2 sin 20º sin70º 8) $\cos^2 15^\circ - \sin^2 15^\circ$ 9) $2\cos^2 15^\circ - 1$

10) $1 - 2\sin^2 15^\circ$ 11) $\cos^2 22\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

12) $1 - 2\sin^2 75^\circ$ 13) sin75ºcos75º 14) 4sin15ºcos15

6 ***Simplify / Vereenvoudig :***

a) $\dfrac{\sin(-63^\circ).\sin 27^\circ}{\sin 126^\circ.\tan 225^\circ}$ *b)* $\dfrac{\tan 240^\circ.\cos 405^\circ}{\cos(-30^\circ)}$

c) $\dfrac{\cos 300^\circ.\sin 140^\circ.}{\tan 765^\circ.\sin 160^\circ.\sin 290^\circ}$ *d)* $\dfrac{\cos 40^\circ}{\sin 25^\circ \cos 25^\circ}$ *e)* $\dfrac{\cos 870^\circ.\tan(-1020^\circ)}{\sin(-270^\circ)}$

Diagrams

A diagram involving a compound angle formula.

7 ***Determine / Bepaal*** $\sin(A+B)$ ***if / indien :***

$\sin A = \frac{4}{5}$; $0^\circ \le A \le 90^\circ$ & $\cos B = \frac{4}{5}$; $270^\circ \le B \le 360^\circ$

TYPICAL EXAMINATION QUESTIONS

MAY / JUNE 2019

QUESTION 5

5.1 **Without using a calculator,** write the following expressions in terms of $\sin 11°$:

5.1.1 $\sin 191°$ (1)

5.1.2 $\cos 22°$ (1)

5.2 Simplify $\cos(x - 180°) + \sqrt{2}\sin(x + 45°)$ to a single trigonometric ratio. (5)

5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90°$

Without using a calculator, determine the value of $\sin 2P$.

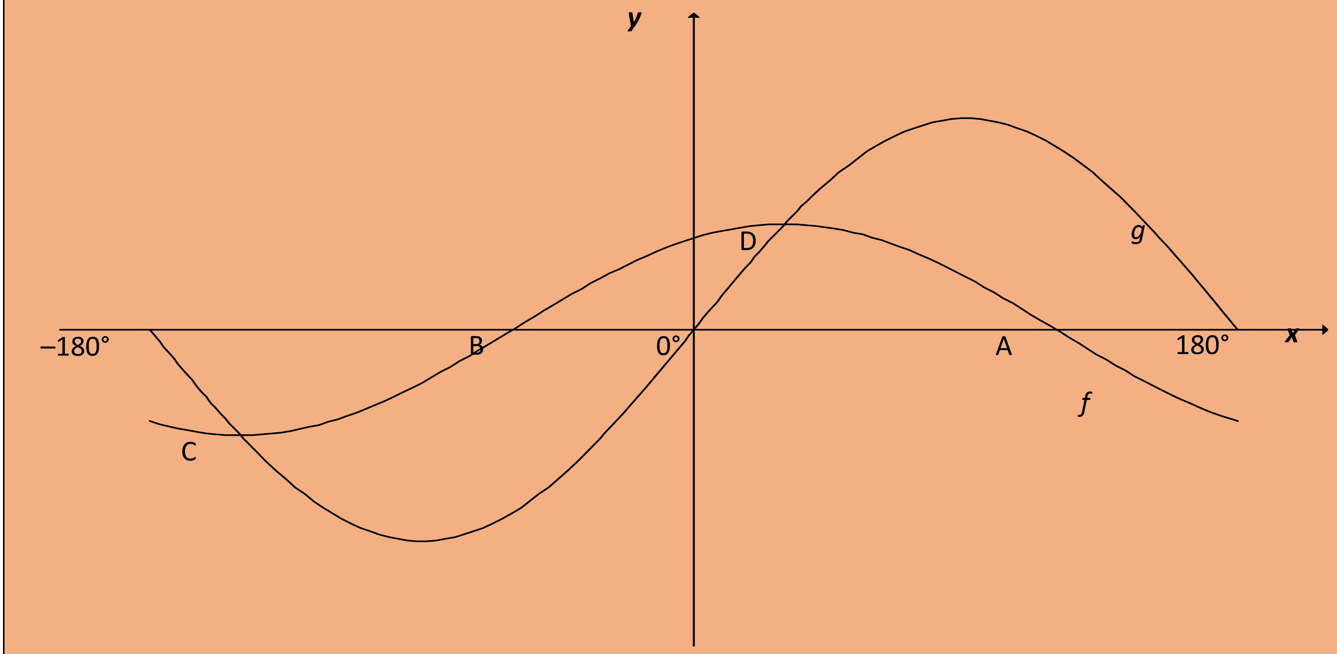
(5)

[12]

QUESTION 6

6.1 Determine the general solution of $\cos(x-30°)=2\sin x$. (6)

6.2 In the diagram, the graphs of $f(x)=\cos(x-30°)$ and $g(x)=2\sin x$ are drawn for the interval $x\in[-180°;180°]$. A and B are the x–intercepts of *f*. The two graphs intersect at C and D, the minimum and maximum turning points respectively of *f*.



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180°; 180°]$, for which:

(a) Both graphs are increasing (2)

(b) $f(x+10°) > g(x+10°)$ (2)

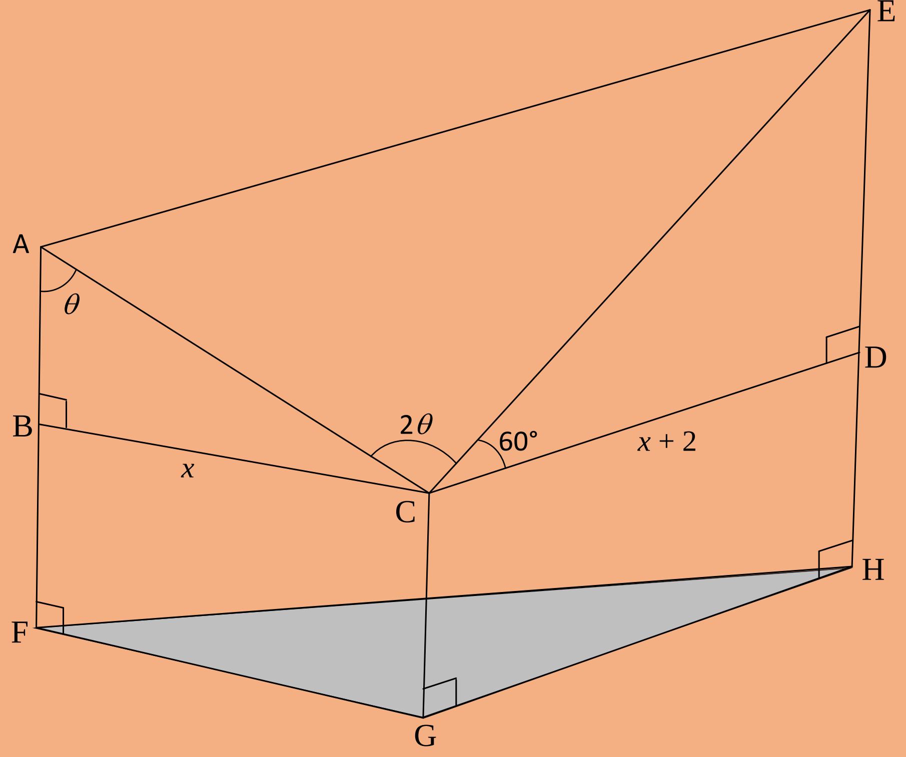
6.2.3 Determine the range of $y = 2^{2\sin x + 3}$ (5)

[18]

QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. ΔACE forms the roof of an entertainment centre.

BC = x, CD = x + 2, $B\hat{A}C = \theta$, $A\hat{C}E = 2\theta$ and $E\hat{C}D = 60°$



7.1 Calculate the length of:

7.1.1 AC in terms of x and θ (2)

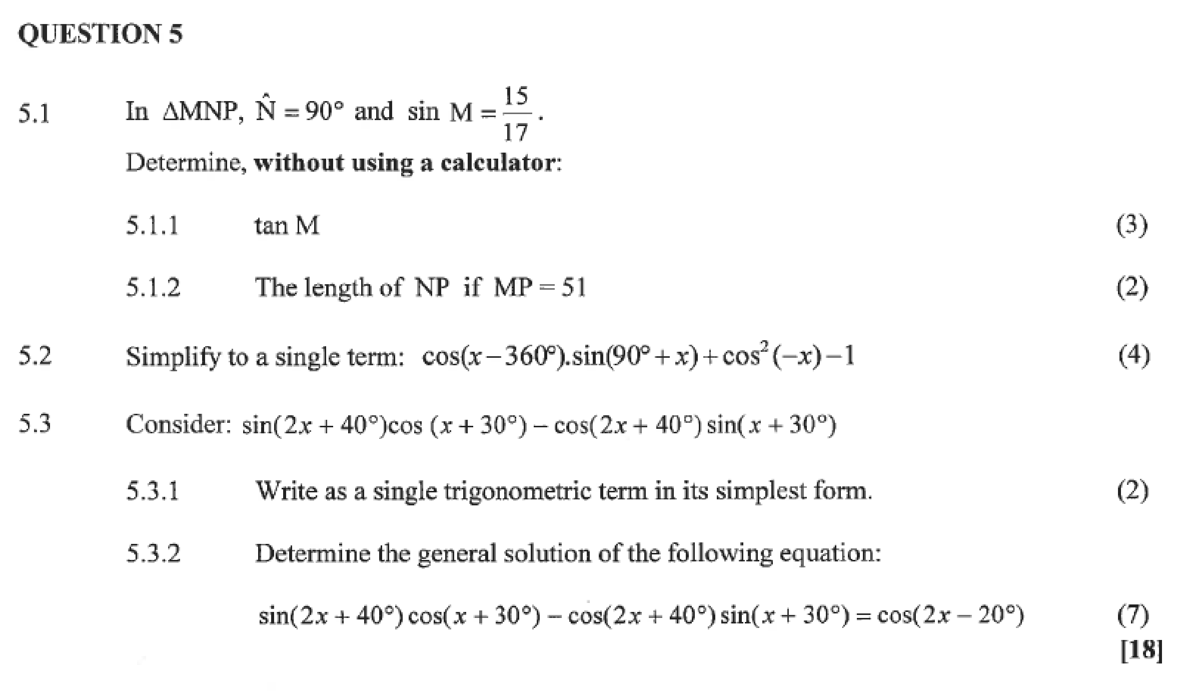
7.1.2 CE in terms of x (2)

7.2 Show that the area of the roof ΔACE is given by $2x(x+2)\cos\theta$. (3)

7.3 If $\theta = 55°$ and BC = 12 metres, calculate the length of AE. (4)

[11]

MAY / JUNE 2018



QUESTION 5

5.1 In ΔMNP, $\hat{N} = 90°$ and $\sin M = \frac{15}{17}$.

Determine, **without using a calculator**:

5.1.1 tan M (3)

5.1.2 The length of NP if MP = 51 (2)

5.2 Simplify to a single term: $\cos(x-360°).\sin(90°+x)+\cos^2(-x)-1$ (4)

5.3 Consider: $\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

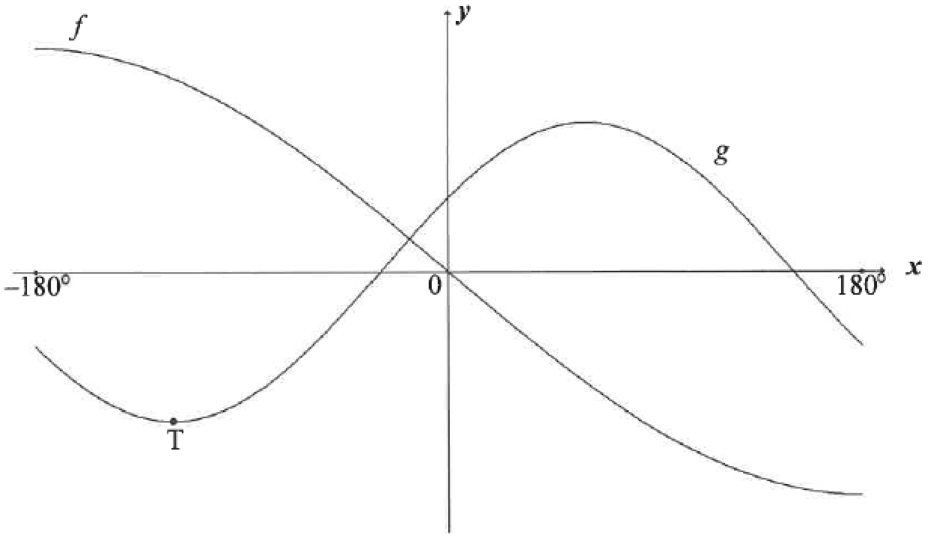
5.3.2 Determine the general solution of the following equation:

$$\sin(2x+40°)\cos(x+30°)-\cos(2x+40°)\sin(x+30°)=\cos(2x-20°)$$ (7)

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = -3\sin\frac{x}{2}$ and $g(x) = 2\cos(x - 60°)$ are drawn in the interval $x \in [-180°;180°]$. T$(p\ ;q)$ is a turning point of g with $p < 0$.



6.1 Write down the period of f. (1)

6.2 Write down the range of g. (2)

6.3 Calculate $f(p) - g(p)$. (3)

6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180°;180°]$ for which:

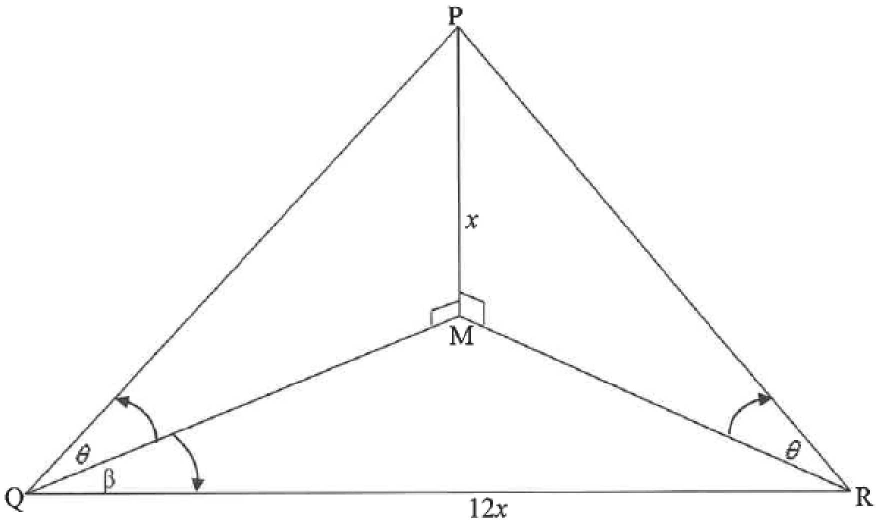
6.4.1 $g(x) > 0$ (3)

6.4.2 $g(x).g^{/}(x) > 0$ (4)

[13]

QUESTION 7

The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ. Q, M and R lie in the same horizontal plane.



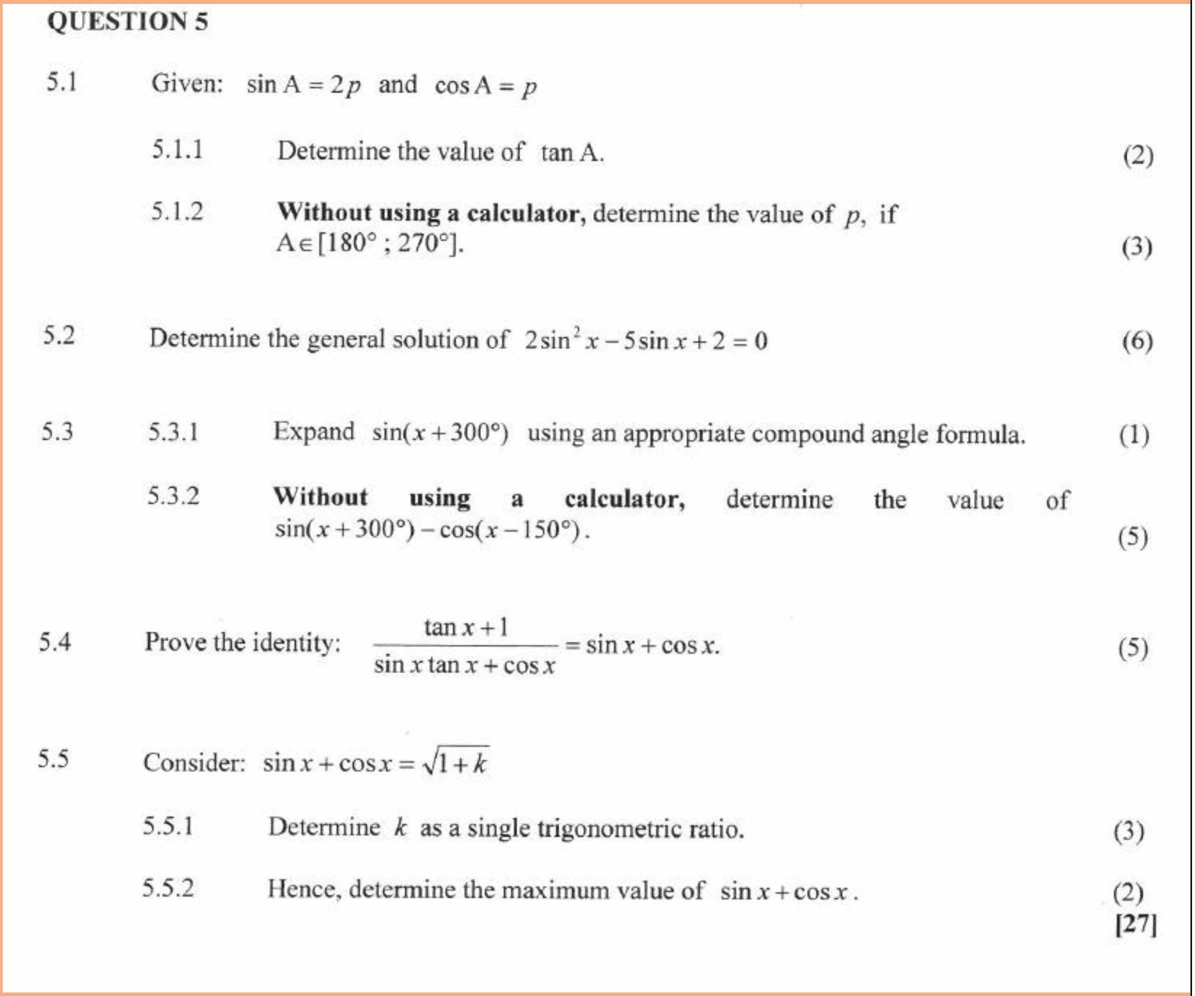
7.1 Write QM in terms of x and θ. (2)

7.2 Prove that $\tan\theta = \dfrac{\cos\beta}{6}$. (4)

7.3 If $\beta = 40°$ and QM = 60 metres, calculate the height of the lighthouse **to the nearest metre.** (3)

[9]

MAY / JUNE 2017



QUESTION 5

5.1 Given: $\sin A = 2p$ and $\cos A = p$

5.1.1 Determine the value of $\tan A$. (2)

5.1.2 **Without using a calculator,** determine the value of p, if $A \in [180° ; 270°]$. (3)

5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)

5.3 5.3.1 Expand $\sin(x + 300°)$ using an appropriate compound angle formula. (1)

5.3.2 **Without using a calculator,** determine the value of $\sin(x + 300°) - \cos(x - 150°)$. (5)

5.4 Prove the identity: $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$. (5)

5.5 Consider: $\sin x + \cos x = \sqrt{1 + k}$

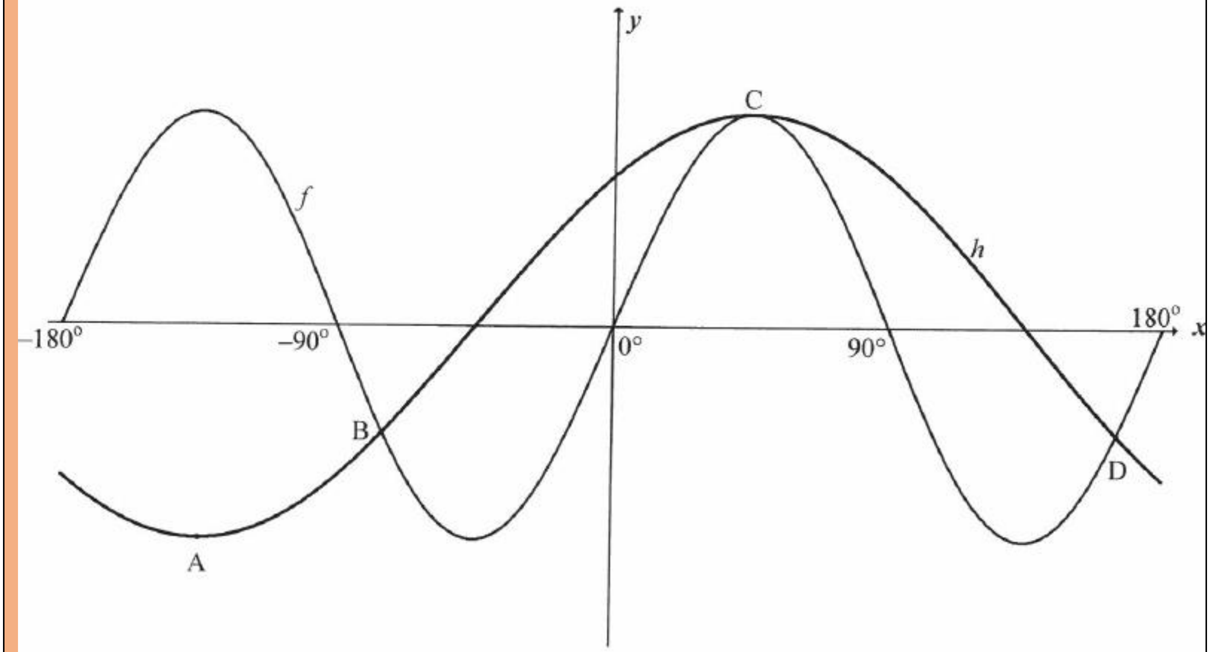
5.5.1 Determine k as a single trigonometric ratio. (3)

5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)

[27]

QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45°)$ for the interval $x \in [-180° ; 180°]$. A$(-135° ; -1)$ is a minimum point on graph h and C$(45° ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and D$\left(165°;-\frac{1}{2}\right)$.



6.1 Write down the period of f. (1)

6.2 Determine the x-coordinate of B. (1)

6.3 Use the graphs to solve $2\sin x.\cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180° ; 180°]$. Show ALL working. (4)

[6]

QUESTION 7

A rectangular box with lid ABCD is given in FIGURE (i) below. The lid is opened through 60° to position HKCD, as shown in the FIGURE (ii) below. EF = 12 cm, FG = 6 cm and BG = 8 cm.

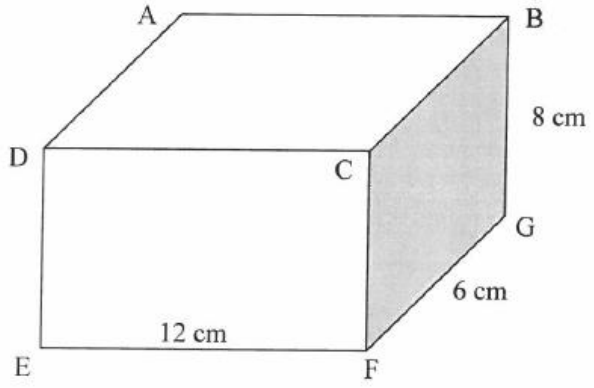


FIGURE (I)

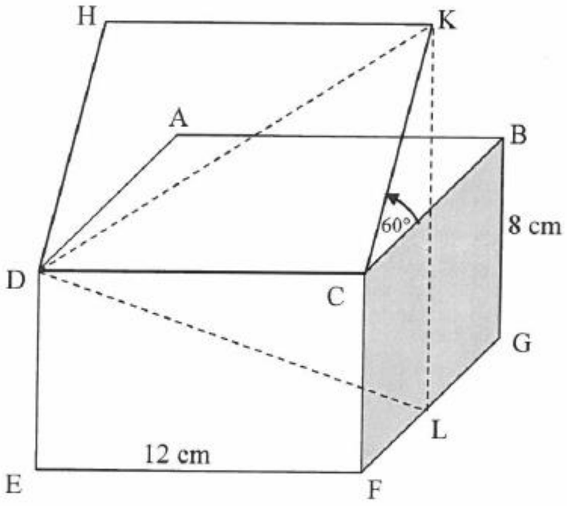


FIGURE (II)

7.1 Write down the length of KC. (1)

7.2 Determine KL, the perpendicular height of K, above the base of the box. (3)

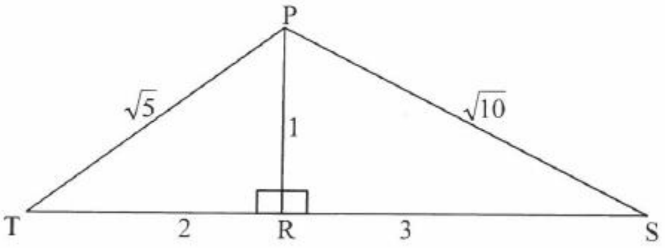
7.3 Hence, determine the value of $\frac{\sin K\hat{D}L}{\sin D\hat{L}K}$. (4)

[8]

MAY / JUNE 2016

QUESTION 5

5.1 In the diagram PR ⊥ TS in obtuse triangle PTS.
$PT = \sqrt{5}$; $TR = 2$; $PR = 1$; $PS = \sqrt{10}$ and $RS = 3$



5.1.1 Write down the value of:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

5.1.2 Calculate, WITHOUT using a calculator, the value of $\cos(\hat{T} + \hat{S})$ (5)

5.2 Determine the value of:

$$\frac{1}{\cos(360° - \theta).\sin(90° - \theta)} - \tan^2(180° + \theta)$$

(6)

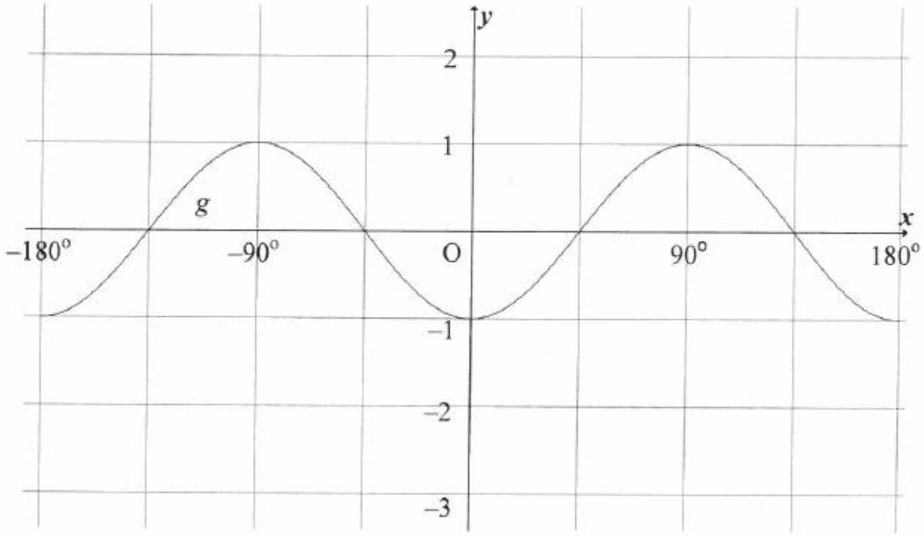
5.3 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)

[18]

QUESTION 6

6.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$ (6)

6.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$ is drawn below.



6.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180° ; 180°]$ on the set of axes provided in the ANSWER BOOK. (3)

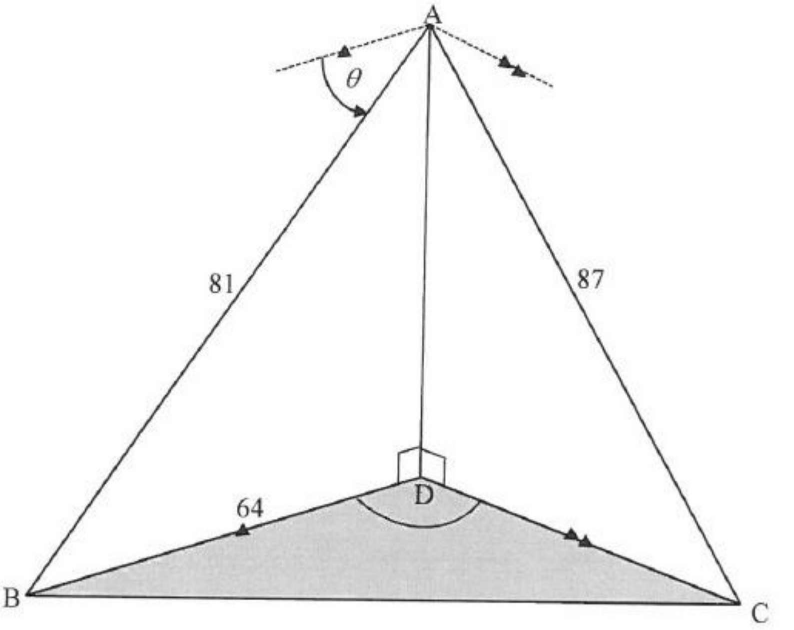
6.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180° ; 0°]$ (2)

6.2.3 Write down the value(s) of x for which $f(x+30°) - g(x+30°) = 0$ for $x \in [-180° ; 180°]$ (2)

[13]

QUESTION 7

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ. D is a point directly below A and is on the same horizontal plane as B and C. BD = 64 m, AB = 81 m and AC = 87 m.



7.1 Calculate the size of θ to the nearest degree. (3)

7.2 If it is given that $B\hat{A}C = 82,6°$, calculate BC, the distance between the boats. (3)

7.3 If $B\hat{D}C = 110°$, calculate the size of $D\hat{C}B$. (3)

[9]

MAY/ JUNE 2015

QUESTION 5

5.1 Given that $\cos \beta = -\frac{1}{\sqrt{5}}$, where $180° < \beta < 360°$.

Determine, with the aid of a sketch and without using a calculator, the value of $\sin \beta$. (5)

5.2 Determine the value of the following expression:

$$\frac{\tan(180° - x).\sin(x - 90°)}{4\sin(360° + x)}$$ (6)

5.3 If $\sin A = p$ and $\cos A = q$:

5.3.1 Write $\tan A$ in terms of p and q (1)

5.3.2 Simplify $p^4 - q^4$ to a single trigonometric ratio (4)

5.4 Consider the identity: $\frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin\theta.\cos\theta} = \tan\theta$

5.4.1 Prove the identity. (5)

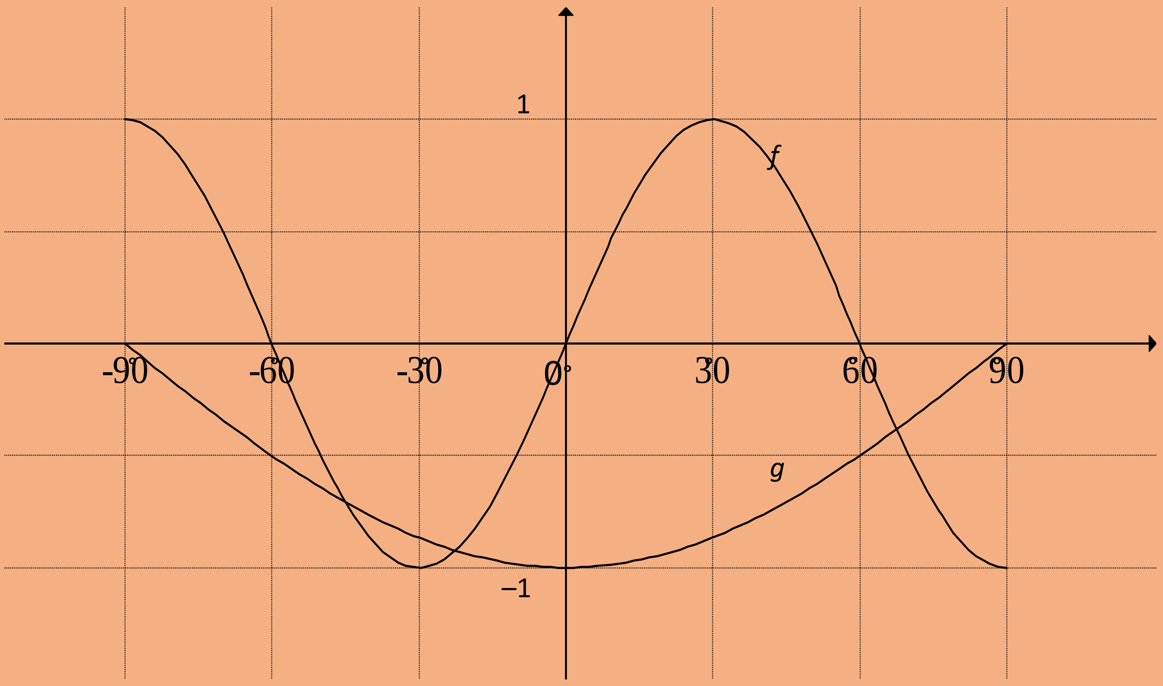
5.4.2 For which value(s) of θ in the interval $0° < \theta < 180°$ will the identity be undefined? (2)

5.5 Determine the general solution of $2 \sin 2x + 3 \sin x = 0$ (6)

[29

QUESTION 6

In the diagram below the graphs of $f(x) = \sin bx$ and $g(x) = -\cos x$ are drawn for $-90° \leq x \leq 90°$. Use the diagram to answer the following questions.



6.1 Write down the period of f. (1)

6.2 Determine the value of b. (1)

6.3 The general solutions of the equation $\sin bx = -\cos x$ are $x = 67{,}5° + k.90°$ or $x = 135° + k.180°$ where $k \in Z$.

Determine the x-values of the points of intersection of f and g for the given domain. (3)

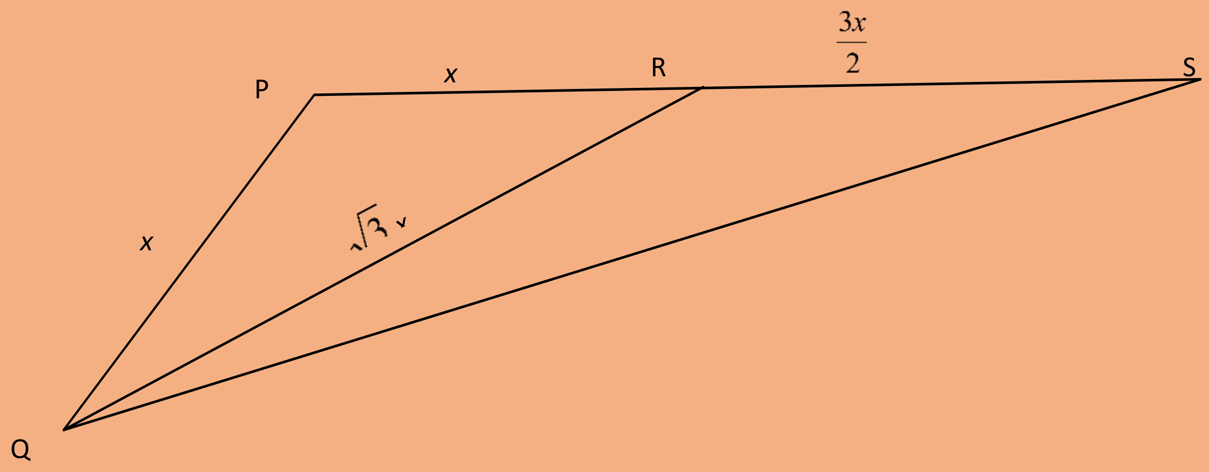
6.4 Write down the values of x for which $\sin bx + \cos x < 0$ for the given domain. (4)

[9]

QUESTION 7

Triangle PQS forms a certain area of a park. R is a point on PS and QR divides the area of the park into two triangular parts, as shown below, for a festive event.

PQ = PR = x units, RS = $\frac{3x}{2}$ units and RQ = $\sqrt{3}\,x$ units.



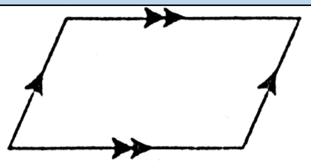
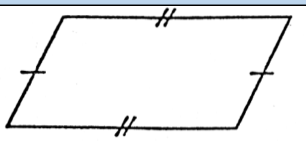
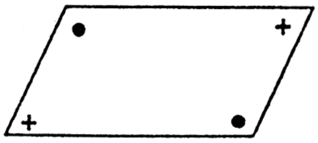
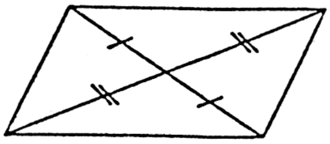
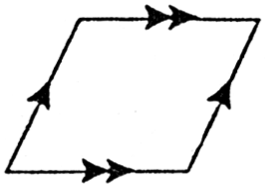
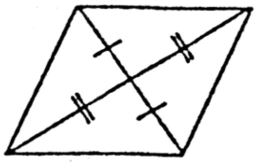
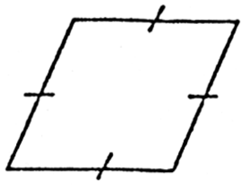
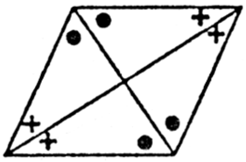
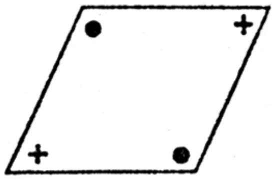
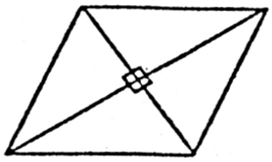
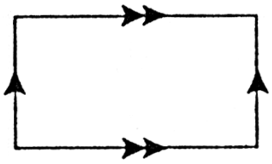
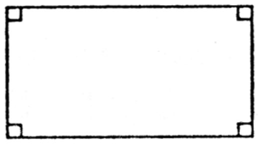
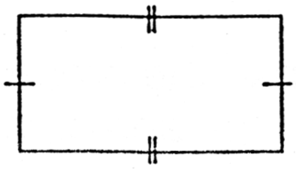
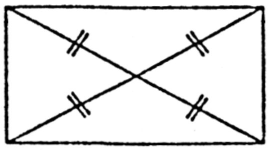
7.1 Calculate the size of $\hat{P}$. (4)

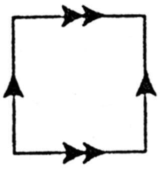
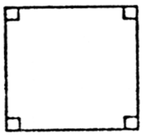
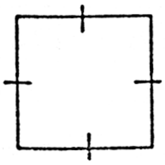
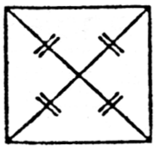
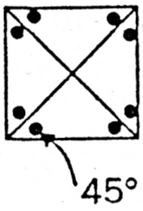
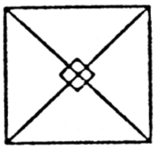
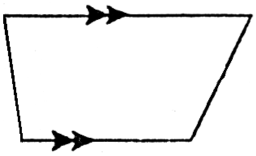
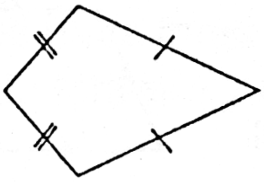
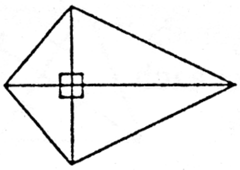
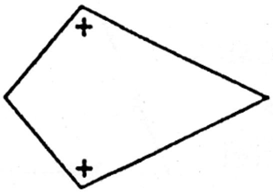
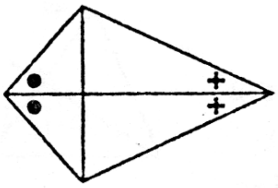
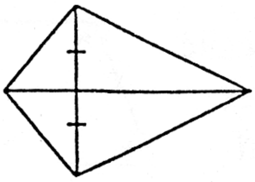
7.2 Hence, calculate the area of triangle QRS in terms of x in its simplest form. (5)

[9]

EUCLIDEAN GEOMETRY

NOTES TO USE

PROPERTIES OF THE SPECIAL QUADRIALTERALS			
1. THE PARALLELOGRAM			
Opposite sides parallel		Opposite sides equal in length	
Opposite angles equal in size		Diagonals bisect each other	
2. THE RHOMBUS			
Opposite sides parallel		Diagonals bisect each other	
All sides equal in length		Diagonals bisect corner angles	
Opposite angles equal in size		Diagonals cross at right angles	
3. THE RECTANGLE			
Opposite sides parallel		All angles equal in size (90°)	
Opposite sides equal in length		Diagonals equal in length and bisect each other	

4. THE SQUARE			
Opposite sides parallel		All angles equal in size (90°)	
All sides equal in length		Diagonals equal in length and bisect each other	
Diagonals bisect corner angles		Diagonals cross at right angles	
5. **THE TRAPEZIUM**			
One pair of parallel sides			
6. **THE KITE**			
Adjacent sides equal in length		Diagonals cross at right angles	
One pair of opposite angles equal in size		Only one pair of opposite angles is bisected	
Only one diagonal is bisected			

1. **The following proofs of theorems are examinable (NB. know them by heart)**

- ✓ The line drawn from the centre of a circle perpendicular to a chord bisects the chord; **(From Gr.11)**
- ✓ The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); **(From Gr.11)**
- ✓ The opposite angles of a cyclic quadrilateral are supplementary; **(From Gr.11)**
- ✓ The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment; **(From Gr.11)**
- ✓ that a line drawn parallel to one side of a triangle divides the other two sides proportionally; **(From Gr.12)**
- ✓ Equiangular triangles are similar. **(From Gr.12)**
- ✓

2. Corollaries derived from the theorems and axioms are necessary in solving riders:

- ✓ Angles in a semi-circle
- ✓ Equal chords subtend equal angles at the circumference
- ✓ Equal chords subtend equal angles at the centre
- ✓ In equal circles, equal chords subtend equal angles at the circumference
- ✓ In equal circles, equal chords subtend equal angles at the centre.
- ✓ The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
- ✓ If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
- ✓ Tangents drawn from a common point outside the circle are equal in length.

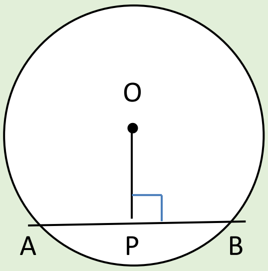
NB: The theory of quadrilaterals will be integrated into questions in the examination

CIRCLE GEOMETRY

GROUP 1: CENTRE THEOREMS

THEOREM 1A

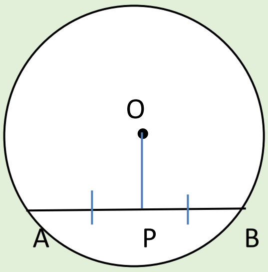
The line drawn from the centre of the circle perpendicular to the chord bisect the chord



Given: $OP \perp AB$
RTP: AP = PB

CONVERSE OF THEOREM 1A

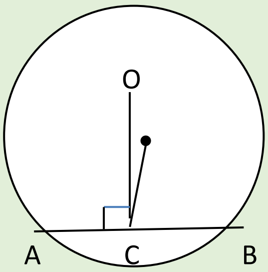
The line drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord.



Given: AP = PB
RTP: $OP \perp AB$

THEOREM 1B

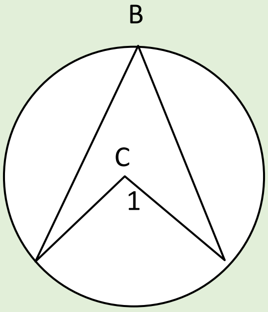
The perpendicular bisector of a chord passes through the centre of the circle.

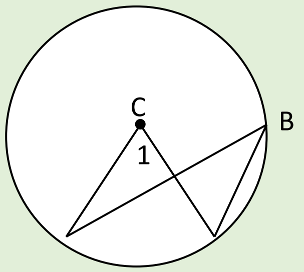


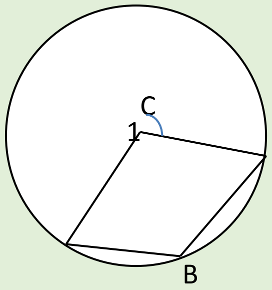
Given: $OC \perp AB$, AC = CB
RTP: OC passes through the centre of the circle.

THEOREM 2

The angle subtended by an arc at the centre of the circle is twice at the angle subtended by the same arc at the circumference of the circle.



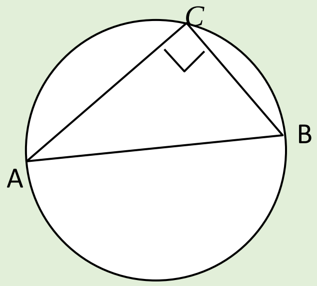




RTP: $\hat{C}_1 = 2B$

THEOREM 3

The angle in the semi-circle is a right angle

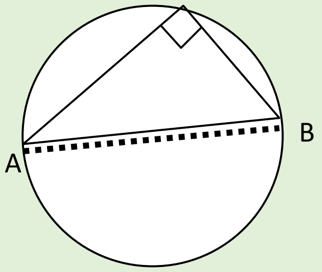


Given: Diameter AB

RTP: $\hat{C} = 90\circ$

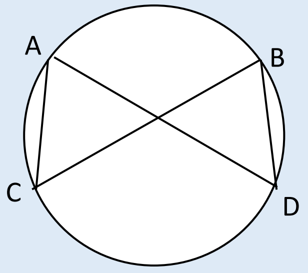
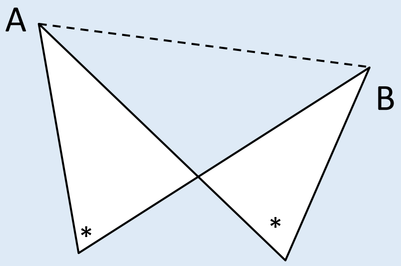
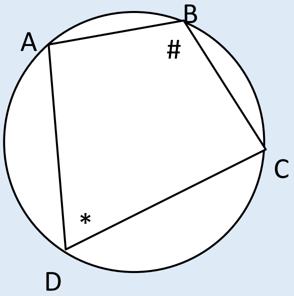
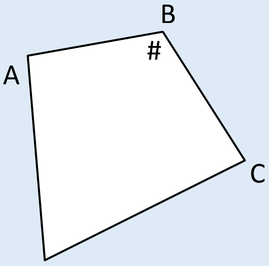
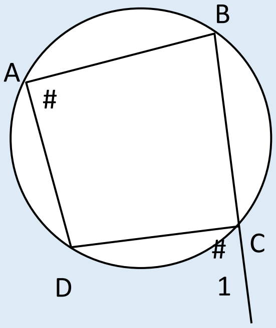
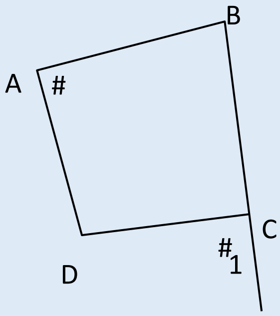
CONVERSE OF THEOREM 3

If the chord of a circle subtends a right angle at a point on the circumference then the chord is a diameter of the circle.



RTP : AB is a diameter

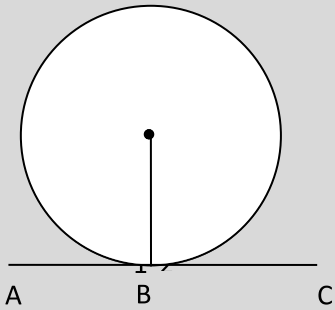
GROUP 2: CYCLIC QUADRLATERAL THEOREMS

THEOREM 4 An arc or a chord of a circle subtends equal angles at the circumference of the circle. (We say angles in the same segment are equal.)  Given: Circle centre O RTP: $\hat{A} = \hat{B}$ and $\hat{C} = \hat{D}$	**CONVERSE OF THEOREM 4** If two equal angles are subtended by the same line, then the four points are concyclic (lie on the circumference).  RTP: ABCD lie on the circle
THEOREM 5 The opposite angles of a cyclic quadrilateral are supplementary  Given: ABCD lie on the circumference of the circle RTP: $\hat{B} + \hat{D} = 180^\circ$	**CONVERSE OF THEOREM 5** If a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic  Given: $\hat{B} + \hat{D} = 180^\circ$ RTP: ABCD is a cyclic quad
THEOREM 6 An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.  Given: ABCD are points that lie on the circumference of the circle RTP: $\hat{C}_1 = \hat{A}$	**CONVERSE OF THEOREM 6** If an exterior angle of a quadrilateral is equal to the interior opposite angle, then the quadrilateral is cyclic.  If $\hat{C}_1 = \hat{A}$ RTP: then ABCD is a cyclic quadrilateral

GROUP 3 TANGENTS THEOREM

THEOREM 7

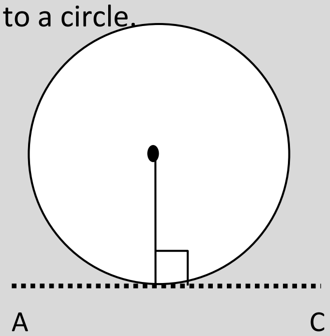
A tangent to a circle is perpendicular to the radius at the point of contact



If AC is a tangent to the circle at B then the radius OB ⊥ AC, i.e. $\hat{B}_1 = \hat{B}_2 = 90^\circ$

CONVERSE OF THEOREM 7

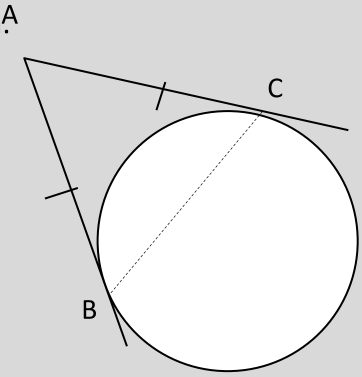
If a line is drawn perpendicular to the radius at a point of contact, then the line is a tangent to a circle.



RTP: AC is a tangent.

THEOREM 8

Two tangents drawn to a circle from the same point outside the circle are equal in length.

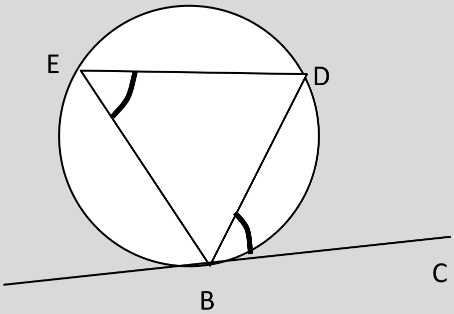


Given AB and AC as tangents

RTP:AB = AC

THEOREM 9

The angle between a chord and a tangent to a circle at a point of contact is equal to the angle in the alternate segment.

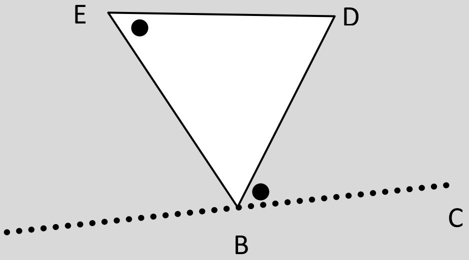


Given : ABC as tangent at B

RTP: $D\hat{B}C = \hat{E}$

CONVERSE OF THEOREM 9

If an angle between a chord and line at the point of contact is equal to the angle in the alternate segment then the line is a tangent.

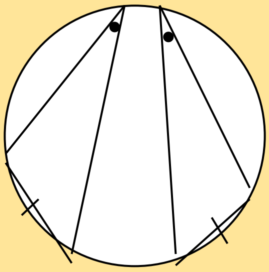


Given : : $D\hat{B}C = \hat{E}$

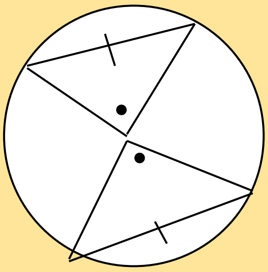
RTP:ABC is a tangent

COROLLARIES ON THEOREM 4:

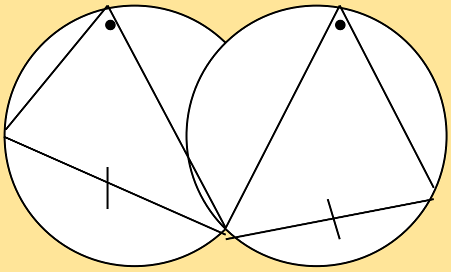
1. Equal chords subtend equal angles at the circumference



2. Equal chords subtend equal angles at the center.



3. Equal chords of equal circles subtend equal angles at the circumference



PROPORTION

REMEMBER THAT:

- $\frac{a}{b} = \frac{c}{d}$ is a proportion statement written in **Fractional** form.
- The statement can also be written $ad = bc$ in **Product** form or
- $a : b = c : d$ in **Colon** form the statement is read : *a* is to *b* as *c* is to *d*
- In $\frac{a}{b} = \frac{b}{c}$ *a* is the 1^{st}, *c* is the 3^{rd} proportional and *b* the mean proportional. The mean proportion in product form is $b^2 = ac$
- If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}, \frac{a}{c} = \frac{b}{d}, \frac{a+b}{b} = \frac{c+d}{d} (\frac{a}{b} + 1 = \frac{c}{d} + 1)$ and $\frac{a-b}{b} = \frac{c-d}{d}$

SIMILARITY

A polygon is a closed figure with three or more sides. In a polygon there are as many vertices as there are sides. Triangles and quadrilaterals are three- and four-sided polygons respectively.

Two polygons, having the same number of sides are similar when:

1. All the pairs of corresponding angles are equal

AND

2. All pairs of corresponding sides are in the same proportion

This means that

a) Given 1 and 2, then the polygons are similar

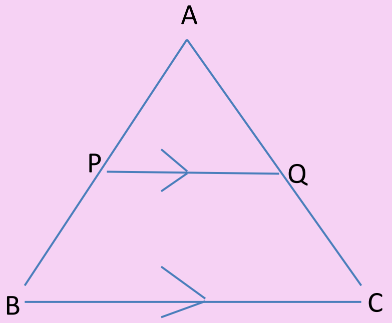
b) Given that two polygons are similar then 1 and 2 are both true

NB: BOTH CONDITIONS MUST HOLD AT THE SAME TIME

1. PROPORTIONALITY

Theorem

A line drawn parallel to one side of a triangle divides the other two sides in the same proportion.

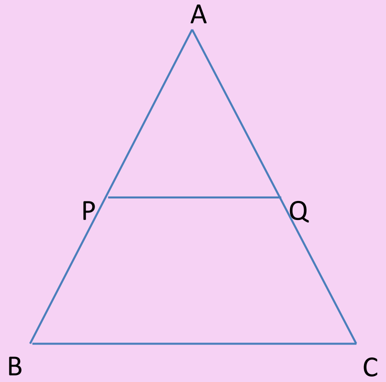


Given: Triangle ΔABC with PQ|| $\boldsymbol{AB}$

RTP: $\frac{AP}{PB} = \frac{AQ}{QC}$

Converse Theorem

A line dividing two sides of a triangle proportionally is parallel to the third side.

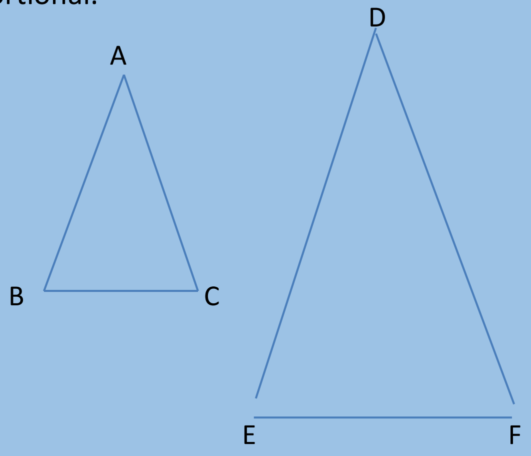


Given: ΔABC with $\frac{AP}{PB} = \frac{AQ}{QC}$

RTP: PQ||BC

2. SIMILARITY

If two triangles are equiangular to one another the lengths of their corresponding sides are proportional.

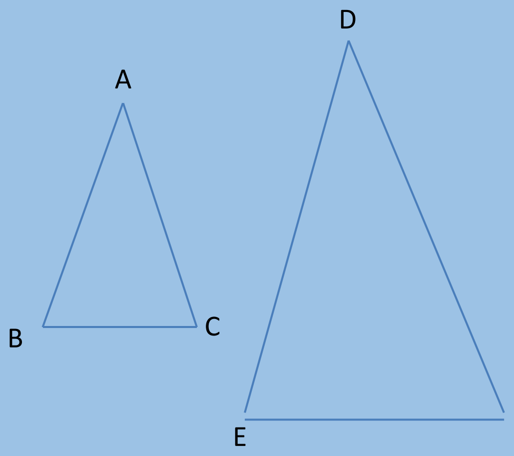


Given: ΔABC and ΔDEF with $\hat{A}=\hat{D},\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$

RTP: $\frac{AD}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

Converse Theorem:

If the corresponding sides of two triangles are proportional then their corresponding angles are equal



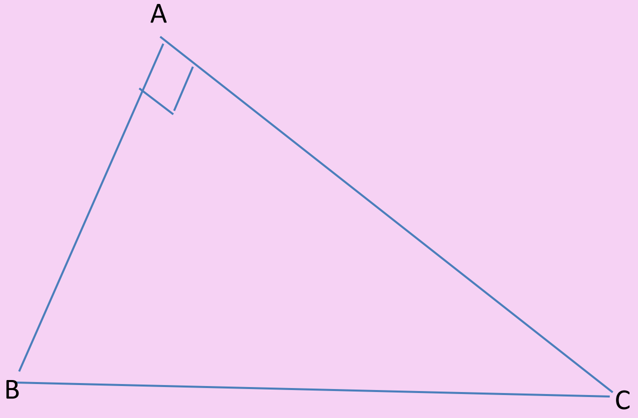
Given: ΔABC and ΔDEF with $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

RTP: $\hat{A}=\hat{D},\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$

3. PYTHAGORAS

Theorem

In a right-angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. (Pyth)

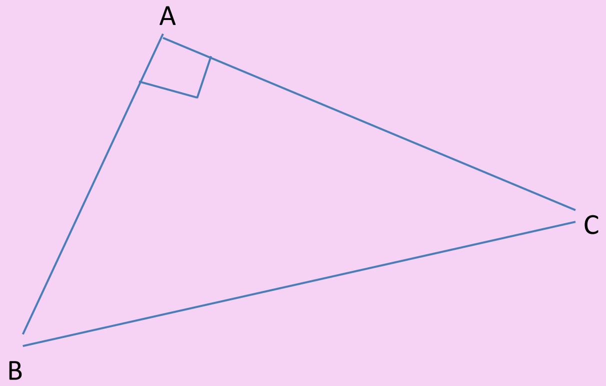


Given: ΔABC with $\hat{A}=90^{\circ}$

RTP: $BC^{2}=AB^{2}+AC^{2}$

Converse Theorem

If the square of one side of triangle equals the sum of the squares of the other two sides, then the angle contained by these two sides is a right angle.



Given: If in ΔABC , $BC^{2}=AB^{2}+AC^{2}$

RTP: $\hat{A}=90^{\circ}$

TECHNICAL REPORT

Common errors and misconceptions

Learners writing incorrect reasons or naming angles incorrectly.

Learners struggling to differentiate between quadrilateral and a cyclic quadrilateral.

Learners assuming certaing diagrams and information which is not correct e.g assuming that and angle is $90°$ by just looking at a diagram.

Learners cannot recognise that angles can be named using three alphabets or one alphabet e.g $\hat{Q}_2$ or $P\widehat{QW}$.

Differentiating between alternate angles and corresponding angle is still a challenge.

When proving using converse theorems learners still write the incorrect reasons.

Linking of statements is still a challenge.

Learners not showing or writing a construction when proving a theorem.

Learners writing incorrect proportions.

Suggestions for improvement

Learners should be encouraged to scrutinise the given information and the diagram for clues about which theorems could be used in answering the question.

Teachers must cover the basic work thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram.

Learners are encouraged to use the list of reasons provided in the Examination Guidelines

Teachers to insist that learners name the angles correctly. The fact that learners are naming angles incorrectly in Grade 12 level indicates that this issue has not been dealt with effectively in earlier grades.

Learners should be taught that all statements should be accompanied by reasons. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternating angles are equal, the sum of co – interior angles is 180° or when stating the proportional intercept theorem.

Learner should be **given exercises where the converse of the theorems are used** on solving questions.

Learners should **be taught that all four vertices of a quadrilateral** should lie on the same circumference of a circle to be a cyclic quadrilateral.

Learners should be **discouraged from writing correct statements that are not** related to the solution .No marks are awarded for statements that do not lead to solving the question.

Learners should be forced to use acceptable reasons in Euclidean Geometry. Teachers should explain the **difference between a theorem and its converse** .They should also explain the conditions for which theorems are applicable and when the converse will apply.

Learners should be told that success in answering Euclidean Geometry comes from **regular practice** , starting off with easy and progressing to the difficult.

More time needs to be spent on the teaching of Euclidean Geometry in all grades

Learners need to be told that there is **no short – cut to mastering the skills** required in answering questions on Euclidean Geometry. This requires continuous and deliberate practise.

Learners should be taught **to refrain from assumptions**.

Learners to be **exposed to question** in Euclidean Geometry that includes theorems and converses.

HINTS FOR LEARNERS WHEN ANSWERING A RIDER

Some important points about proof in Geometry

1. When solving a geometry rider, one has to have a thorough understanding of the theorems, their conditions and the respective conclusions
2. Read the problem carefully for understanding. You may need to underline important points and make sure you understand each term in the given and conclusion.
3. **There are certain keywords when reading the information** supplied with the rider that should trigger concept images of the geometric tools that you will use
4. **Draw the sketch if it is not already drawn**. The sketch need not be accurately drawn but must as close as possible to what is given i.e. lines and angles which are equal must look equal or must appear parallel etc. Also indicate further observations based on previous theorems.
5. Indicate on the figure drawn or **given** all the equal lines and angles, lines which are parallel, drawing in circles, measures of angles given if not already indicated in the question. It might be more helpful to have a variety of colour pens or highlighters for this purpose.
6. Usually you can see the conclusion before you actually start your **formal proof** of a rider. Don't forget to write the reason for each important statement you make, quoting in brief the theorem or another result as you proceed.
7. Sometimes you may need to work backwards, asking yourself what I need to show to prove this **conclusion** (required to be proved) and then see if you can prove that as you reverse.
8. As you read this information, **pause**, **fill in the *translated* information onto your diagram**, and then continue to read, doing the same until you have used all the information that you were supplied with. If you have done this effectively, you will soon *see* questions that are possible to ask. By the time you get to the actual questions, all the answers should be on your diagram.

9. Each paragraph that accompanies a rider has information that needs to be *translated* and indicated on the accompanying diagram using a **simple symbol** system to represent **equal entities**. Once these have been placed on the diagram, it is time to look for the disclosed (but not necessarily mentioned) information. These are usually cyclic quadrilaterals that are present in the figure, and are usually not stated in the information paragraph. Remember, that **four points on a circle determines a cyclic quadrilateral**, just as **three points determines a cyclic triangle**

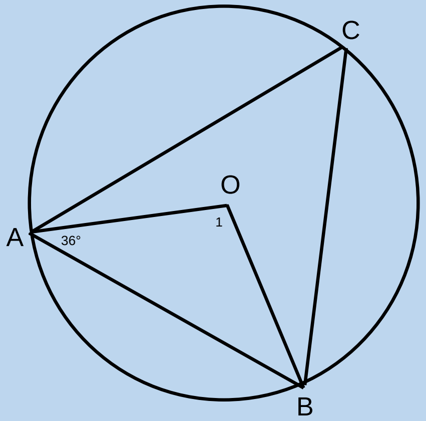
EXAMPLES ON (DIAGRAM ANALYSIS)

EXAMPLE 1

A, B and C are points on the circumference of circle O. $\hat{A}_1 = 36°$. Calculate $\hat{O}_1$ and $\hat{C}$

DIAGRAM ANALYLIS
Key Word: Centre O.

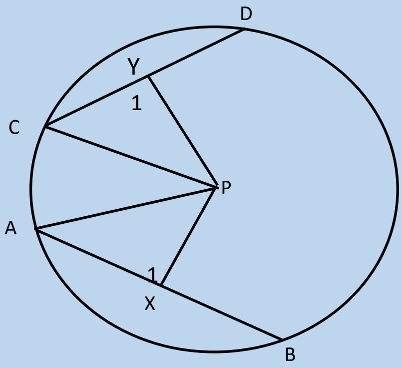
Reasoning: O is an angle at the centre. Support theorems are *radii are equal*, *sum of the angles of a triangle equal to 180º*. Which circle theorem is applicable?



R.T.P	STATEMENT	REASON
(a) $\hat{O}_1$=?	$\hat{O}_1 + \hat{A}_1 + \hat{B}_1 = 180^\circ$ $But\ \hat{B}_1 = \hat{A}_1 = 36^\circ$ $\hat{O}_1 + 36^\circ + 36^\circ = 180^\circ$ $\hat{O}_1 = 108^\circ$	…. ….
(b) $\hat{C}$ =?	$\hat{C} = \frac{1}{2}\hat{O}_1 = \frac{1}{2}\left(108^\circ\right) = 54^\circ$	∠ at centre = 2< at circle

EXAMPLE 2

AB and CD are equal chords of ⊙ P. PX ⊥ AB and PY ⊥ CD.



Prove that PX = PY

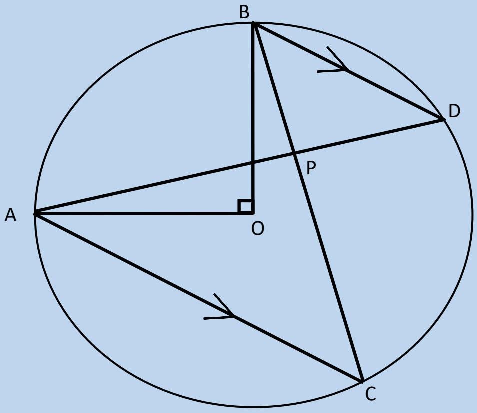
Reasoning: We start with the **R.T.P.**: PX and PY is in two different triangles. In circles where the centre and the chords are given with perpendicular lines from the centre we know that the auxiliary theorems are congruency, Pythagoras and that radii are equal. Pythagoras is not an option because no lengths are given. Congruency looks viable as PX and PY are in two different triangles that may be congruent. What do we know about perpendicular lines from the centre to the chord?

R.T.P.	**STATEMENT**	**REASON**
PX = PY.	AX = BX	….
	CY = DY But AB = CD $\frac{1}{2}AB = \frac{1}{2}CD$ $\therefore$ AX = CY In Δ‘s APX and CPY $AX = CY$ $CP = AP$ $X_1 = \hat{Y}_1$ $\therefore \quad \Delta APX \equiv \Delta CPY$ $\therefore \quad PX = PY$	…. …. …. …. …. …. ….

EXAMPLE 3

AO and BO are two radii of ⊙ O such that $\hat{O} = 90^\circ$ .AC and BD are two parallel chords. AD and BC intersect in P.

Prove that (a) AP = CP
(b) AC is the diameter of circle APC.



Reasoning: Auxiliary theorem: parallel lines. When will AP be equal to CP? (isosceles triangles). Which circle theorems are applicable in (a) and (b)? When will a line segment be a diameter?

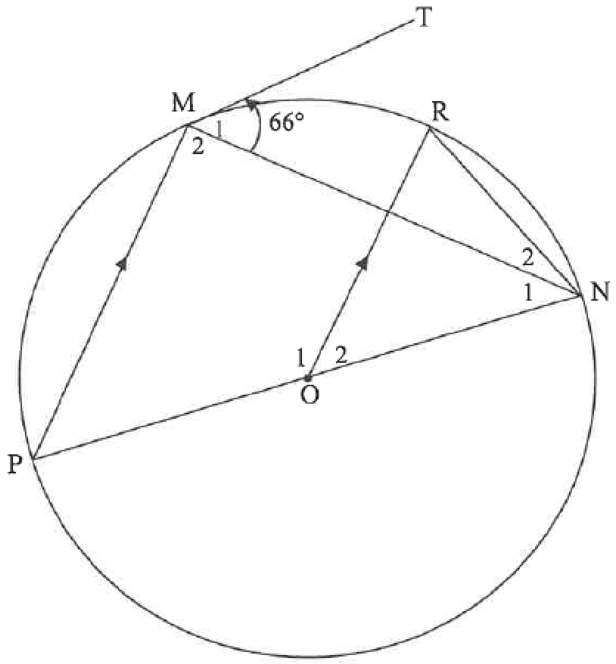
R.T.P	**STATEMENT/**	**REASON**
(a) AP=CP	$\hat{C} = \hat{D}$ $\hat{A} = \hat{D}$ $\therefore \hat{C} = \hat{A}.$ $\therefore AP = CP$	…. …. ....
(b) AC is the diameter of circle APC	$\hat{O} = 90^\circ$ $\frac{1}{2}\hat{O} = \hat{C} = 45^\circ$ $= \hat{A}$ $\therefore \hat{C} + \hat{A} = 90^\circ$ $\therefore A\hat{P}C = 90^\circ$ AC is the diameter of circle APC	.... ….

TYPICAL EXAM QUESTIONS

FEB/MARCH 2018

QUESTION 8

8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

8.1.1 $\hat{P}$ (2)

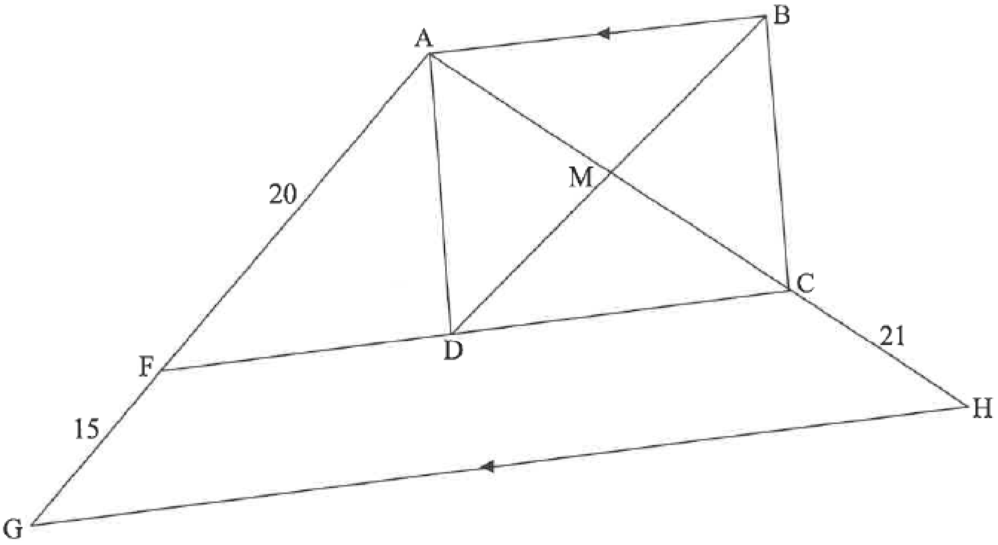
8.1.2 $\hat{M}_2$ (2)

8.1.3 $\hat{N}_1$ (1)

8.1.4 $\hat{O}_2$ (2)

8.1.5 $\hat{N}_2$ (3)

8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.



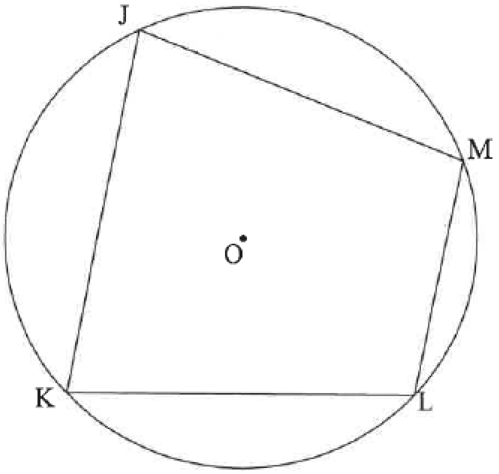
8.2.1 Explain why FC || GH. (1)

8.2.2 Calculate, with reasons, the length of DM. (5)

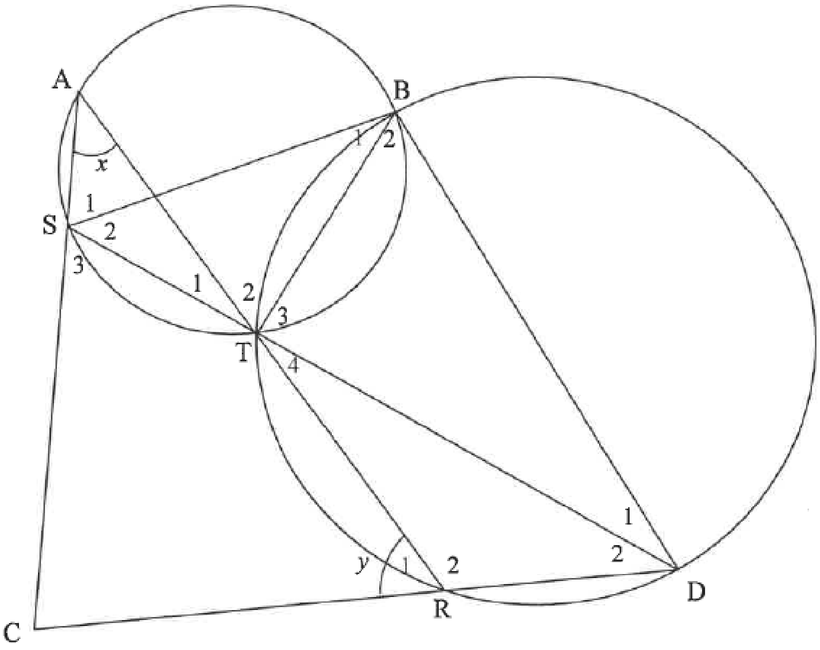
[16]

QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O. Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)

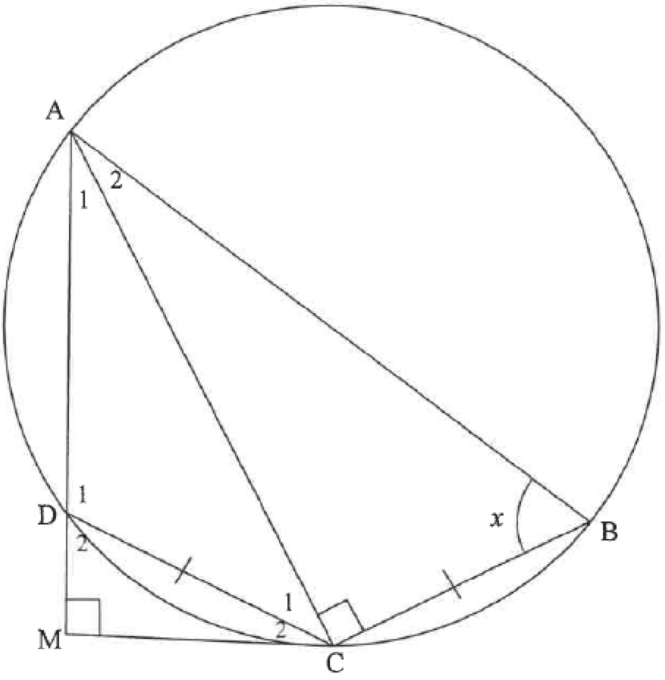
9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)

[16]

QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 ΔACB ||| ΔCMD (3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

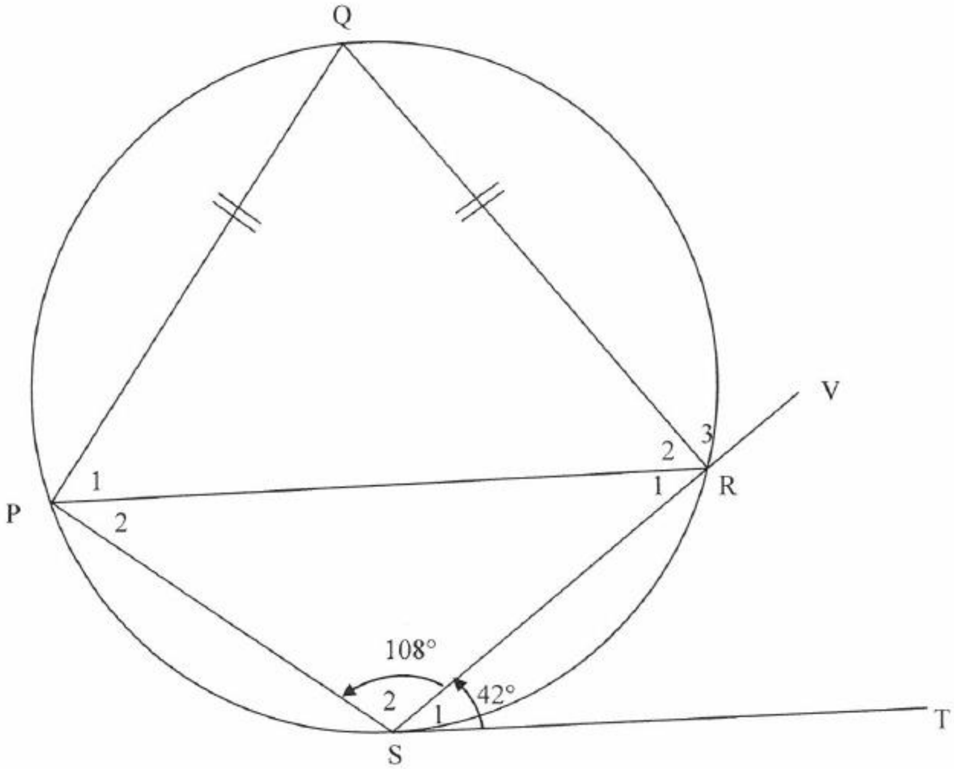
10.2.2 $\frac{AM}{AB} = \sin^2 x$ (2)

[16]

FEB/ MARCH 2017

QUESTION 8

In the diagram, PQRS is a cyclic quadrilateral. ST is a tangent to the circle at S and chord SR is produced to V. PQ = QR, $\hat{S}_1 = 42°$ and $\hat{S}_2 = 108°$.



Determine, with reasons, the size of the following angles:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

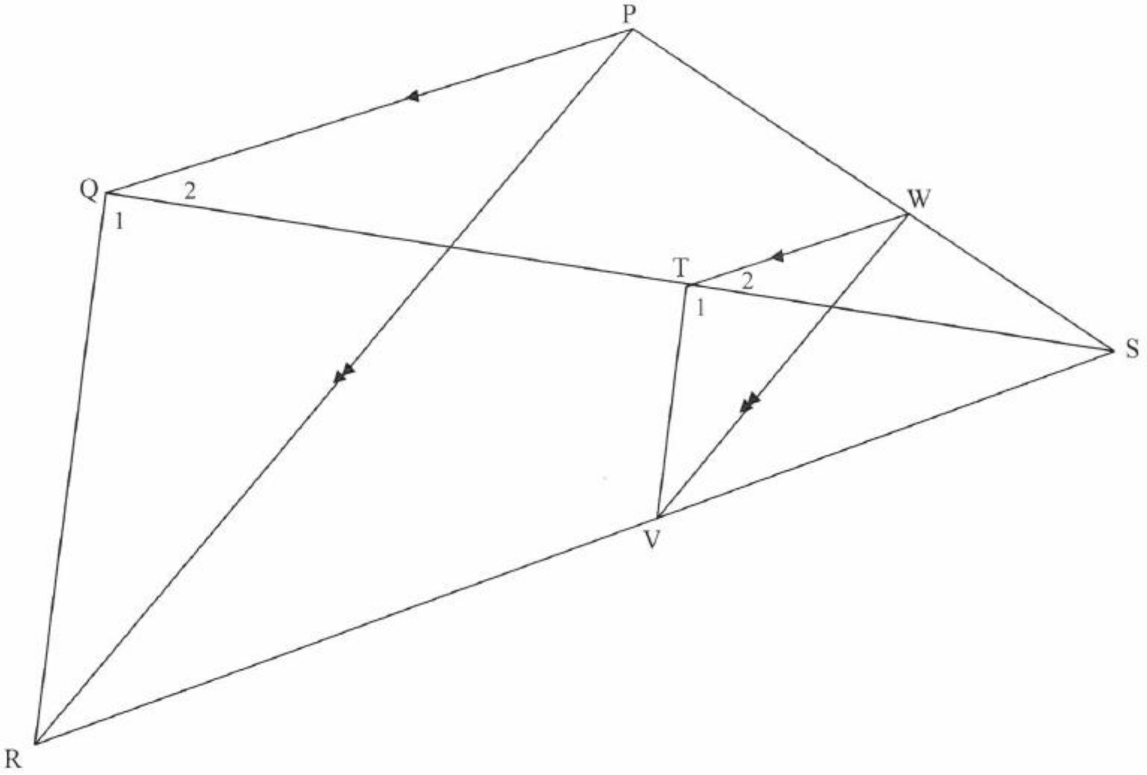
8.3 $\hat{P}_2$ (2)

8.4 $\hat{R}_3$ (2)

[8]

QUESTION 9

In the diagram, PQRS is a quadrilateral with diagonals PR and QS drawn. W is a point on PS. WT is parallel to PQ with T on QS. WV is parallel to PR with V on RS. TV is drawn. PW : WS = 3 : 2.



9.1 Write down the value of the following ratios:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

9.2 Prove that $\hat{T}_1 = \hat{Q}_1$. (4)

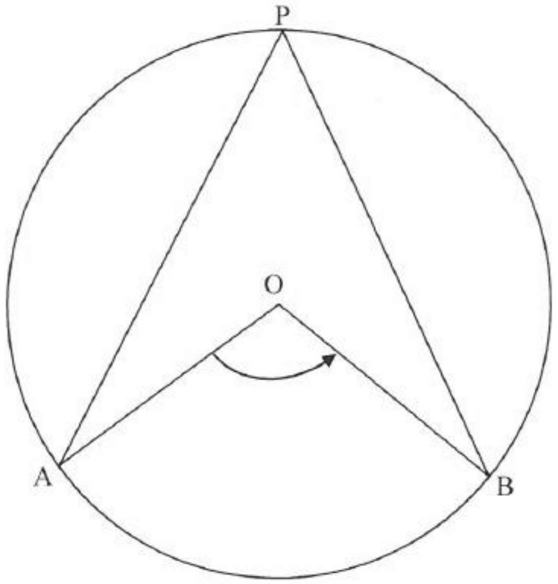
9.3 Complete the following statement: $\Delta VWS \mathbin{|||} \Delta \ldots$ (1)

9.4 Determine WV : PR. (2)

[10]

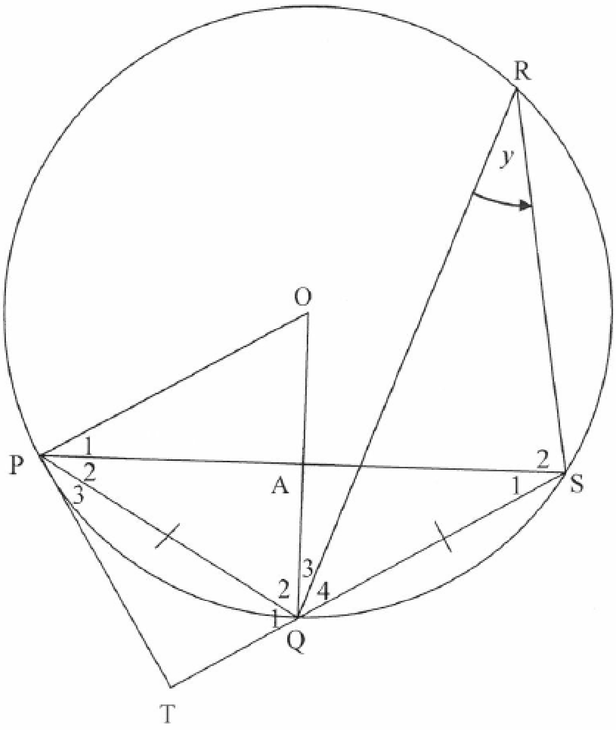
QUESTION 10

10.1 In the diagram, O is the centre of the circle and P is a point on the circumference of the circle. Arc AB subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the circumference of the circle.



Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$. (5)

10.2 In the diagram, O is the centre of the circle and P, Q, S and R are points on the circle. PQ = QS and $Q\hat{R}S = y$. The tangent at P meets SQ produced at T. OQ intersects PS at A.



10.2.1 Give a reason why $\hat{P}_2 = y$. (1)

10.2.2 Prove that PQ bisects $T\hat{P}S$. (4)

10.2.3 Determine $P\hat{O}Q$ in terms of y. (2)

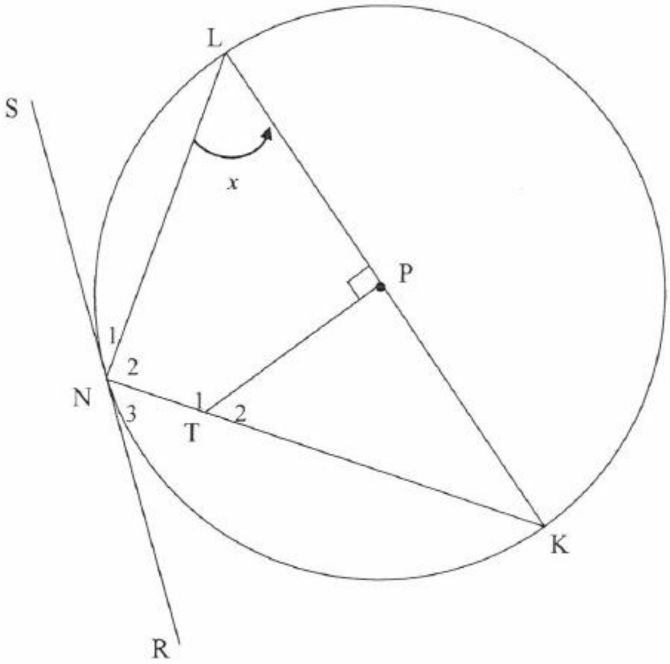
10.2.4 Prove that PT is a tangent to the circle that passes through points P, O and A. (2)

10.2.5 Prove that $O\hat{A}P = 90°$. (5)

[19]

QUESTION 11

In the diagram, LK is a diameter of the circle with centre P. RNS is a tangent to the circle at N. T is a point on NK and $TP \perp KL$. $P\hat{L}N = x$.



11.1 Prove that TPLN is a cyclic quadrilateral. (3)

11.2 Determine, giving reasons, the size of $\hat{N}_1$ in terms of x. (3)

11.3 Prove that:

11.3.1 $\Delta KTP \parallel\!| \Delta KLN$ (3)

11.3.2 $KT \cdot KN = 2KT^2 - 2TP^2$ (5)

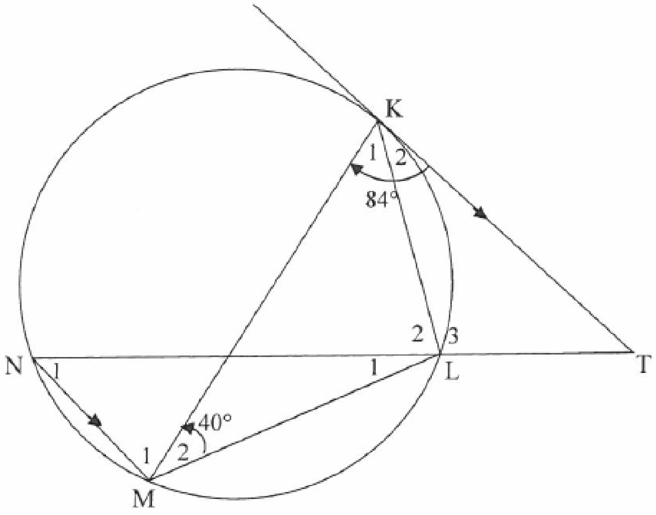
[14]

TOTAL: 150

FEB/ MARCH 2016

QUESTION 8

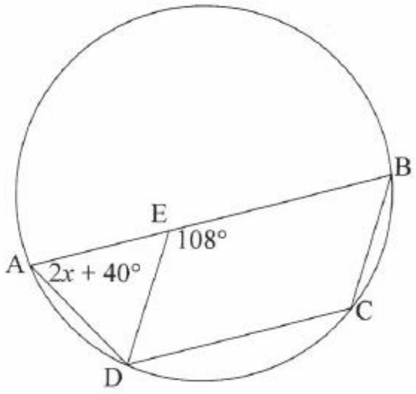
8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1	$\hat{K}_2$	(2)
8.1.2	$\hat{N}_1$	(3)
8.1.3	$\hat{T}$	(2)
8.1.4	$\hat{L}_2$	(2)
8.1.5	$\hat{L}_1$	(1)

8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.

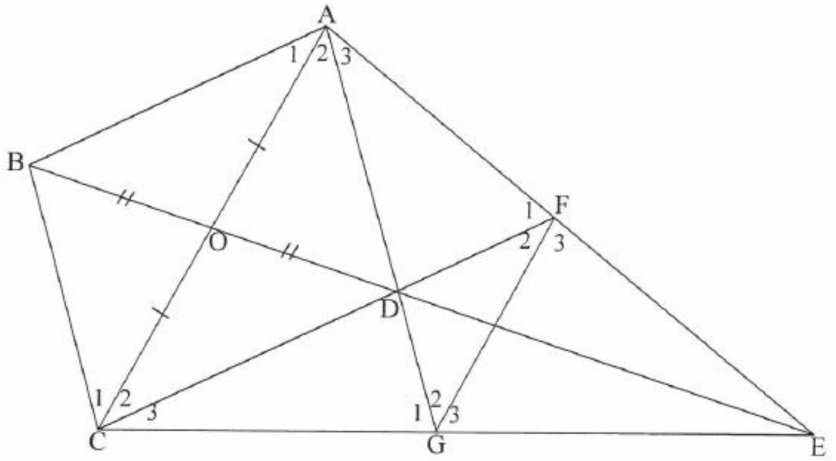


Calculate, giving reasons, the value of x. (5)

[15]

QUESTION 9

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that $BO = OD$. AD and CD produced meet EC and EA at G and F respectively.



9.1 Give a reason why ABCD is a parallelogram. (1)

9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)

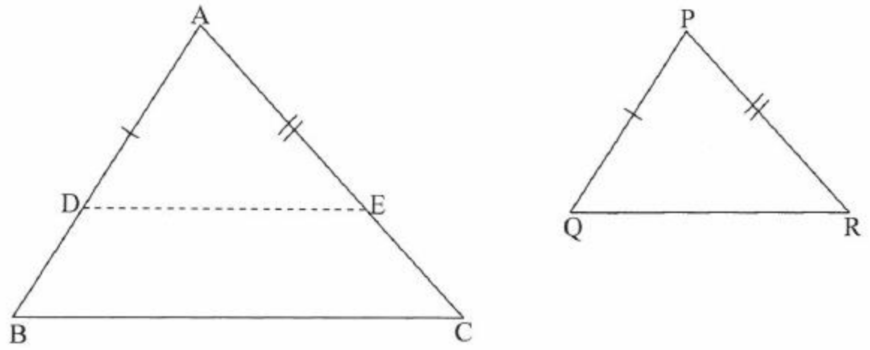
9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)

9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)

[13]

QUESTION 10

10.1 In the diagram below, ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



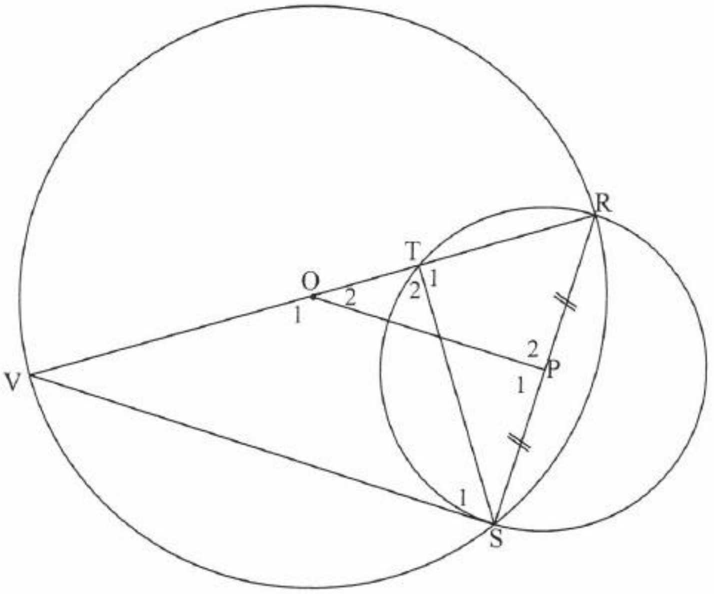
DE is drawn such that $AD = PQ$ and $AE = PR$.

10.1.1 Prove that $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Prove that $DE \parallel BC$. (3)

10.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1 Why is $OP \perp PS$? (1)

10.2.2 Prove that $\Delta ROP ||| \Delta RVS$. (4)

10.2.3 Prove that $\Delta RVS ||| \Delta RST$. (3)

10.2.4 Prove that $ST^2 = VT . TR$. (6)

[21]

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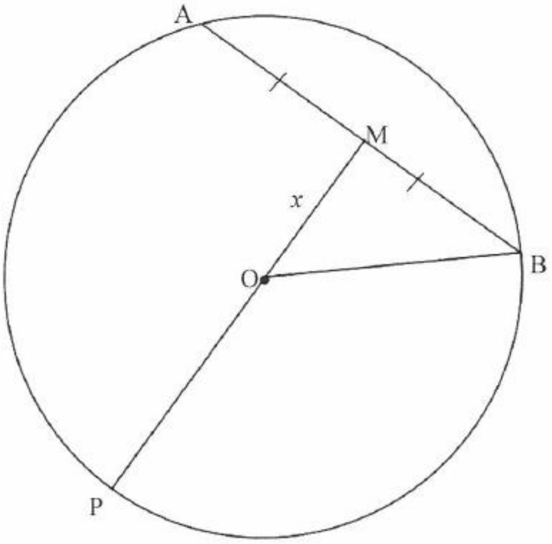
QUESTION 7

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = x units, AB = 20 units and $\frac{PM}{OM} = \frac{5}{2}$.



7.1 Write down the length of MB. (1)

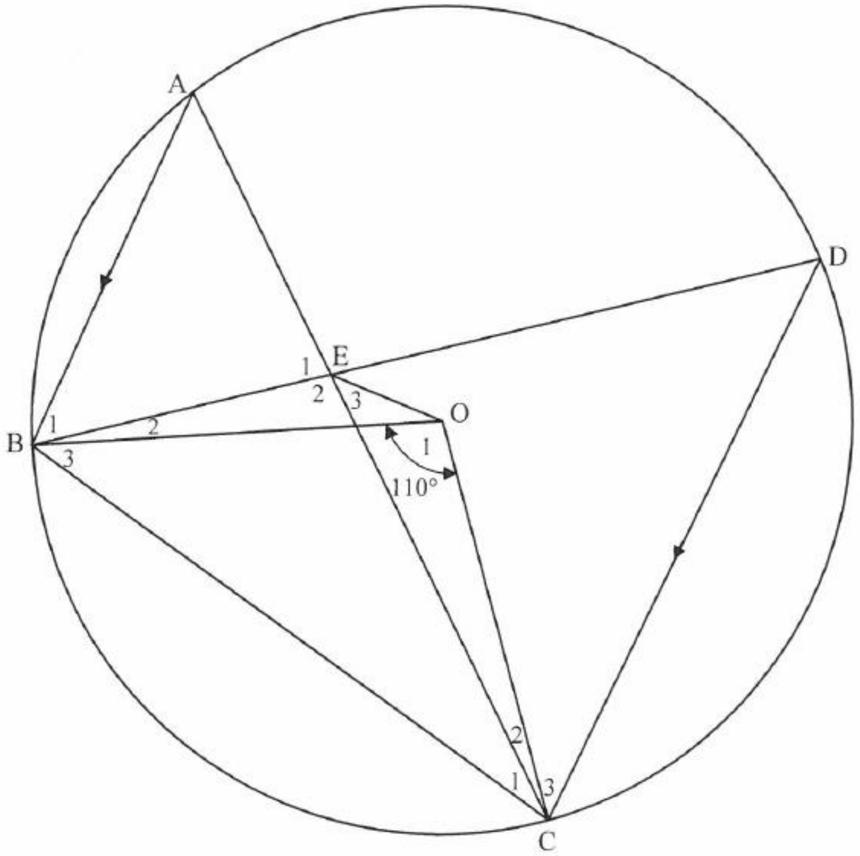
7.2 Give a reason why OM $\perp$ AB. (1)

7.3 Show that OP $= \frac{3x}{2}$ units. (2)

7.4 Calculate the value of x. (3)
[7]

QUESTION 8

In the diagram below, the circle with centre O passes through A, B, C and D.
$AB \parallel DC$ and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

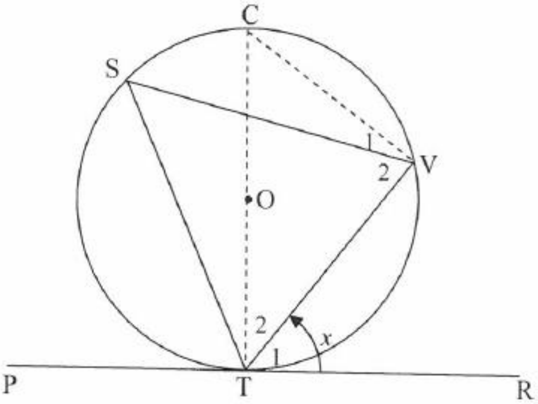
8.2 Prove that BEOC is a cyclic quadrilateral. (2)
[10]

QUESTION 9

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



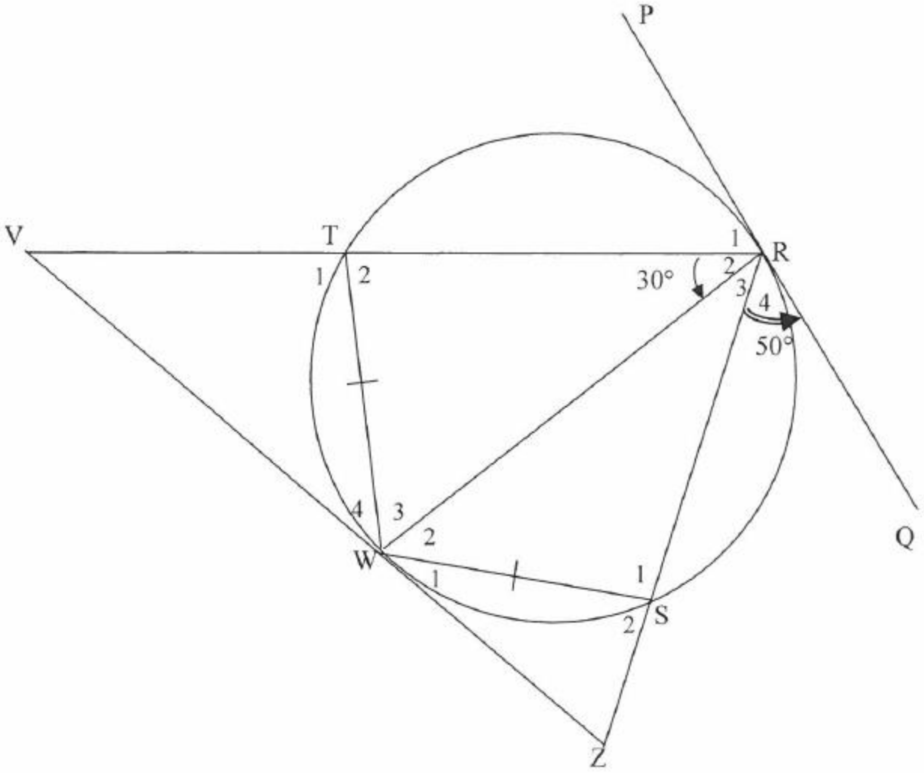
Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$.
Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.

Construction: Draw diameter TC and join CV.

Statement	**Reason**
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$..........	....
$\hat{T}_2 = 90° - x$	....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	....
$\therefore V\hat{T}R = \hat{S}$	

(5)

9.3 In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



9.3.1 Give a reason why $\hat{R}_3 = 30°$. (1)

9.3.2 State, with reasons, TWO other angles equal to 30°. (3)

9.3.3 Determine, with reasons, the size of:

(a) $\hat{S}_2$ (3)

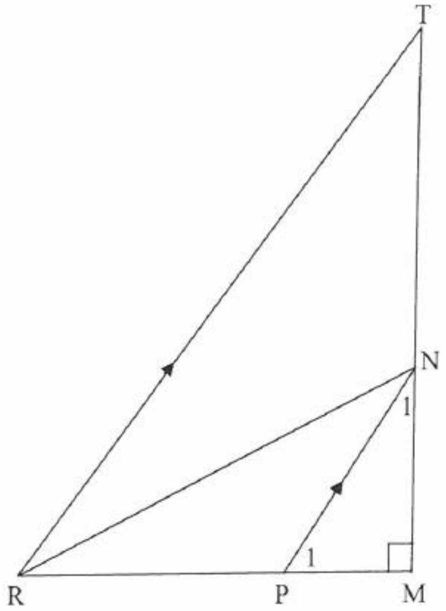
(b) $\hat{V}$ (4)

9.3.4 Prove that $WR^2 = RV \times RS$. (5)

[22]

QUESTION 10

In ΔTRM, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that $RT = 3PN$.



10.1 Give reasons for the statements below.
Use DIAGRAM SHEET 5.

	Statement	**Reason**
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	....
	$\hat{M}$ is common	
10.1.2	$\therefore \Delta PNM \ ||| \ \Delta RTM$	....

(2)

10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$. (2)

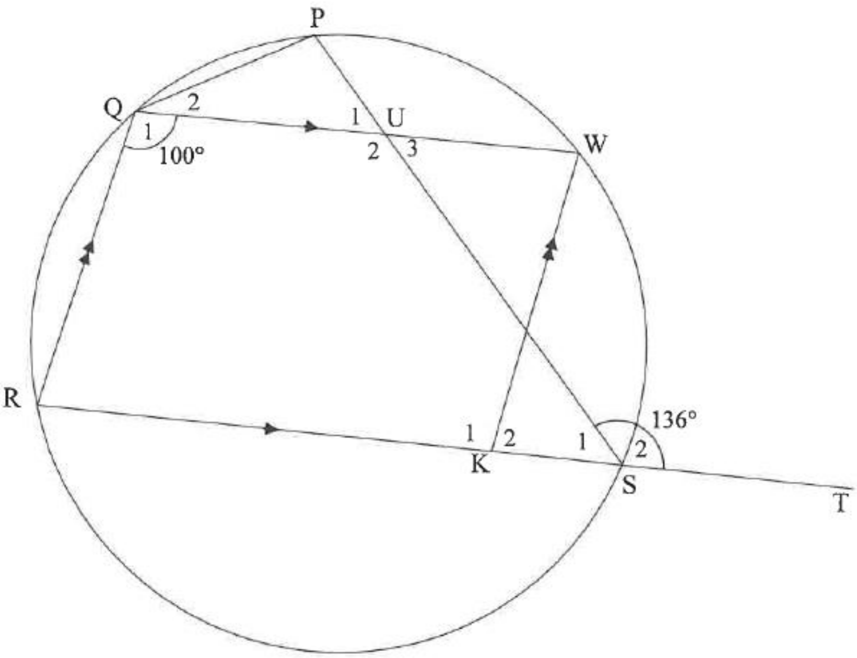
10.3 Show that $RN^2 - PN^2 = 2RP^2$. (4)

[8]

OCT/NOV 2019

QUESTION 8

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

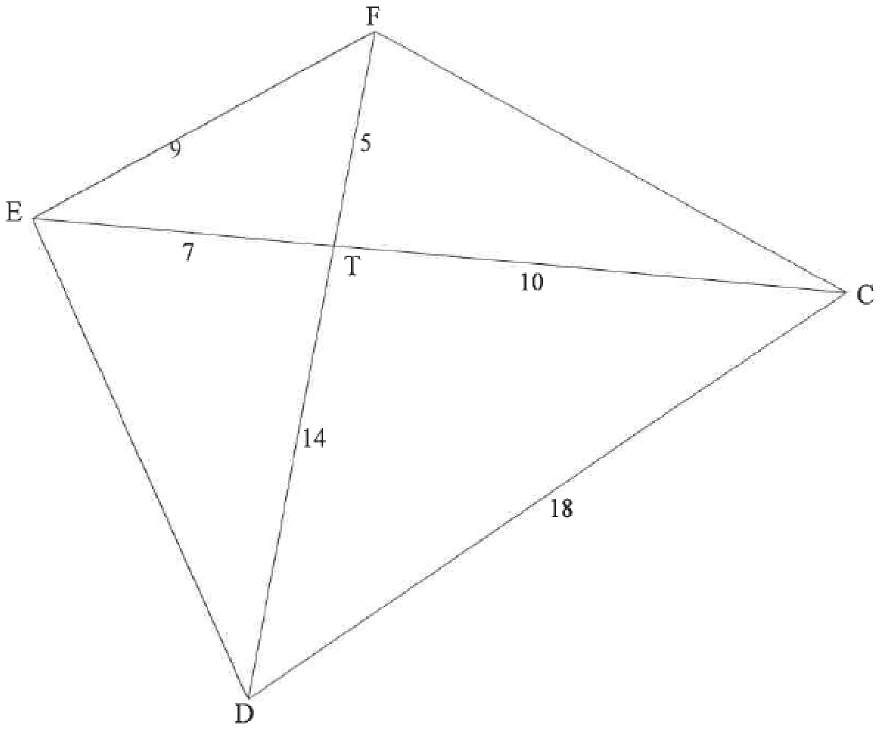
8.1.1 $\hat{R}$ (2)

8.1.2 $\hat{P}$ (2)

8.1.3 $P\hat{Q}W$ (3)

8.1.4 $\hat{U}_2$ (2)

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T. EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



Prove, with reasons, that:

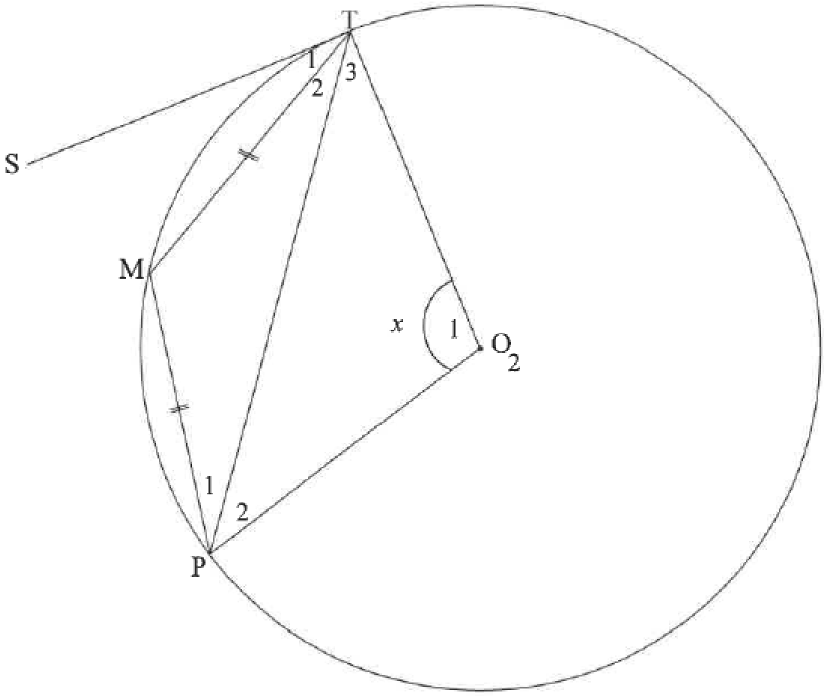
8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)

[16]

QUESTION 9

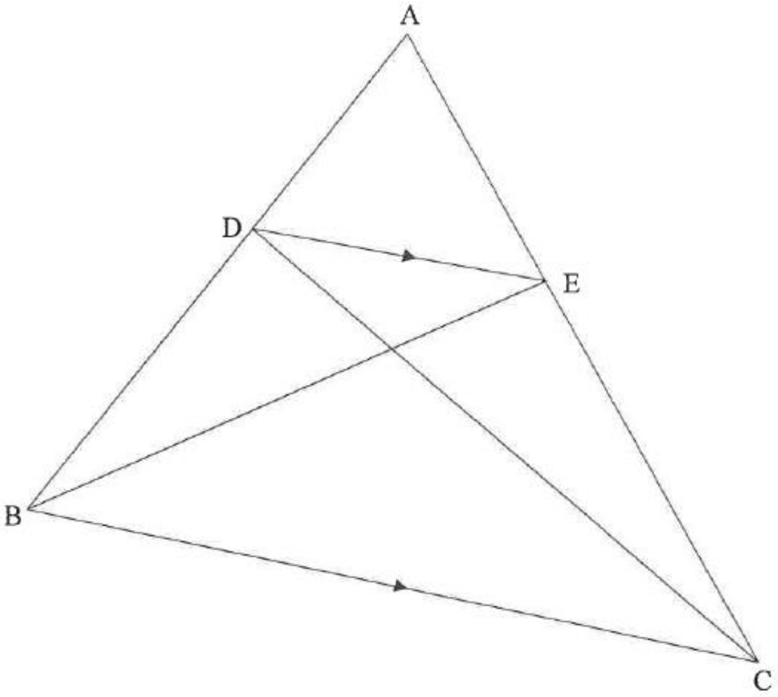
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. **[7]**

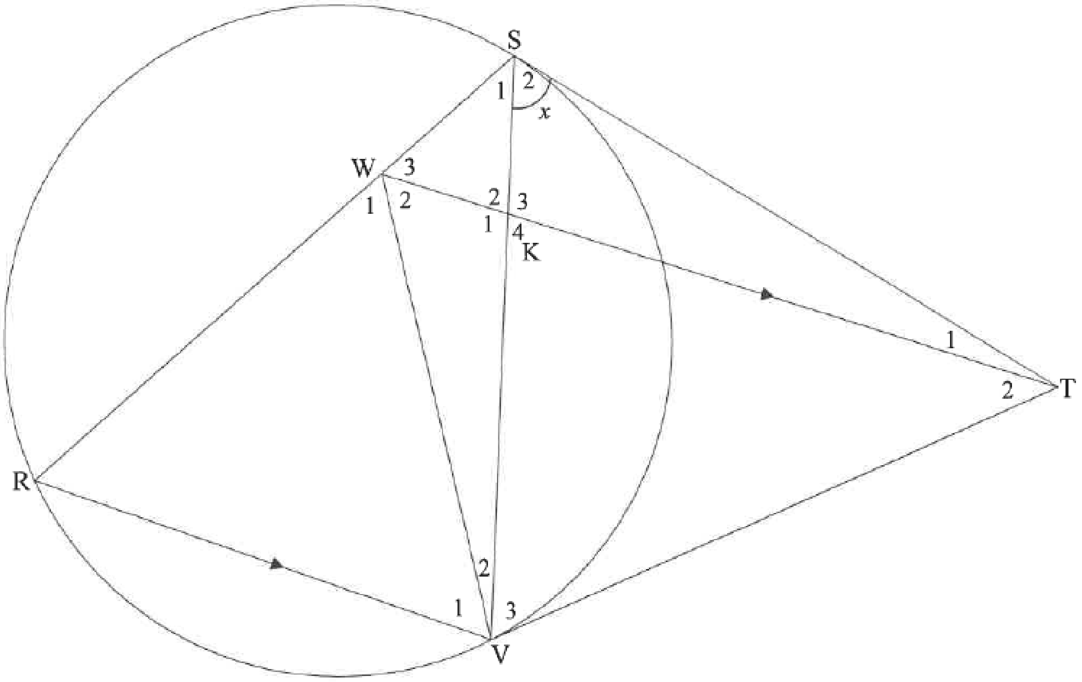
QUESTION 10

10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that $DE \parallel BC$. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

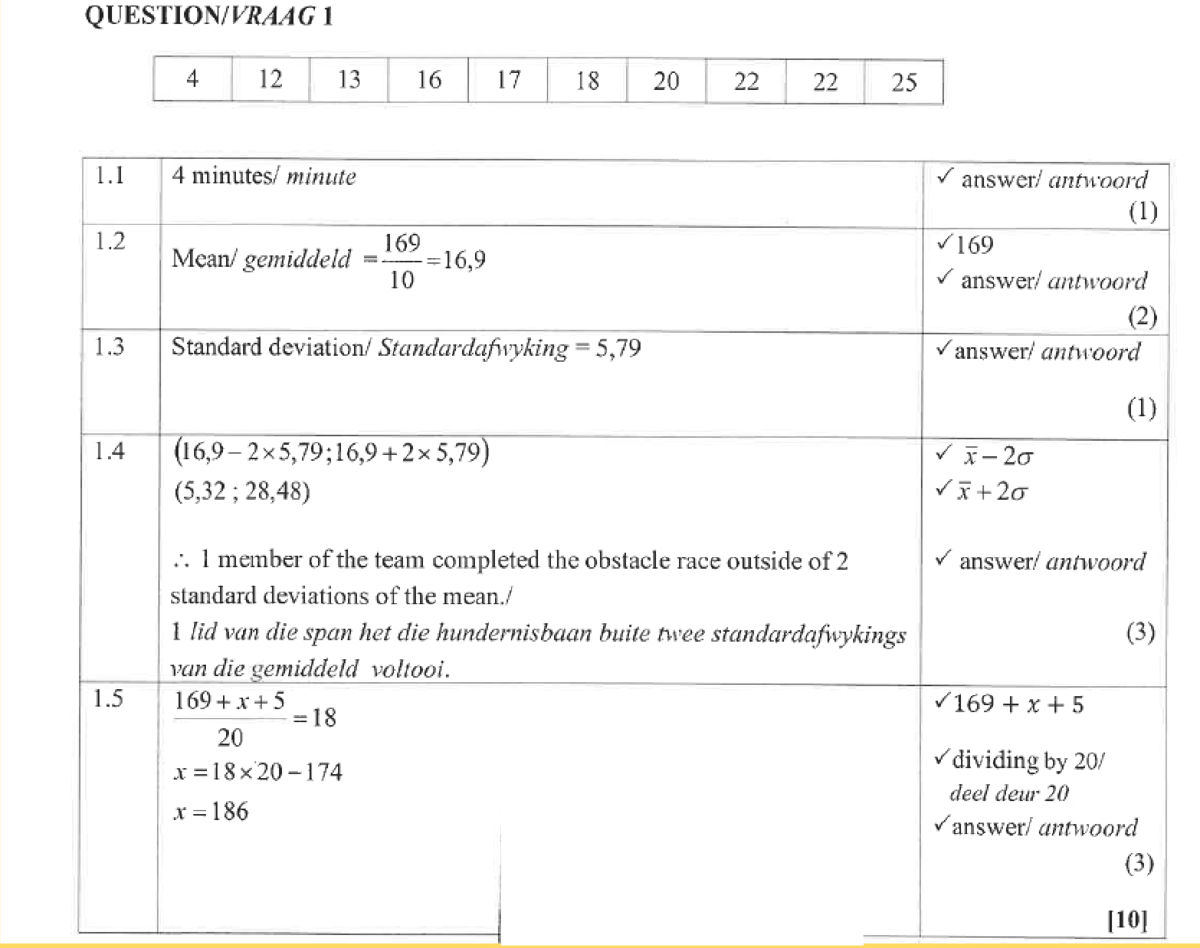
(d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

TOTAL: 150

SOLUTIONS
STATISTICS

NOVEMBER 2018 GRADE 11

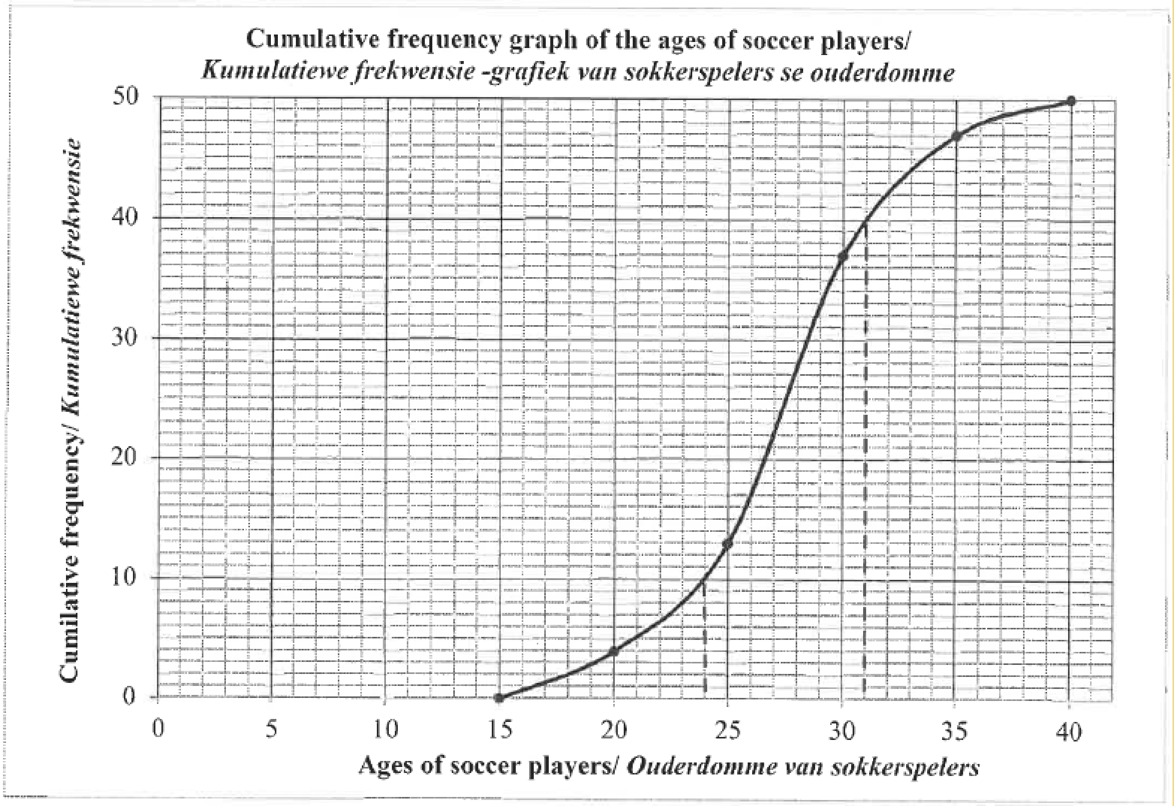


QUESTION/*VRAAG* 1

4	12	13	16	17	18	20	22	22	25

1.1	4 minutes/ *minute*	✓ answer/ *antwoord* (1)
1.2	Mean/ *gemiddeld* $=\frac{169}{10}=16{,}9$	✓169 ✓ answer/ *antwoord* (2)
1.3	Standard deviation/ *Standardafwyking* = 5,79	✓answer/ *antwoord* (1)
1.4	$(16{,}9-2\times5{,}79;16{,}9+2\times5{,}79)$ (5,32 ; 28,48) $\therefore$ 1 member of the team completed the obstacle race outside of 2 standard deviations of the mean./ 1 *lid van die span het die hundernisbaan buite twee standardafwykings van die gemiddeld voltooi.*	✓ $\bar{x}-2\sigma$ ✓ $\bar{x}+2\sigma$ ✓ answer/ *antwoord* (3)
1.5	$\frac{169+x+5}{20}=18$ $x=18\times20-174$ $x=186$	✓169 + x + 5 ✓dividing by 20/ *deel deur 20* ✓answer/ *antwoord* (3) **[10]**

QUESTION/*VRAAG* 2

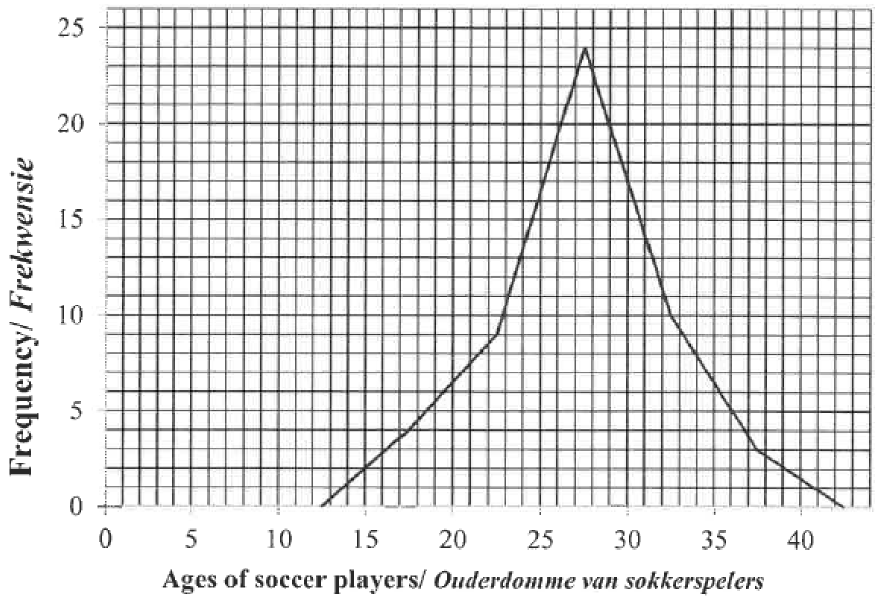


<table>
<tr><td>2.1.1</td><td>50 players/ <i>spelers</i></td><td>✓ answer/ <i>antwoord</i>
(1)</td></tr>
<tr><td>2.1.2</td><td>40 – 10 = 30 players/ <i>spelers</i></td><td>✓40 and/ <i>en</i> 10
✓answer/ <i>antwoord</i>
(2)</td></tr>
<tr><td>2.1.3</td><td>
<table>
<tr><th>Class interval/
<i>Klas interval</i></th><th>Frequency/
<i>Frekwensie</i></th><th>Cumulative frequency
<i>Kumulatiewe frekwensie</i></th></tr>
<tr><td>$15 \leq x < 20$</td><td>4</td><td>4</td></tr>
<tr><td>$20 \leq x < 25$</td><td>9</td><td>13</td></tr>
<tr><td>$25 \leq x < 30$</td><td>24</td><td>37</td></tr>
<tr><td>$30 \leq x < 35$</td><td>10</td><td>47</td></tr>
<tr><td>$35 \leq x < 40$</td><td>3</td><td>50</td></tr>
</table>
</td><td>✓ two correct values/ <i>twee korrekte waardes</i>
✓ three correct values/ <i>drie korrekte waardes</i>
✓ all correct values/ <i>al die waardes korrek</i>
(3)</td></tr>
</table>

2.1.4

Class interval/ *Klas- interval*	Class midpoint/ *Klas- middelpunt*	Frequency/ *Frekwensie*
$15 \le x < 20$	17,5	4
$20 \le x < 25$	22,5	9
$25 \le x < 30$	27,5	24
$30 \le x < 35$	32,5	10
$35 \le x < 40$	37,5	3

Frequency polygon of the ages of soccer players/
Frekwensie- veelhoek van sokkerspelers se oudermme



✓ using midpoints / *gebruik middelpunte*

✓plotting the points correctly/ *korrekte punte geplot*

✓points joined by straight line/ *punte verbind met 'n reguitlyn*

✓ grounding at/ *geanker by* (12,5;0) and/ *en* (42,5 ; 0)

(4)

2.2 The claim is not valid. / *Die bewering is nie geldig nie*

Range of class/ *Omvang van klas* A = 60
Range of class/ *Omvang van klas* B = 40

The range of class A is bigger than the range of class B. Therefore the marks of class A are more spread out than the class B./
Die omvang van klas A is groter as die omvang van klas B. Dus is die punte in klas A meer verspreid as klas B

At least 25% of class A have lower marks than any learner in class B./
ten minste 25% van klas A het laer punte as enige leerder in klas B.

Class A performed worse at the bottom end. /
Klas A het slegter gevorder aan die onderste groep

✓ claim not valid/ *bewering nie geldig nie*

✓ comment on the overall spread/ *kommentaar oor die algehele verspreiding*

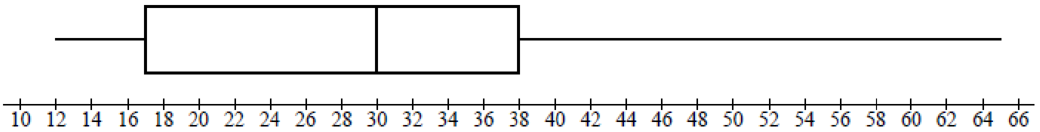
✓ comparison of the lower marks/ *vergelyk laer punte*

(3)

[13]

GRADE 11 NOVEMBER 2017

QUESTION/*VRAAG* 1



1.1.1	min = 12 $Q_1 = 17$ median / *mediaan* = 30 $Q_3 = 38$ max = 65	✓ min + max ✓ median, Q_1 and/en Q_3 (2)
1.1.2	$IQR = Q_3 - Q_1$ $= 38 - 17$ $= 21$	✓ answer/*antw* (1)
1.1.3	Skewed to the right OR positively skewed *Skeef na regs* *OF positief skeef*	✓ answer/*antw* (1)

5	8	10	17	20	29	32	48	50	50	63	y	107

1.2.1	Mean/*Gemiddeld* $= \dfrac{439+y}{13}$ $41 = \dfrac{439+y}{13}$ $439 + y = 533$ $y = 94$	✓ $41 = \dfrac{439+y}{13}$ ✓ answer/*antw* (2)
1.2.2	$\sigma = 30{,}94$	✓ answer/*antw* (1)

1.2.3	$41 \times 13 = 533$ $6 \times 18 = 108$ $\dfrac{533+108}{19} = \dfrac{641}{19} = 33{,}74$	✓108 ✓ $533 + 108 = 641$ ✓ answer/*antw* (3) **[10]**

QUESTION/*VRAAG* 2

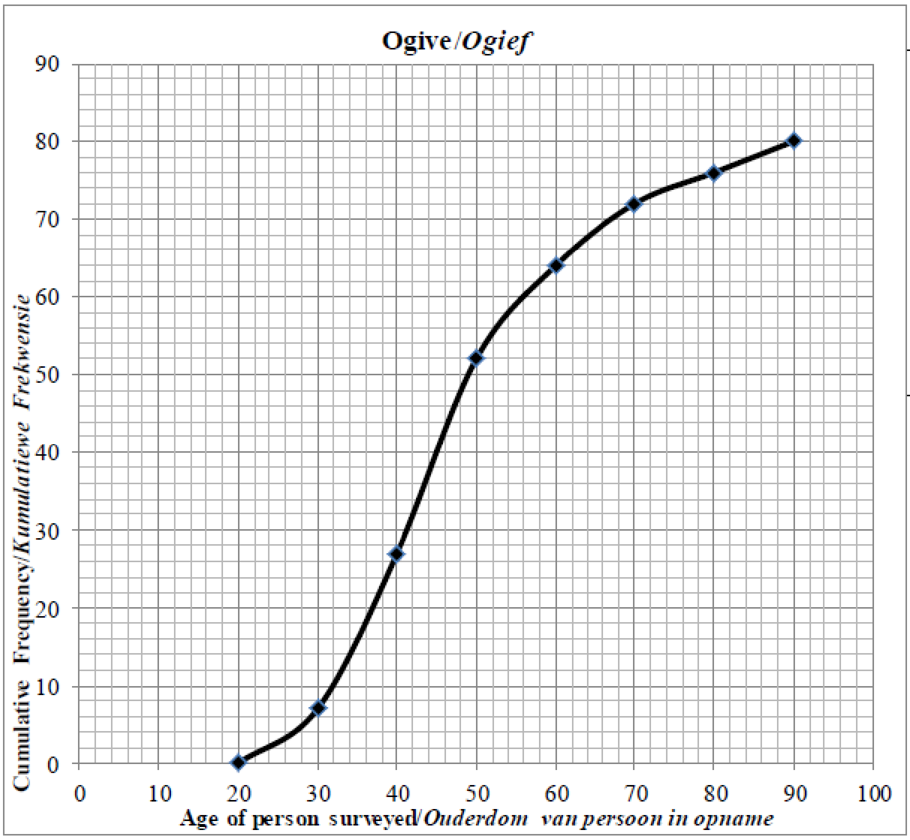
2.1

AGE OF PERSON SURVEYED/*OUDERDOM VAN PERSOON IN OPNAME*	FREQUENCY/*FREKWENSIE*	CUMULATIVE FREQUENCY/ *KUMULATIEWE FREKWENSIE*
$20 < x \leq 30$	7	7
$30 < x \leq 40$	**20**	27
$40 < x \leq 50$	25	**52**
$50 < x \leq 60$	**12**	64
$60 < x \leq 70$	**8**	72
$70 < x \leq 80$	4	**76**
$80 < x \leq 90$	**4**	80

✓ 20, 12
✓ 8, 4
✓ 52
✓76
(4)

2.2	$n = 80$	✓answ/*antw* (1)
2.3	$40 < x \leq 50$	✓ answ/*antw* (1)

2.4



✓Grounding (20; 0)
/*Geanker by (20; 0)*

✓upper limits/
boonste limiete

✓ shape
(smooth curve)/
vorm
(gladde kurwe)
(3)

2.5	$80 - 58 = 22$ **Accept/*aanvaar*:** 56 – 59 calls/*oproepe* $\frac{22}{80} \times 100 = 27{,}5\%$	✓58 calls/*oproepe* ✓22 ✓ 27,5% (3)
		[12]

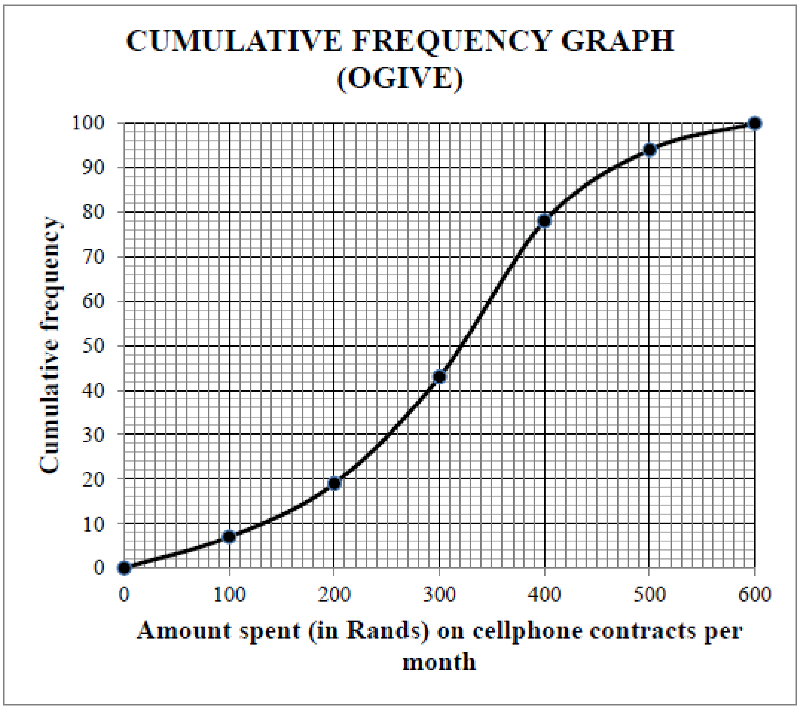
GRADE 12 SOLUTIONS
NOVEMBER 2019

QUESTION/*VRAAG* 1

Monthly income (in rands) **Maandelikse inkomste (in rand)**	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) **Maandelikse paaiement (in rand)**	2 000	3 000	3 500	5 200	5 500	6 000

1.1	$a = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation (3)
1.2	Monthly repayment ≈ R3 727,16 (calculator) *Maandelikse paaiement* ≈ R3 727,16 **OR** $\hat{y} = -1946,88 + 0,41(14000)$ ≈ R3 793,12	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0,95$	✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D *Spandeer nie R9 000 per maand nie, want die punt (18 000 ; 9 000) lê baie ver van die kleinste-kwadrate regressielyn.* ***OF D***	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\dfrac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\dfrac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$	✓ $\sum x = 100$ ✓ $a = 40 - b$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200b = 3200$ (5)
2.3	Modal class/*modale klas:* $300 < x \leq 400$	✓ answer (1)
2.4		✓ grounded at (0 ; 0) ✓ upper limits for *x*-coordinates ✓ cumulative frequencies for *y*-coordinates ✓ smooth shape (4)
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] *18 mense betaal meer as R420 per maand* Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

NOVEMBER 2018

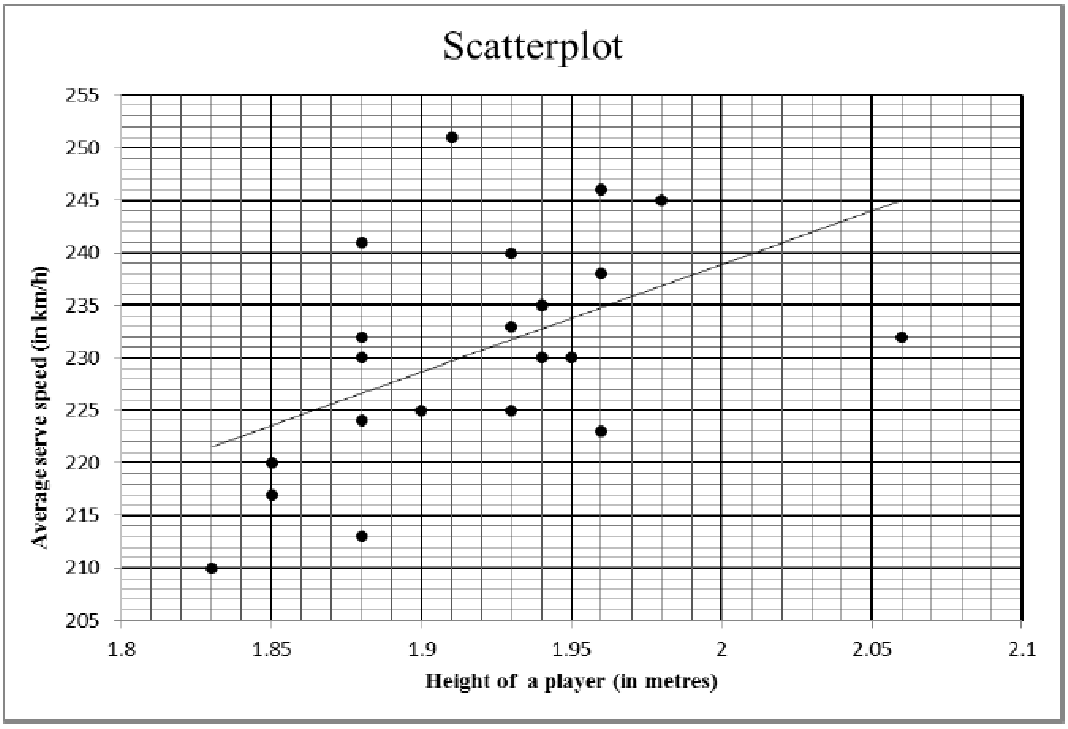
QUESTION/*VRAAG* 1

1.1.1	140 items	✓ answer (1)
1.1.2	Modal class/*modale klas:* $20 < x \le 30$ minutes **OR**/***OF*** $20 \le x < 30$ minutes	✓ answer (1) ✓ answer (1)
1.1.3	Number of minutes taken = 20 minutes	✓ answer (1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) Answer only: Full marks	✓ 126 ✓ answer (2)
1.1.5	75th percentile is at 105 items =37 minutes [accept 36 – 38 minutes] Answer only: Full marks	✓ 105 ✓ answer (2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] Answer only: Full marks	✓ lower quartile (Q_1) ✓ answer (2)

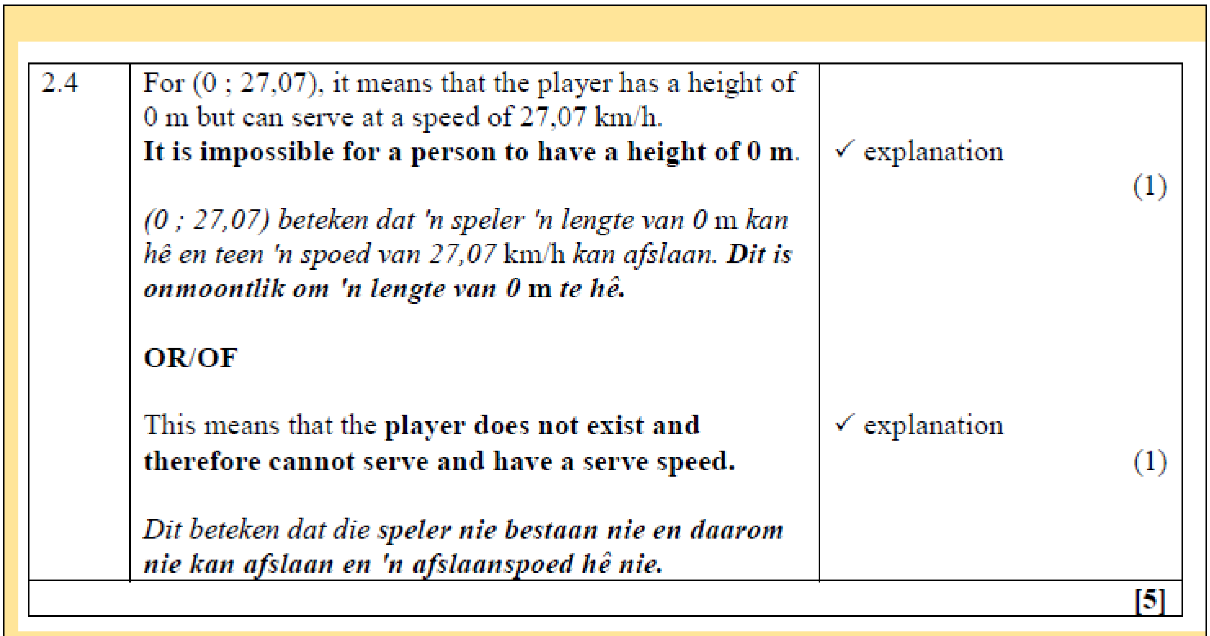
35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ $= R94{,}666.. = R94{,}67$ Answer only: Full marks	✓ 1420 ✓ answer (2)
1.2.1(b)	$\sigma = R22{,}691... = R22{,}69$	✓✓ answer (2)
1.2.2(a)	They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period. *Hulle albei het* ***dieselfde bedrag*** *met footjies ontvang, nl. R1 420 oor die 15 dae-tydperk*	✓ answer (1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day. *Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar* ***groter variasie/verspreiding in die footjies was wat Reggie elke dag ontvang*** *het in vergelyking met die getal footjies wat Marie elke dag ontvang het.*	✓ explanation (1)
		[15]

QUESTION/*VRAAG* 2



2.1	251 km/h	✓ answer (1)
2.2.1	$r = 0{,}52$ **OR** C	✓ answer (1)
2.2.2	The points are **fairly scattered** and the least squares regression line is increasing. *Die punte is **redelik verspreid** en die kleinstekwadrate-regressielyn neem toe.*	✓ reason (1)
2.3	There is a weak positive relation hence the height could have an influence *Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.* **OR/*OF*** There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. *Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.* **OR/*OF*** There is no conclusive evidence that a taller person will serve faster than a shorter person. *Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.*	✓ answer (1) ✓ answer (1) ✓ answer (1)

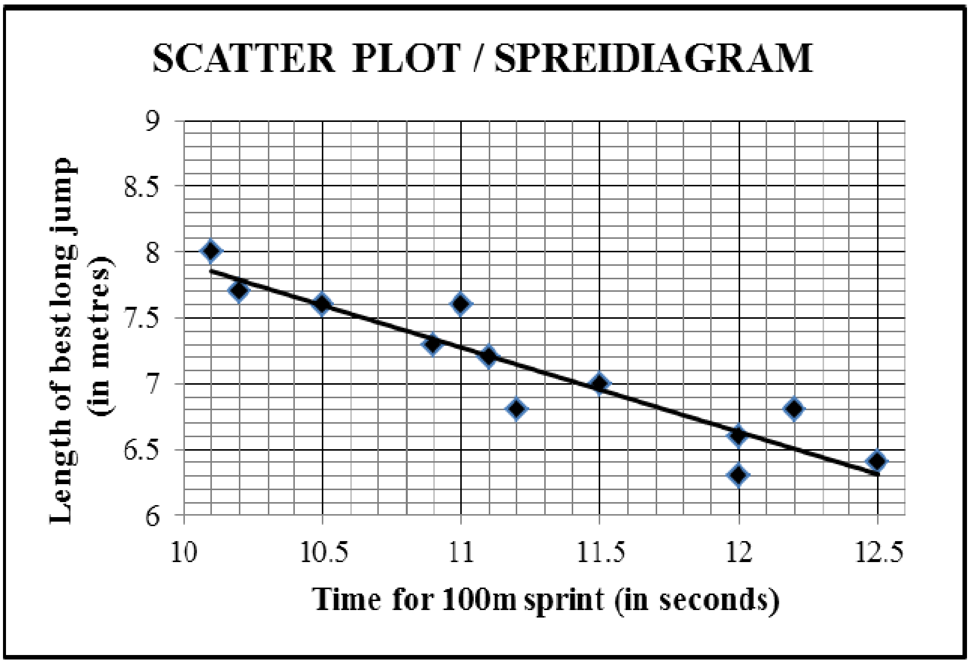


2.4	For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. **It is impossible for a person to have a height of 0 m.** *(0 ; 27,07) beteken dat 'n speler 'n lengte van 0* m *kan hê en teen 'n spoed van 27,07* km/h *kan afslaan.* ***Dit is onmoontlik om 'n lengte van 0* m *te hê.*** **OR/OF** This means that the **player does not exist and therefore cannot serve and have a serve speed.** *Dit beteken dat die **speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.***	✓ explanation (1) ✓ explanation (1)
		[5]

NOVEMBER 2017

QUESTION/*VRAAG* 1

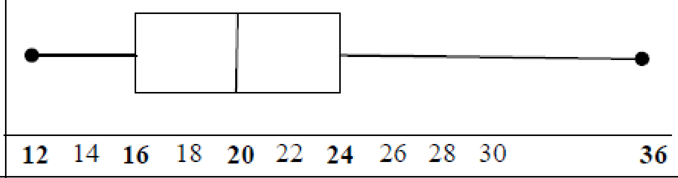
Time for 100 m sprint (in seconds) ***Tyd vir 100 m-naelloop (in sekondes)***	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) ***Afstand van beste sprong in verspring (in meter)***	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



1.1	$a = 14{,}343\ldots\ = 14{,}34$ $b = -0{,}642\ldots\ = -\,0{,}64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. */Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) [7]

QUESTION/*VRAAG* 2

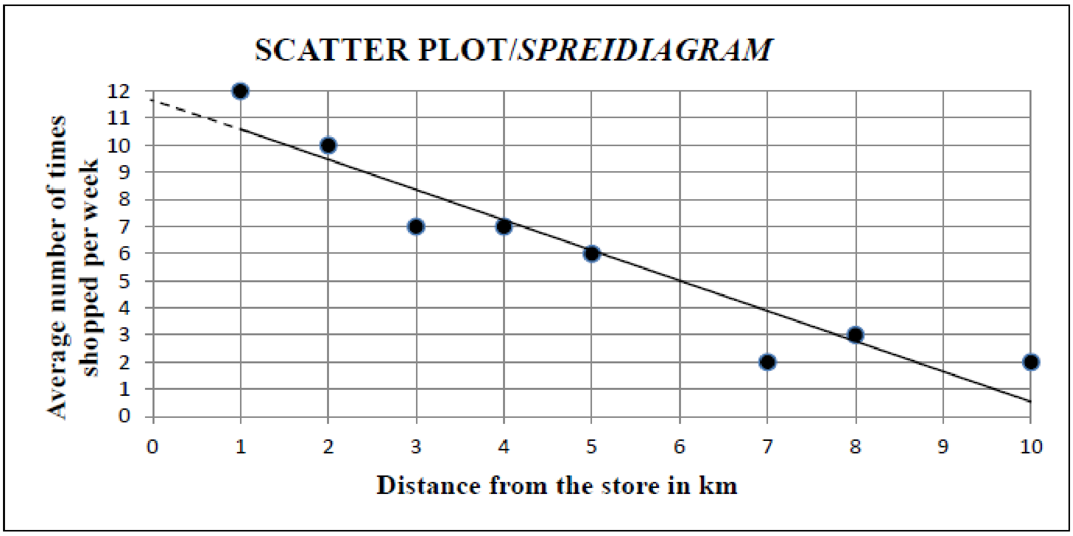
12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* $= Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ ∴ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3		✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een van die seuns nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

NOVEMBER 2016

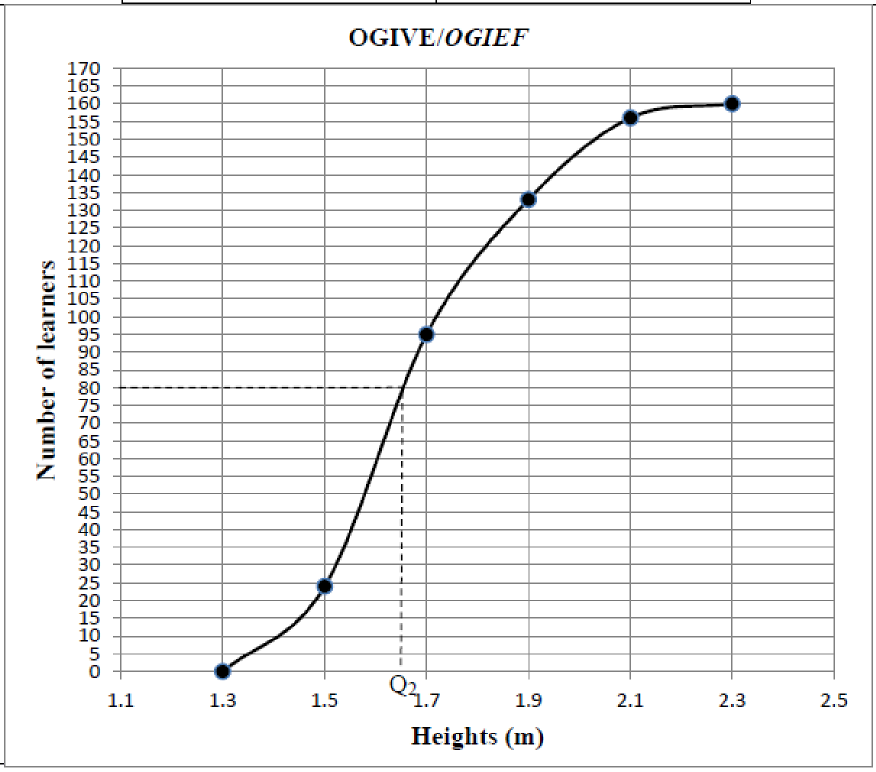
QUESTION/*VRAAG* 1

Distance from the store in km *Afstand vanaf die winkel in* km	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers die winkel per week besoek*	12	10	7	7	6	2	3	2



1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71\ \ (11{,}7132\ldots)$ $b = -1{,}12\ \ (-1{,}1176\ldots)$ $\hat{y} = -1{,}12x + 11{,}71$	✓ value of *a* ✓ value of *b* ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ $= 5$ times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) [9]

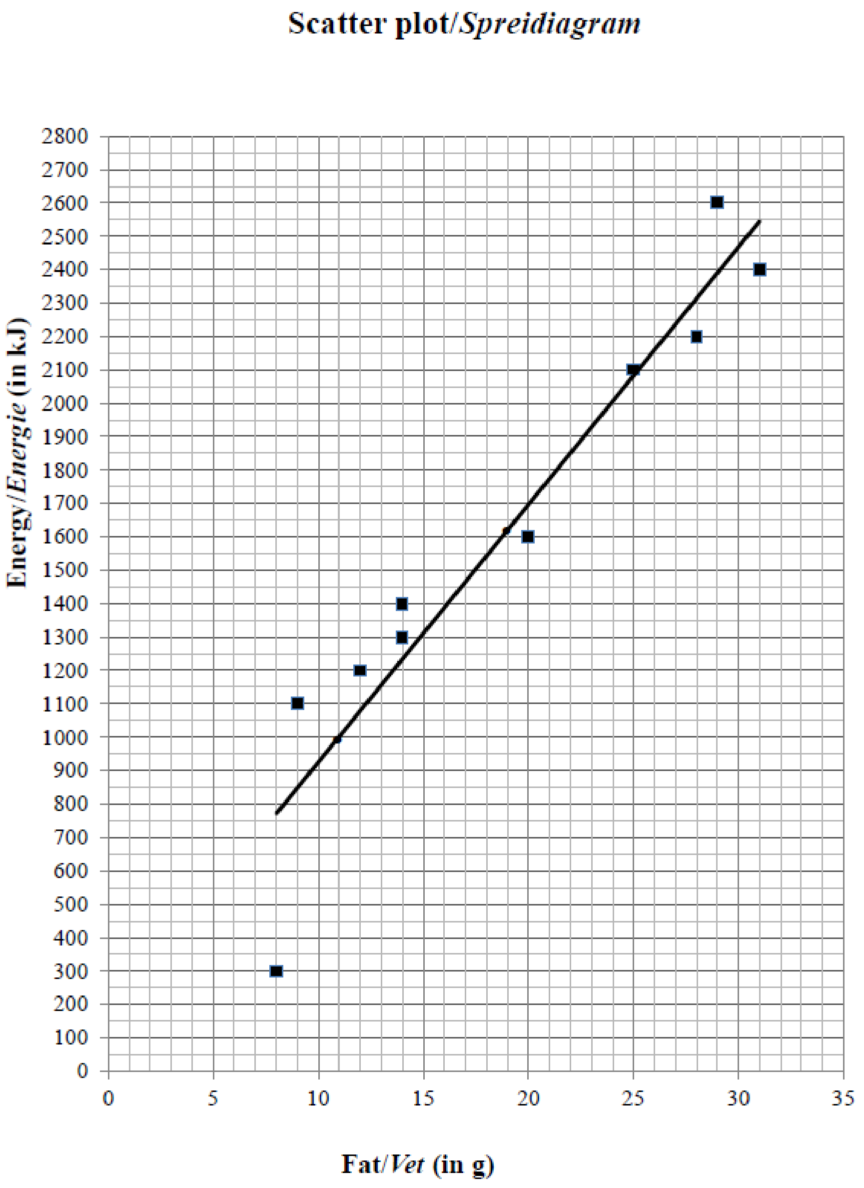
QUESTION/*VRAAG* 2

<table>
<tr><td>2.1</td><td>Positively skewed OR skewed to the right/<i>positief skeef OF skeef na regs</i></td><td>✓ answer
(1)</td></tr>
<tr><td>2.2</td><td>Range/<i>Omvang</i> = 2,21 – 1,39 = 0,82 m</td><td>✓ subtract values
✓answer
(2)</td></tr>
<tr><td>2.3</td><td>
<table>
<tr><th>Intervals
<i>Klasse</i></th><th>Cumulative frequency
<i>Kumulatiewe frekwensie</i></th></tr>
<tr><td>$1,3 \le x < 1,5$</td><td>24</td></tr>
<tr><td>$1,5 \le x < 1,7$</td><td>95</td></tr>
<tr><td>$1,7 \le x < 1,9$</td><td>133</td></tr>
<tr><td>$1,9 \le x < 2,1$</td><td>156</td></tr>
<tr><td>$2,1 \le x < 2,3$</td><td>160</td></tr>
</table>
</td><td>✓95 , 133, 156
✓160
(2)</td></tr>
<tr><td>2.4</td><td>
</td><td>✓ upper limits / <i>boonste limiete</i>
✓ cum f/ <i>kum f</i>
✓ shape/ <i>vorm</i>
✓ grounded <i>geanker</i>
(4)</td></tr>
<tr><td>2.5</td><td>method (using 80 to determine the height)
1,65 (accept any value between 1,6 and 1,69)</td><td>✓ method
✓ answer
(2)</td></tr>
<tr><td>2.6.1</td><td>The mean would change by 0,1 <i>m</i>
<i>Die gemiddelde sal met 0,1 m verander</i></td><td>✓ answer
(1)</td></tr>
<tr><td>2.6.2</td><td>No influence/change as there is no difference in variation of data./<i>Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.</i></td><td>✓ answer
(1)
[13]</td></tr>
</table>

NOVEMBER 2015

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1		1.1 no marks: 0 – 2 points correctly ✓plotting 3 – 5 points correctly ✓✓plotting 6 – 9points correctly ✓✓✓plotting all 10 points correctly *geen punte: 0 – 2 punte korrek* ✓ *strip 3 – 5 pte korrek* ✓✓*strip 6 –9 pte korrek* ✓✓✓ *strip al 10 pte korrek* (3)
1.2.2		1.2.2 ✓ y – int close to (0 ; 150) ✓ one pt close to (25 ; 2100) or (20 ; 1700) (2)

1.2.1	$\hat{y} = 154{,}60 + 77{,}13(18)$ $= 1\ 542{,}94 \approx 1\ 500$ kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	$r = 0{,}9520\ldots \approx 0{,}95$	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

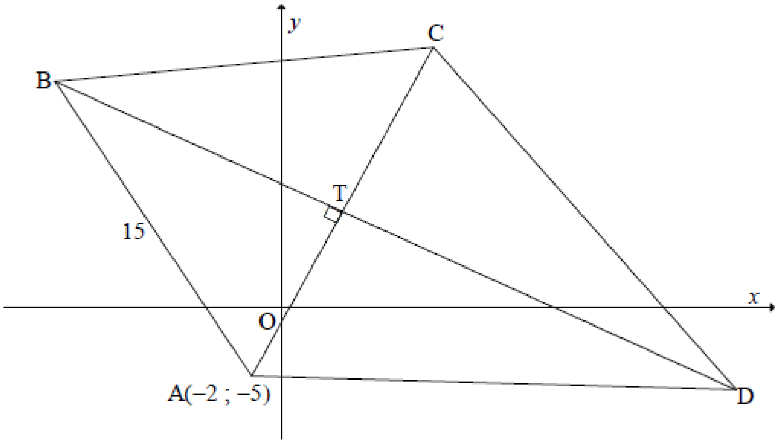
QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ ***Som van die waardes op boonste vlakke***	**Frequency/** ***Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1	mean/*gemiddelde* = $\frac{2(0)+3(3)+4(2)+\dots 12(1)}{30}=\frac{202}{30}$ $= 6{,}73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $=\frac{T_{15}+T_{16}}{2}=\frac{7+7}{2}=7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* = 2,264... ≈ 2,26	✓✓ answ/*antw* (2)
2.4	(6,73 – 2,26 ; 6,73 + 2,26) = (4,47 ; 8,99) ∴ 4 + 4 + 8 + 3 = 19 times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

ANALYTICAL GEOMETRY
SOLUTIONS NOVEMBER 2019 GRADE 11

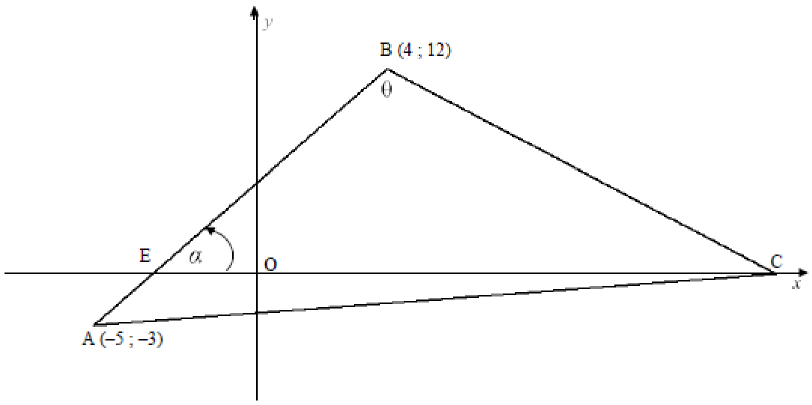
QUESTION/*VRAAG* 3



3.1	$BD \quad y=-\frac{1}{2}x+9$ $\therefore m_{BD}=-\frac{1}{2}$ $\therefore m_{AC}=2$	✓ Standard form/*vorm* ✓ answ/*antw* (2)
3.2	$y-y_1=m(x-x_1)$ $y-(-5)=2(x-(-2))$ $y=2x-1$	✓ subst (−2 ; −5) ✓ answ/*antw* (2)
3.3	$2x-1=-\frac{1}{2}x+9$ $\frac{5}{2}x=10$ $x=4$ $y=2(4)-1$ $y=7$ $\text{T}(4\,;\,7)$	✓ $2x-1=-\frac{1}{2}x+9$ ✓ $x=4$ ✓ $y=7$ (3)

3.4.1	$4=\frac{-2+x}{2}$ $8=-2+x$ $x=10$ $7=\frac{-5+y}{2}$ $14=-5+y$ $y=19$ C$(10\,;19)$	✓ $x=10$ ✓ $y=19$ (2)
3.4.2	$AT=\sqrt{(4-(-2))^2+(7-(-5))^2}$ $=\sqrt{180}$ $=6\sqrt{5}$ $BT^2+AT^2=AB^2$ $(Pythagoras)$ $BT=\sqrt{15^2-(\sqrt{180})^2}$ $=3\sqrt{5}$	✓ subst. in distance/*afstand* form. ✓ answer/*antw* ✓ subst. in pyth ✓ answer/*antw* (4)
3.4.3	BC is the diameter/ *middellyn* [subt. right / *ondersp. reg* ∠] or/o [conv. ∠^{s} in semi - circle/ *omgk.* ∠^{s} *in halfsirkel*] Radius $=\frac{15}{2}=7{,}5$ units/ *eenh.*	✓ S ✓ answ/*antw* (2) **[15]**

QUESTION/*VRAAG* 4

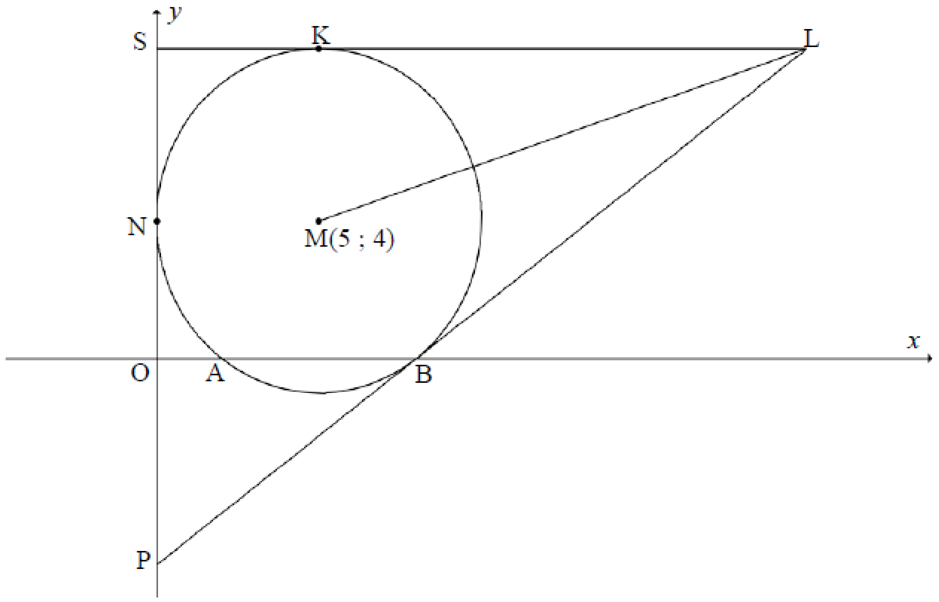


4.1	$m_{AB}=\frac{12-(-3)}{4-(-5)}=\frac{5}{3}$ **OR/*OF*** $m_{AB}=\frac{-3-12}{-5-4}=\frac{5}{3}$	✓ subst. in gradient form ✓ answ/*antw* (2)
4.2	$y-12=\frac{5}{3}(x-4)$ $0-12=\frac{5}{3}(x-4)$ $x=-\frac{16}{5}$ $\mathrm{E}\left(-\frac{16}{5}\ ;\ 0\right)$ **OR/*OF*** $\frac{0-12}{x-4}=\frac{5}{3}$ $-36=5x-20$ $-16=5x$ $x=\frac{-16}{5}$ $\frac{0+3}{x+5}=\frac{5}{3}$ $9=5x+25$ $-16=5x$ $x=\frac{-16}{5}$ $\mathrm{E}\left(-\frac{16}{5}\ ;\ 0\right)$	✓equation/*verg.* ✓ $y=0$ ✓ answ/*antw* (3) ✓equating/*verg.* ✓ $y=0$ ✓ answ/*antw* (3)

4.3	$\tan\alpha = m_{AB}$ $\tan\alpha = \frac{5}{3}$ $\alpha = 59°$	✓ $\tan\alpha = \frac{5}{3}$ ✓ $\alpha = 59°$ (2)
4.4	$76° + 59° = 135°$ [ext $\angle$ of Δ] $B\hat{C}X = 135°$ $\tan 135° = m_{BC}$ $m_{BC} = -1 = m_{//}$ $y-(-3) = -1(x-(-5))$ $y = -x-8$	✓ 135° ✓ $\tan 135° = m_{BC}$ ✓ answer/*antw* ✓ subst (–3 ; –5) ✓ answer/*antw* (5) **[12]**

NOVEMBER 2014

QUESTION/*VRAAG* 3

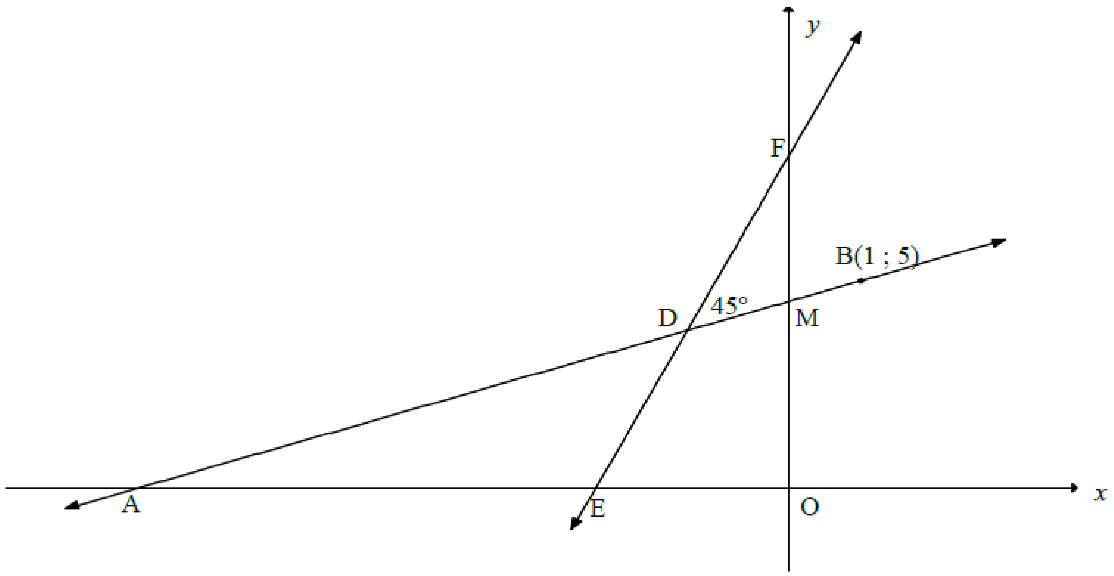


3.1	$r = \text{MN} = 5$	✓ answer/*antw* (1)
3.2	$(x-5)^2+(y-4)^2=25$	✓equation/*vgl* (1)
3.3	A$(x ; 0)$ $(x-5)^2+(0-4)^2=25$ $x^2-10x+25+16=25$ $x^2-10x+16=0$ $(x-8)(x-2)=0$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0) **OR/*OF*** $(x-5)^2+(0-4)^2=25$ $(x-5)^2+16=25$ $(x-5)^2=9$ $(x-5)=\pm 3$ $\therefore x=8$ or/*of* $x=2$ $\therefore$ A(2 ; 0)	✓ substitute into eq/ *vervang in vgl* $y=0$ ✓ standard form/ *standaardvorm* or perfect square form/*kwadr vorm* ✓ answer/*antw* (3)
3.4.1	$m_{\text{MB}} = \dfrac{4-0}{5-8}$ $= -\dfrac{4}{3}$	✓ subst M and B into form/*vervang M and B in form* ✓ $m_{\text{MB}} = -\dfrac{4}{3}$ (2)

3.4.2	$m_{MB} \times m_{PB} = -1$ (tangent ⊥ radius/ *rkl ⊥ radius*) $m_{PB} = \frac{3}{4}$ $y = \frac{3}{4}x + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$ $0 = \frac{3}{4}(8) + c$ $y - 0 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 6$ $y = \frac{3}{4}x - 6$	✓ $m_{MB} \times m_{PB} = -1$ ✓ $m_{PB} = \frac{3}{4}$ ✓ equation/*vgl* (3)
3.5	$y_K = y_M + r = 4 + 5$ $y = 9$	✓ 9 ✓ equation/*vgl* (2)
3.6	At/*By* L: $\frac{3}{4}x - 6 = 9$ $3x - 24 = 36$ $3x = 60$ $x = 20$ $\therefore$ L(20 ; 9)	✓ equating simultaneously ✓ simplification (2)
3.7	L(20 ; 9) $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **OR/OF** $ML = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(20-5)^2 + (9-4)^2}$ $= \sqrt{(15)^2 + (5)^2}$ $= \sqrt{225 + 25}$ $= \sqrt{(5)^2(9+1)}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$ $= \sqrt{250}$ *or / of* $5\sqrt{10}$	✓ correct subst into distance formula/ *korrekte subst in afstand-formule* ✓ answer in surd form/*antw in wortelvorm* (2)
3.8	**MK ⊥ KL** **OR/OF** $M\hat{K}L = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*) $\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn* $r = \frac{ML}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or 7,91 Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML* $x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$ Centre/*midpt*: (12,5 ; 6,5) Equation of the circle KLM / *Vgl van sirkel KLM*: $\therefore (x - 12{,}5)^2 + (y - 6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$ **OR/*OF***	✓ S ✓ value of/*waarde van r* ✓ $x = 12{,}5$ ✓ $y = 6{,}5$ ✓ answer in correct form/ *antw in korrekte vorm* (5)

	MK ⊥ KL OR/OF $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*)	✓ S
	$\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn*	
	Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML*	
	$x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $\qquad y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$	✓ $x = 12{,}5$ ✓ $y = 6{,}5$
	Centre/*midpt*: (12,5 ; 6,5)	
	Equation of the circle KLM /*Vgl van sirkel KLM*:	
	$(x-12{,}5)^2 + (y-6{,}5)^2 = r^2$	
	subst (5 ; 4): $(5-12{,}5)^2 + (4-6{,}5)^2 = r^2$ $62{,}5 = r^2$	✓ value of/*waarde van* r^2
	$\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ answer in correct form/*antw in korrekte vorm* (5)
	OR/*OF*	
	By symmetry about LM/*deur simmetrie om LM*:	
	MK ⊥ KL OR/OF $\mathbf{M\hat{K}L} = 90°$ (radius ⊥ tangent/*radius ⊥ rkl*)	✓ S
	$\therefore$ ML is a diameter as it subtends a right angle/*ML is middellyn*	
	ML is a diameter /*ML is 'n middellyn*	
	$r = \frac{\text{ML}}{2} = \frac{\sqrt{250}}{2} = \sqrt{\frac{125}{2}}$ or /of 7,91	✓ value of/*waarde van r*
	Centre of circle = midpoint of ML/*Midpt van sirkel = midpt v ML*	
	$x = \frac{5+20}{2} = \frac{25}{2} = 12{,}5$ $\qquad y = \frac{4+9}{2} = \frac{13}{2} = 6{,}5$	✓ $x = 12{,}5$ ✓ $y = 6{,}5$
	Centre/*midpt*: (12,5 ; 6,5)	
	Equation of the circle KLM /*Vgl van sirkel KLM*:	
	$\therefore \quad (x-12{,}5)^2 + (y-6{,}5)^2 = \frac{250}{4} = \frac{125}{2} = 62{,}5$	✓ answer in correct form/*antw in korrekte vorm* (5) **[21]**

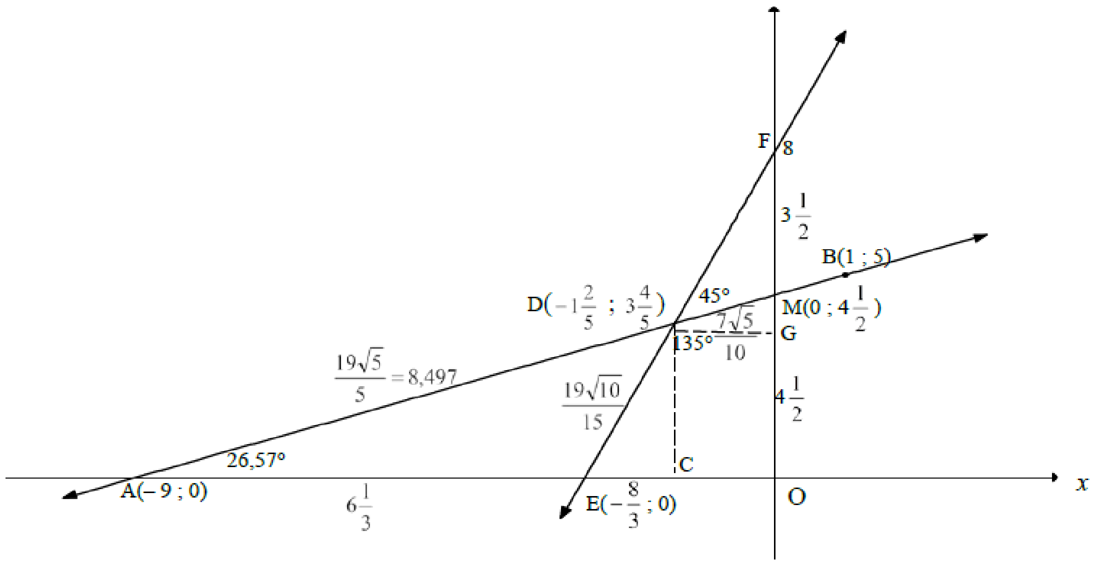
QUESTION/VRAAG 4



4.1	$y = 0$: $3x + 8 = 0$ $x = -\frac{8}{3}$ $\therefore \text{E}\left(-2\frac{2}{3}; 0\right)$ **OR/*OF*** $\text{E}\left(-\frac{8}{3}; 0\right)$	✓ y-value/*waarde* ✓ x-value/*waarde* (2)
4.2	$\tan \text{D}\hat{\text{E}}\text{O} = m_{\text{DE}} = 3$ $\therefore \text{D}\hat{\text{E}}\text{O} = 71{,}565... = 71{,}57°$ $\text{D}\hat{\text{A}}\text{E} = 71{,}565...° - 45°$ $= 26{,}57°$	✓ $\tan \text{D}\hat{\text{E}}\text{O} = 3$ ✓ $71{,}565\ldots°$ ✓ $26{,}57°$ (3)
4.3	$m_{\text{AB}} = \tan 26{,}57°$ $= \frac{1}{2}$ $y = \frac{1}{2}x + c$ **OR/*OF*** $y - y_1 = \frac{1}{2}(x - x_1)$ $5 = \frac{1}{2}(1) + c$ $\qquad$ $y - 5 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x + 4\frac{1}{2}$ $\qquad$ $y = \frac{1}{2}x + \frac{9}{2}$	✓ $m_{\text{AB}} = \tan 26{,}57°$ ✓ $m_{\text{AB}} = \frac{1}{2}$ ✓ subst of m and (1 ; 5)into formula/ *subst m en (1 ; 5) in formule* ✓ equation/*vgl* (4)

4.4	Solve $x - 2y + 9 = 0$ and $y = 3x + 8$ simultaneously: $x - 2(3x+8) + 9 = 0$	✓ subst/*vervang*
	$x - 6x - 16 + 9 = 0$ $-5x = 7$ $x = -1\frac{2}{5}$	✓ x-value/*waarde*
	$\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $-1\frac{2}{5} - 2y + 9 = 0$	✓ subst/*vervang*
	$y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$	✓ y-value/*waarde* (4)
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	
	OR/OF	
	$x = 2y - 9$ $y = 3(2y - 9) + 8$	✓ subst/*vervang*
	$y = 6y - 27 + 8$ $\therefore y = 3\frac{4}{5}$	✓ y value/*waarde*
	$x = 2(3\frac{4}{5}) - 9$ **OR/OF** $3\frac{4}{5} = 3x + 8$	✓ subst/*vervang*
	$x = -1\frac{2}{5}$ $x = -1\frac{2}{5}$	✓ x-value/*waarde*
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	
	OR/*OF*	(4)
	$3x + 8 = \frac{1}{2}x + 4\frac{1}{2}$	✓ equating/*gelyk stel*
	$6x + 16 = x + 9$ $5x = -7$ $\therefore x = -1\frac{2}{5}$	✓ x value/*waarde*
	$\therefore y = 3(-1\frac{2}{5}) + 8$ **OR/*OF*** $y = \frac{1}{2}(-1\frac{2}{5}) + 4\frac{1}{2}$	✓ subst/*vervang*
	$y = 3\frac{4}{5}$ $y = 3\frac{4}{5}$	✓ y-value/*waarde*
	$\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	(4)
	OR/*OF*	

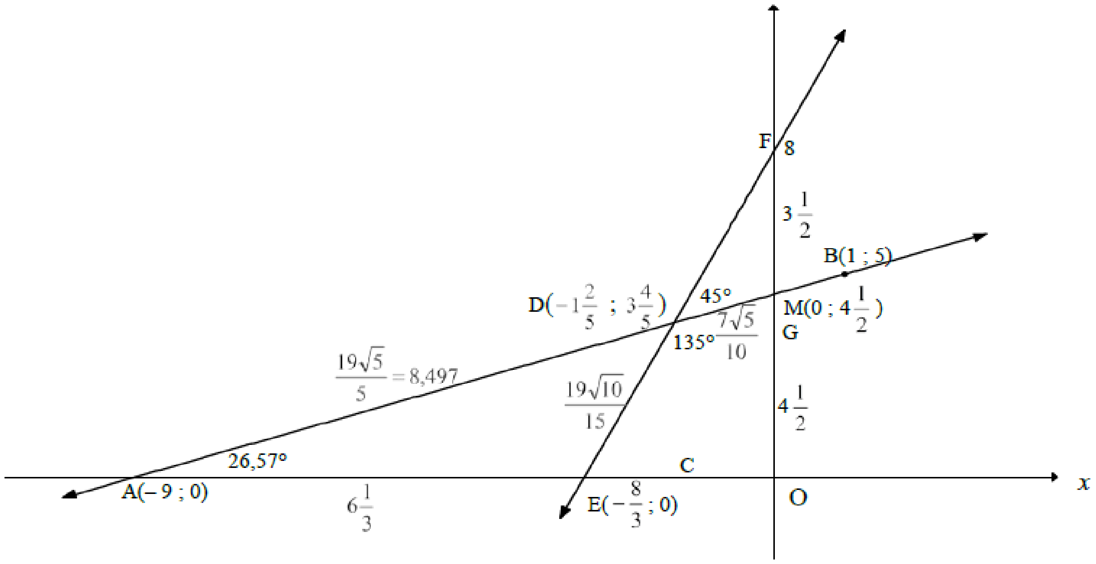
	$x-2y=-9$ ……(1) $-6x+2y=16$ ……(2) (1) + (2): $-5x \quad = 7$	✓ adding/*optelling*
	$\therefore x=-1\frac{2}{5}$	✓ x-value/*waarde*
	$\therefore -1\frac{2}{5}-2y=-9$ **OR/*OF*** $y=3(-1\frac{2}{5})+8$	✓ subst/*vervang*
	$y=3\frac{4}{5}$ $\quad$ $y=3\frac{4}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ y-value/*waarde*
	OR/*OF*	(4)
	$y=3x+8$ ………(1) $6y=3x+27$ …….(2) (1) – (2): $-5y = -19$	✓ subtracting/*aftrekking*
	$\therefore y=3\frac{4}{5}$	✓ y-value/*waarde*
	$3\frac{4}{5}=3x+8$ **OR/*OF*** $x=2(3\frac{4}{5})-9$	✓ subst/*vervang*
	$x=-1\frac{2}{5}$ $\quad$ $x=-1\frac{2}{5}$ $\therefore D(-1\frac{2}{5}\ ;\ 3\frac{4}{5})$	✓ x-value/*waarde* (4)



4.5	area DMOE = area ΔAMO – area ΔADE $x_A = 2(0) - 9 \quad \therefore A(-9\ ;\ 0)$	✓ correct method/ *korrekte metode* ✓ $x_A = -9$
	area ΔAMO $= \frac{1}{2}.\ AO.\ OM$ $= \frac{1}{2}(9)(4\frac{1}{2})$ $= 20{,}25$ area ΔADE $= \frac{1}{2}.\ AE\ .\ y_D$ $= \frac{1}{2}.\ (AO - EO).\ y_D$ $= \frac{1}{2}\left(9 - 2\frac{2}{3}\right)\left(3\frac{4}{5}\right)$ $= 12{,}03$	✓ $\frac{1}{2}(9)(4\frac{1}{2})$ ✓ $AE = 9 - 2\frac{2}{3} = 6\frac{1}{3}$ ✓ $y_D = 3\frac{4}{5}$
	OR/OF	**OR/OF**
	area ΔADE $= \frac{1}{2}AD.AE.\sin\ D\hat{A}E$ $= \frac{1}{2}\ (\frac{19\sqrt{5}}{5}).6\frac{1}{3}.\sin 26{,}57°$ $= 12{,}03$	✓ $AD = \frac{19\sqrt{5}}{5}$ ✓ $AE = 6\frac{1}{3}$
	∴ area DMOE = 8,22 square units/*vk eenh*	✓ answer/*antw* (6)
	OR/*OF*	

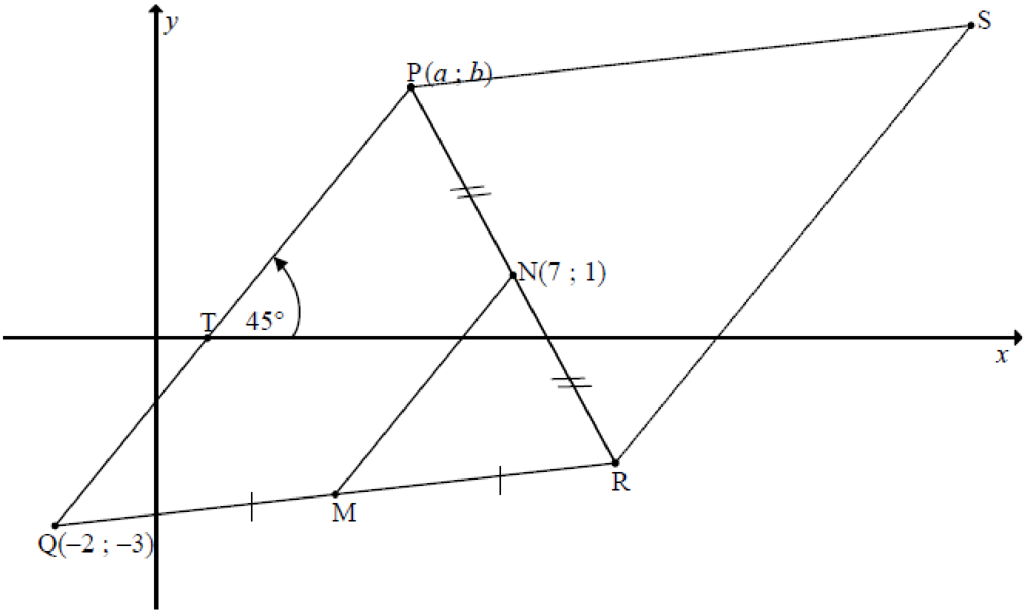
	area DMOE = area rectangle DCOG + area ΔDMG + area ΔDEC $= (1\frac{2}{5} \times 3\frac{4}{5}) + \frac{1}{2}\left(1\frac{2}{5}\right)(\frac{7}{10}) + \frac{1}{2}(3\frac{4}{5})(\frac{19}{15})$ = 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $3\frac{4}{5}$ ✓ $1\frac{2}{5}$ ✓ 0,7 ✓ $\frac{19}{15}$ ✓ answer (6)
	OR/*OF*	
	area DMOE = area ΔEDO + area ΔODM $= \frac{1}{2}(\text{EO} \times y_D) + \frac{1}{2}(\text{OM} \times -x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times \frac{19}{5}\right) + \left(\frac{9}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{304+189}{30}\right)$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_D = \frac{19}{5}$ or $3\frac{4}{5}$ ✓ EO = $\frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ OM = $\frac{9}{2}$ or $4\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	
	area DMOE = area ΔEOF – area ΔDMF $= \frac{1}{2}(\text{EO} \times \text{OF}) - \frac{1}{2}(\text{OF} - \text{OM})(-x_D)$ $= \frac{1}{2}\left[\left(\frac{8}{3} \times 8\right) + \left(\frac{7}{2} \times \frac{7}{5}\right)\right]$ $= \frac{1}{2}\left(\frac{640-147}{30}\right)$ $= \frac{493}{60}$ or $8\frac{13}{60}$ or 8,22 square units/*vk eenh*	✓ correct method/ *korrekte metode* ✓ $y_F = 8$ ✓ EO = $\frac{8}{3}$ ✓ $-x_D = \frac{7}{5}$ ✓ FM = $3\frac{1}{2}$ ✓ answer/*antw* (6)
	OR/*OF*	

	area $\Delta EOM = \frac{1}{2}(EO \times OM)$ $= \frac{1}{2}\left(\frac{8}{3} \times \frac{9}{2}\right)$ $= 6$ sq units/*vk eenh*	✓ area ΔEOM
	$ED = \sqrt{\left(-\frac{7}{5}+\frac{8}{3}\right)^2+\left(\frac{19}{5}\right)^2}$ and $DM = \sqrt{\left(\frac{7}{5}\right)^2+\left(\frac{9}{2}-\frac{19}{5}\right)^2}$ $= \frac{19\sqrt{10}}{15}$ or 4,005... $= \frac{7\sqrt{5}}{10}$ or 1,565..	✓ $ED = \frac{19\sqrt{10}}{15}$ ✓ $DM = \frac{7\sqrt{5}}{10}$
	area $\Delta EDM = \frac{1}{2}(ED \times DM \times \sin E\hat{D}M)$ $= \frac{1}{2}\left(\frac{19\sqrt{10}}{15}\right)\left(\frac{7\sqrt{5}}{10}\right)\sin 135°$ $= \frac{133}{60}$ or 2,216...	✓ area ΔEDM
	∴ area DMOE = area ΔEOM + area ΔEDM $= 6 + 2{,}216...$ $= \frac{493}{60}$ or/*of* $8\frac{13}{60}$ or/*of* 8,22 square units/*eenh*^{2}	✓ correct method/ *korrekte metode* ✓ answer/*antw* (6) **[19]**



NOVEMBER 2015

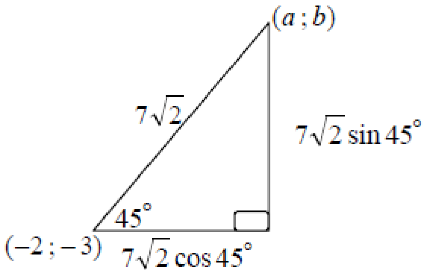
QUESTION/*VRAAG* 3



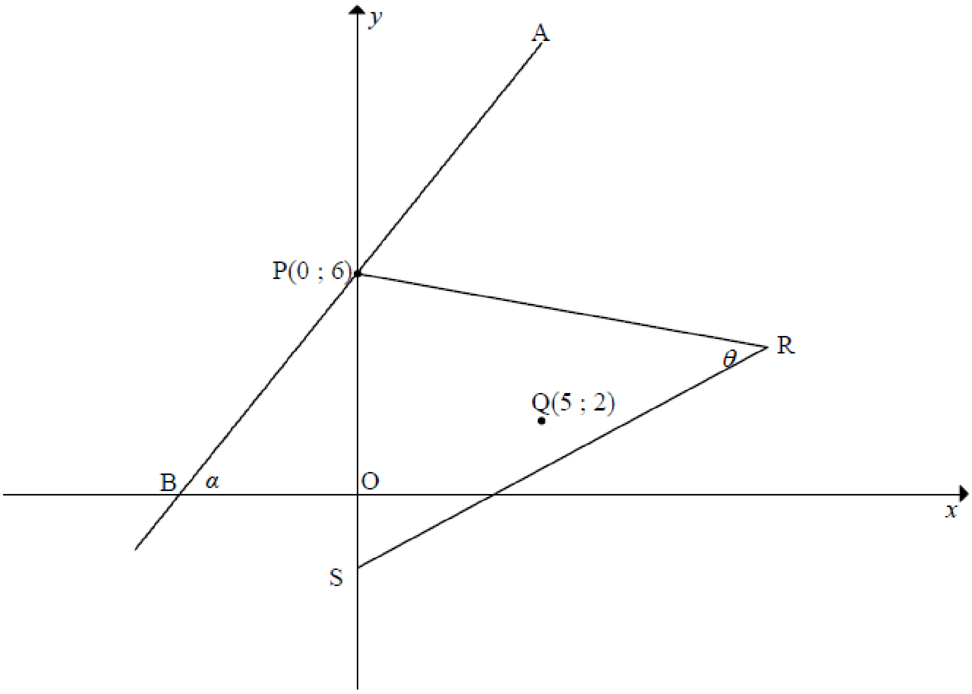
3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN | | QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore \quad y = x - 6$ **OR/*OF*** MN | | PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore \quad y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst m and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$\text{MN} = \frac{1}{2}\text{PQ}$ [midpoint theorem/midp stelling] $\therefore \text{MN} = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

<table>
<tr>
<td>3.5</td>
<td>QN = NS [diag of ||m/hoekl van ||m]
$\frac{-2+x_S}{2}=7$ and/en $\frac{-3+y_S}{2}=1$
∴ $x_S=16$ ∴ $y_S=5$
OR/<i>OF</i>
QN = NS [diag of ||m/hoekl van ||m]
∴ by inspection/deur inspeksie:
S(16 ; 5)</td>
<td>✓ method/metode
✓ <i>x</i>-value/waarde
✓ <i>y</i>-value/waarde
(3)
✓ method/metode
✓ <i>x</i>-value/waarde
✓ <i>y</i>-value/waarde
(3)</td>
</tr>
<tr>
<td>3.6</td>
<td>Equation of/<i>Vgl van PQ</i>: $y=x+c$
$-3=-2+c$
$y=x-1$ $\therefore a=b+1$ .....(1)
From distance formula/<i>Van afstandsformule</i>:
$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$
$\therefore 98=(a+2)^2+(b+3)^2$ .....(2)
Subst (1) into (2):
$98=(b+1+2)^2+(b+3)^2$
$98=b^2+6b+9+b^2+6b+9$
$0=2b^2+12b-80$
$0=b^2+6b-40$
$\therefore 0=(b+10)(b-4)$
$\therefore b=4$ (since $b>0$)
Subst $b=4$ into (1):
$\therefore a=4+1=5$
∴ P(5 ; 4)

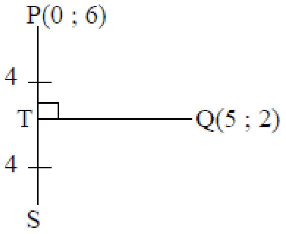
OR/<i>OF</i>
Equation of/<i>Vgl van PQ</i>: $y=x+c$
$-3=-2+c$
$y=x-1$ $\therefore a=b+1$ .....(1)
From distance formula/<i>Van afstandsformule</i>:
$7\sqrt{2}=\sqrt{(a-(-2))^2+(b-(-3))^2}$
$\therefore 98=(a+2)^2+(b+3)^2$ .....(2)
Subst (1) into (2):
$98=(b+1+2)^2+(b+3)^2$
$98=2(b+3)^2$
$49=(b+3)^2$
$\pm 7=b+3$
$\pm 7-3=b$
$\therefore b=4$ (since $b>0$)
Subst $b=4$ into (1):
$\therefore a=4+1=5$
∴ P(5 ; 4)</td>
<td>✓ eq of/<i>vgl van</i> PQ
✓ subst Q & $7\sqrt{2}$ into/<i>in</i> distance formula/ <i>afstandsformule</i>
✓ subst eq of/<i>vgl v.</i> PQ
✓ st form/<i>st vorm</i>
✓ value of/<i>waarde van b</i>
✓ value of/<i>waarde van a</i>
(6)
✓ eq of/<i>vgl van</i> PQ
✓ subst Q & $7\sqrt{2}$ into/<i>in</i> distance formula/ <i>afstandsformule</i>
✓ subst eq of/<i>vgl v.</i> PQ
✓ simplification/ <i>vereenvoudig</i>
✓ value of/<i>waarde van b</i>
✓ value of/<i>waarde van a</i>
(6)</td>
</tr>
</table>

	OR/***OF*** Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1 \qquad \therefore a = b + 1 \qquad .....(1)$ From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2+(b-(-3))^2}$ $98 = (a+2)^2 + (a-1+3)^2$ $= 2(a+2)^2$ $\therefore\ a+2 = 7$ (since/*aangesien* $a > 0$) $\therefore\ a = 5$ Subst $a = 4$ into (1): $\therefore b = 5 - 1 = 4$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van a* ✓ value of/*waarde van b* (6)
	OR/***OF***  $a = -2 + 7\sqrt{2}\cos 45^\circ = 5$ $b = -3 + 7\sqrt{2}\sin 45^\circ = 4$	✓✓✓✓ ✓ ✓ (6) **[17]**

QUESTION/*VRAAG* 4



4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $\text{PQ}=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6 \quad or/of \quad y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2=\pm 4$ $y = 2\pm 4$ $y \neq 6$ or / *of* $y=-2$ $\therefore$ S(0 ; –2)	✓ $x=0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3)
	OR/*OF* Draw/*Trek* QT ⊥ PS PT = TS [line from centre ⊥ to chord/ *lyn van midpt ⊥ koord*] PT = $y_P - y_Q = 6-2=4$ $y_Q - y_S = 4$ $y_S = 2-4=-2$ $\therefore$ S(0 ; –2) 	✓ $x=0$ ✓✓ $y=-2$ (3)
4.3	$m_{PQ} = \dfrac{6-2}{0-5}$ $= -\dfrac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* ⊥ radius] $\therefore\ m_{APB} = \dfrac{5}{4}$ $\therefore y = \dfrac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \dfrac{5}{4}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2)
	OR/*OF* B(4,8 ; 0) $\therefore\ \tan\alpha = \dfrac{6}{4{,}8}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = \dfrac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [∠ sum in Δ/∠ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4)
	OR/*OF* PS = 8 $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [∠ at centre = 2 × ∠ circumf] $= 38{,}66°$	✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}PS \times$height/*hoogte* $= \frac{1}{2}(8)(5)$ = 20 sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4)
	OR/*OF* $P\hat{Q}S = 2 \times 38{,}66°$ [∠at centre = 2×∠ at circum/ *midpts∠=2omtreks∠*] $= 77{,}32°$ Area $\Delta PQS = \frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ = 20 sq units/*vk eenh*	✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/*subst korrek* ✓ answ/*antw* (4) **[20]**

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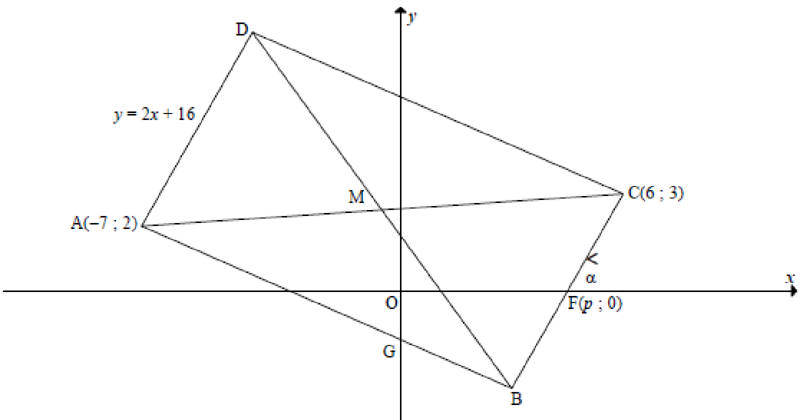
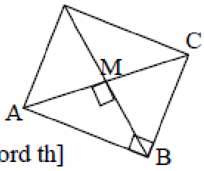
QUESTION/*VRAAG 3*

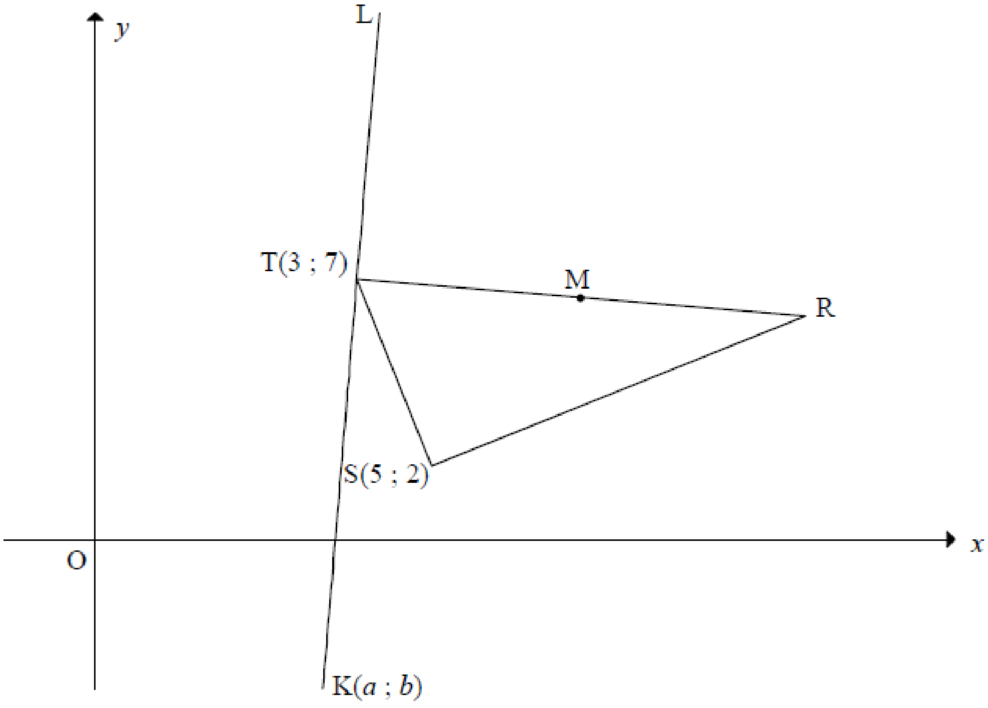


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= M\left(\frac{-7+6}{2};\frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2};\frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{BC}=\frac{3-0}{6-p}=\frac{3}{6-p}$ **OR/*OF*** $m_{BC}=\frac{0-3}{p-6}=\frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{AD}=m_{BC}$ [AD || BC] $m_{BC}=2$ $\frac{3}{6-p}=2$ $3=12-2p$ $p=4\frac{1}{2}$ **OR/OF** $y-y_1=2(x-x_1)$ $C(6;3)$ $y-3=2(x-6)$ $\therefore y=2x-9$ $but\ y=0$ $\therefore x=4\frac{1}{2}=p$ **OR/OF**	✓ $m_{BC}=2$ ✓ equating ✓answer (3) ✓ $m_{BC}=2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \qquad \therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore DB = \sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst ∠e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som ∠e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ ∴GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ ∴ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42{,}5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$\hat{CBM}=\hat{BAM}=45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF***  $\hat{AMB}=90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th]	✓S ✓R (2) ✓S ✓R (2) **[19]**

QUESTION/*VRAAG 4*



4.1	∠ in semi circle/ ∠ at centre = 2∠ on circle ∠ *in halfsirkel* /∠ by middelpt = 2∠ op sirkel	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

	$m_{TS} \times m_{RS} = -1$ [TS⊥SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $\text{TR} = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ r ✓ answer (2)
	OR/OF $\text{TM} = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $\text{TR} = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ substitution ✓ answer (2)
4.4.2	$\text{M}\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore \text{R}(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark)	✓ M ✓ x coordinate ✓ y coordinate (3)
	OR/*OF* $\text{M}\left(9 ; 6\frac{1}{2}\right)$ $\therefore \text{R}\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = \text{R}(15 ; 6)$	✓ M ✓ x coordinate ✓ y coordinate (3)
	OR/OF	

	$m_{TM}=\frac{9-3}{6\frac{1}{2}-7}=-\frac{1}{12}$ $TM:\ \ 7=-\frac{1}{12}(3)+c\quad y=-\frac{1}{12}x+\frac{29}{4}\quad ....(1)$ $SR:\ \ y=\frac{2}{5}x\quad ....(2)$ $\frac{2}{5}x=-\frac{1}{12}x+\frac{29}{4}$ $\frac{29}{60}x=\frac{29}{4}$ $\therefore x=15$ $\therefore y=\frac{2}{5}(15)=6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$\text{ST}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\text{ST}=\sqrt{(5-3)^2+(2-7)^2}$ $\text{ST}=\sqrt{4+25}=\sqrt{29}$ $\sin\text{R}=\frac{\text{TS}}{\text{TR}}=\frac{\sqrt{29}}{\sqrt{145}}\ or\ \frac{\sqrt{5}}{5}\ or\ \frac{1}{\sqrt{5}}\ or\ \ 0{,}45$ **OR/OF** $TS=\sqrt{29}$ $SR=2\sqrt{29}$ $\text{area of }\Delta\text{TSR}=\frac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right)=29$ $29=\frac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R=\frac{\sqrt{5}}{5}\ or\ \frac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{\text{TR}}=\frac{7-6\frac{1}{2}}{3-9}=-\frac{1}{12}$ **OR/OF** $m_{\text{TR}}=\frac{7-6}{3-15}=-\frac{1}{12}$ $m_{\text{TR}}\times m_{\text{KTL}}=-1$ $[r\perp\text{tangent}]$ $m_{\text{KTL}}=12$ $y-y_1=12(x-x_1)$ $y-7=12(x-3)$ $y=12x-29$ $\text{substitute K}(a;b):$ $b=12a-29$ **OR/OF**	✓ $m_{\text{TR}}=-\frac{1}{12}$ ✓ $m_{\text{KTL}}=12$ ✓ $y=12x-29$ (3)

	$m_{\text{TR}}=\dfrac{7-6\frac{1}{2}}{3-9}=-\dfrac{1}{12}$	✓ $m_{\text{TR}}=-\dfrac{1}{12}$
	$m_{\text{TR}}\times m_{\text{KTL}}=-1$ [$r\perp$ tangent] $\dfrac{b-7}{a-3}=12$ $b-7=12(a-3)$ $b=12a-29$	✓ $m_{\text{KTL}}=12$ ✓substitution (3 ; 7) & (*a* ; *b*) (3)
	OR/*OF*	
	$\text{KR}^2=\text{TR}^2+\text{TK}^2$ $(a-15)^2+(b-6)^2=(15-3)^2+(6-7)^2+(a-3)^2+(b-7)^2$	✓subst into Pyth
	$-30a+225-12b+36=144+1-6a+9-14b+49$ $2b=24a-58$ $b=12a-29$	✓multiplication ✓simplification (3)
4.4.5	$\text{TK}=\text{TR}$ $\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$	✓ substitution into distance formula
	Substitute $b=12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$	✓ substitution of $b=12a-29$
	$a^2-6a+9+144a^2-864a+1296-145=0$ $145a^2-870a+1160=0$ $a=\dfrac{870\pm\sqrt{(870)^2-4(145)(1160)}}{290}$ $a=2$ or $a=4$	✓standard form ✓ subst into formula or factorise ✓ values of *a*
	$\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ $\therefore$ K(2 ; –5)	✓ value of *b* (6)
	OR/*OF*	

	TK = TR	
	$\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$	✓ substitution into distance formula
	Substitute $b=12a-29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$	✓ substitution of $b=12a-29$
	$(a-3)^2+144(a-3)^2=145$ $(a-3)^2=1$	✓ $(a-3)^2=1$
	$a-3=\pm1$	✓ ±1
	$a=2$ or 4	✓ values of a
	$\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ $\therefore$ K(2 ; –5)	✓ value of b (6)
	OR/*OF*	
	$KR^2=TR^2+TK^2$ $(a-15)^2+(b-6)^2=145+145$ $(a-15)^2+(12a-29-6)^2=290$	✓ substitution ✓ substitution of $b=12a-29$
	$(a-15)^2+(12a-35)^2=290$ $a^2-30a+225+144a^2-840a+1225=290$	✓standard form
	$145a^2-870a+1160=0$ $a^2-6a+8=0$	✓ factors
	$\therefore(a-2)(a-4)=0$ $a=2$ or $a=4$	✓ values of a
	$\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ K$(2;-5)$	✓ value of b (6)
		[23]

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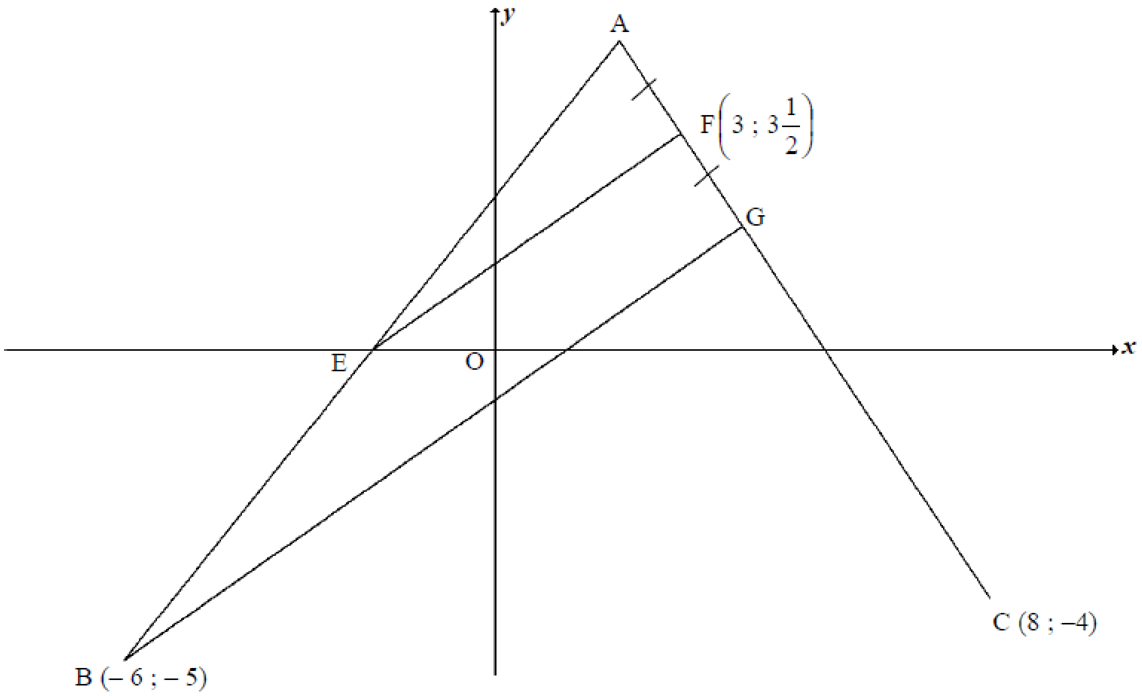
QUESTION/*VRAAG* 3

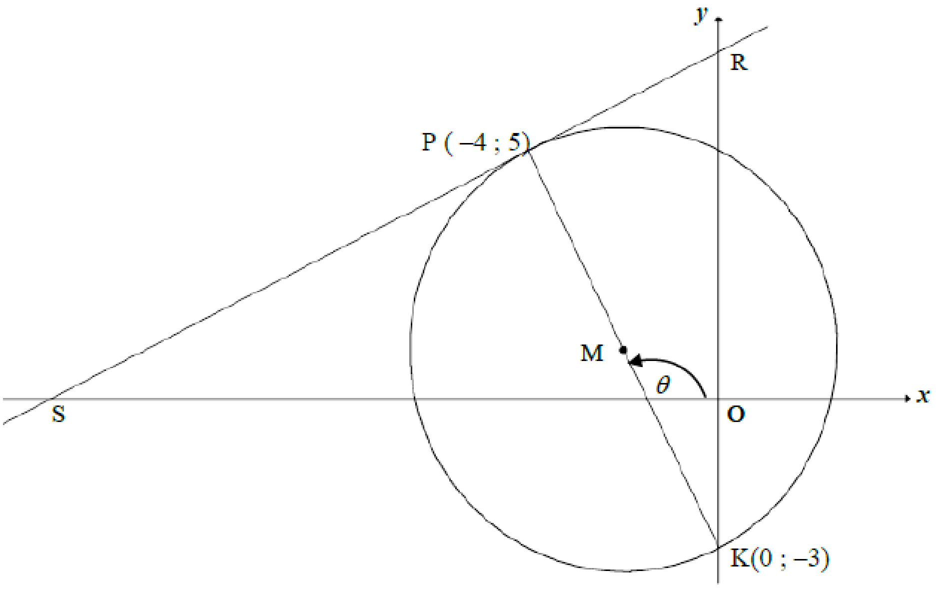


3.1.1	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ $y = mx + c$ $y - y_1 = m(x - x_1)$ $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c$ **OR/*OF*** $(y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$ $y = -\frac{3}{2}x + 8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3\,;3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of *m* and (8 ; –4) ✓ equation of AC (4)

	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4)-\left(3\frac{1}{2}\right)}{8-3}$ $= -\frac{3}{2}$ $y = mx + c$ $\quad\quad$ $y - y_1 = m(x - x_1)$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $\quad\quad$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ $c = 8$ $\quad$ **OR/*OF*** $\quad$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$ $\quad\quad$ $y = -\frac{3}{2}x + 8$	✓ substitution of (8 ; –4) & $\left(3;3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3;3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$ $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2) **OR/*OF*** BG: $7x - 10y = 8$ $\quad \therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8$ $\quad$ [CA from 3.1.1] $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2)	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3)
3.2	$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$ $\therefore$ A(2 ; 5) **OR/*OF*** by translation/*deur translasie*: $x_A = 3 - (4 - 3) = 2$ $y_A = 3\tfrac{1}{2} + (3\tfrac{1}{2} - 2) = 5$ $\therefore$ A(2 ; 5)	✓ equation ito x ✓ equation ito y (2) ✓ equation ito x ✓ equation ito y (2)

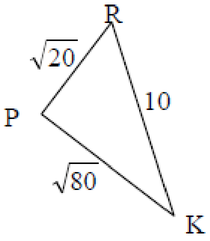
3.4	Midpoint of AC = $\left(5\ ;\ \frac{1}{2}\right)$ $\frac{x_D+(-6)}{2}=5$ and $\frac{y_D+(-5)}{2}=\frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5\ ;\ \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC}=\frac{-5-(-4)}{-6-8}=\frac{1}{14}$ and $m_{AB}=\frac{5-(-5)}{2-(-6)}=\frac{5}{4}$ AD: $y-5=\frac{1}{14}(x-2)\Rightarrow y=\frac{1}{14}x+\frac{34}{7}$ CD: $y+4=\frac{5}{4}(x-8)\Rightarrow y=\frac{5}{4}x-14$ $\frac{5}{4}x-14=\frac{1}{14}x+\frac{34}{7}$ $\therefore$ $x=16$ $y=6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

QUESTION/*VRAAG* 4

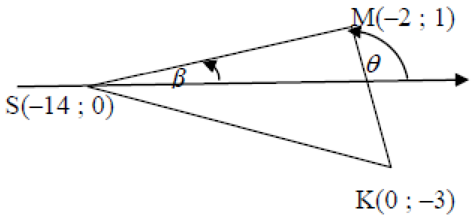
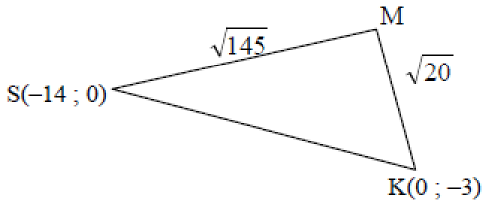


4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK ⊥ SR [radius ⊥ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK ⊥ SR **OR** r ⊥ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of *m* and P ✓ equation (2)

4.1.3	$M\left(\frac{-4+0}{2}\,;\,\frac{5+(-3)}{2}\right)$ $\therefore M(-2\,;\,1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4)
	OR/***OF***	
	$M\left(\frac{-4+0}{2}\,;\,\frac{5+(-3)}{2}\right)\quad \therefore M(-2\,;\,1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$	✓✓ M $(-2\,;\,1)$ $r^2=20$ ✓ equation (4)
	OR/***OF***	
	$M\left(\frac{-4+0}{2}\,;\,\frac{5+(-3)}{2}\right)\quad \therefore M(-2\,;\,1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$	✓✓ M $(-2\,;\,1)$ $r^2=20$ ✓ equation (4)

4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext $\angle$of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF*  In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ $RK = 10$ $\cos P\hat{K}R = \frac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ lengths of PK, PR & RK ✓ correct values into cos rule ✓ answer (3)
	OR/*OF* $\sin P\hat{K}R = \frac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \frac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF* $\tan P\hat{K}R = \frac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

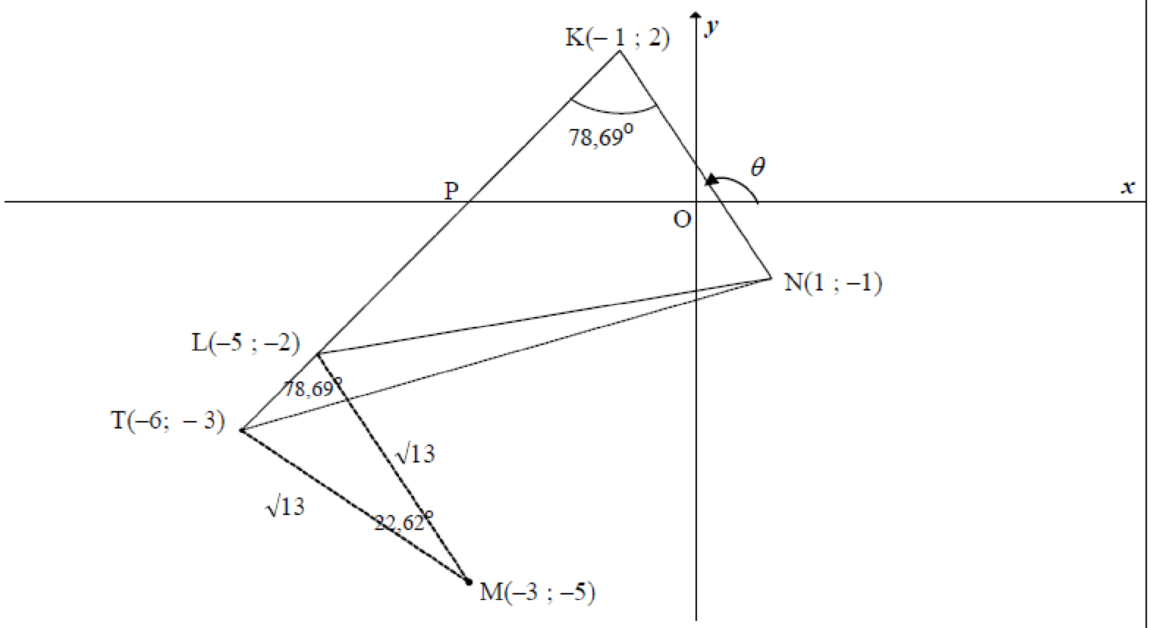
4.1.5	RS ∥ tangent at K$(0\ ;\ -3)$ $\therefore m_{PS}=m_{\text{tang}}=\frac{1}{2}$ $\therefore y=\frac{1}{2}x-3$ **OR/*OF*** $m_{PK}=\frac{1-5}{-2+4}=-2$ $m_{PK}\times m_{\tan g}=-1$ [radius ⊥ tangent/*raaklyn*] $\therefore m_{\tan g}=\frac{1}{2}$ $\therefore\ y=\frac{1}{2}x-3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t\in(-3\ ;\ 7)$ **OR/*OF*** $-3<t<7$	✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y=\frac{1}{2}x+7$ $\therefore$ S$(-14\ ;0)$ $\text{SP}=\sqrt{(-14-(-4))^2+(0-5)^2}=\sqrt{100+25}=\sqrt{125}$ Area ΔSMK $=\frac{1}{2}$. MK . SP $=\frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

	OR/OF	
		
	Let β = inclination of SM/ *inklinasie van SM*	
	RS: $y=\frac{1}{2}x+7$ $\therefore$ S(−14 ; 0)	✓ coordinates of S
	$\text{SM}=\sqrt{(-14-(-2))^2+(0-1)^2}=\sqrt{145}$	✓ length of SM
	$\tan\beta=\frac{1-0}{-2-(-14)}=\frac{1}{12}$ $\therefore$ $\beta=4{,}76°$	
	$\therefore\ \text{S}\hat{\text{M}}\text{K}=116{,}57°-4{,}76°$ [ext ∠ of Δ] $=111{,}81°$	✓ size of/*grootte v* $\text{S}\hat{\text{M}}\text{K}$
	Area $\Delta\text{SMK}=\frac{1}{2}(\text{SM})(\text{MK}).\sin\text{S}\hat{\text{M}}\text{K}$	
	$=\frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$	✓ correct substitution into area rule
	$=24{,}9985\ =25$ square units	✓ answer
		(5)
	OR/*OF*	
		
	RS: $y=\frac{1}{2}x+7$ $\therefore$ S(−14 ; 0)	✓ coordinates of S
	$\text{SK}=\sqrt{(-14-0)^2+(0+3)^2}=\sqrt{205}$	✓ length of SK
	$\cos\text{S}\hat{\text{M}}\text{K}=\frac{(\sqrt{145})^2+(\sqrt{20})^2-(\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})}=-\frac{2\sqrt{29}}{29}$	
	$\text{S}\hat{\text{M}}\text{K}=111{,}80°$	✓ size of /*grootte v* $\text{S}\hat{\text{M}}\text{K}$
	Area $\Delta\text{SMK}=\frac{1}{2}(\text{SM})(\text{MK}).\sin\text{S}\hat{\text{M}}\text{K}$	
	$=\frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$	✓ correct substitution into area rule
	$=24{,}9985\ =25$ square units	✓ answer
		(5)

	OR/***OF*** Produce KS to T RS: $y=\frac{1}{2}x+7 \quad \therefore \text{S}(-14\,;\,0)$ $\text{SK}=\sqrt{(-14-0)^2+(0+3)^2}=\sqrt{205}$ $\text{SM}=\sqrt{(-14-(-2))^2+(0-1)^2}=\sqrt{145}$ $m_{SK}=-\frac{3}{14} \Rightarrow \text{T}\hat{\text{S}}\text{O}=167{,}91°$ $m_{SM}=\frac{1}{12} \Rightarrow \text{M}\hat{\text{S}}\text{O}=4{,}76°$ $\text{M}\hat{\text{S}}\text{K}=180°-167{,}91°+4{,}76°=16{,}85°$ $\text{Area } \Delta\text{SMK}=\frac{1}{2}(\text{SM})(\text{SK}).\sin\text{M}\hat{\text{S}}\text{K}$ $=\frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$ $=24{,}9985 \ = 25 \text{ square units}$	✓ coordinates of S ✓ length of SK & SM ✓ size of /*grootte v* $\text{M}\hat{\text{S}}\text{K}$ ✓ correct substitution into area rule ✓ answer (5)

NOVEMBER 2018

QUESTION/*VRAAG* 3



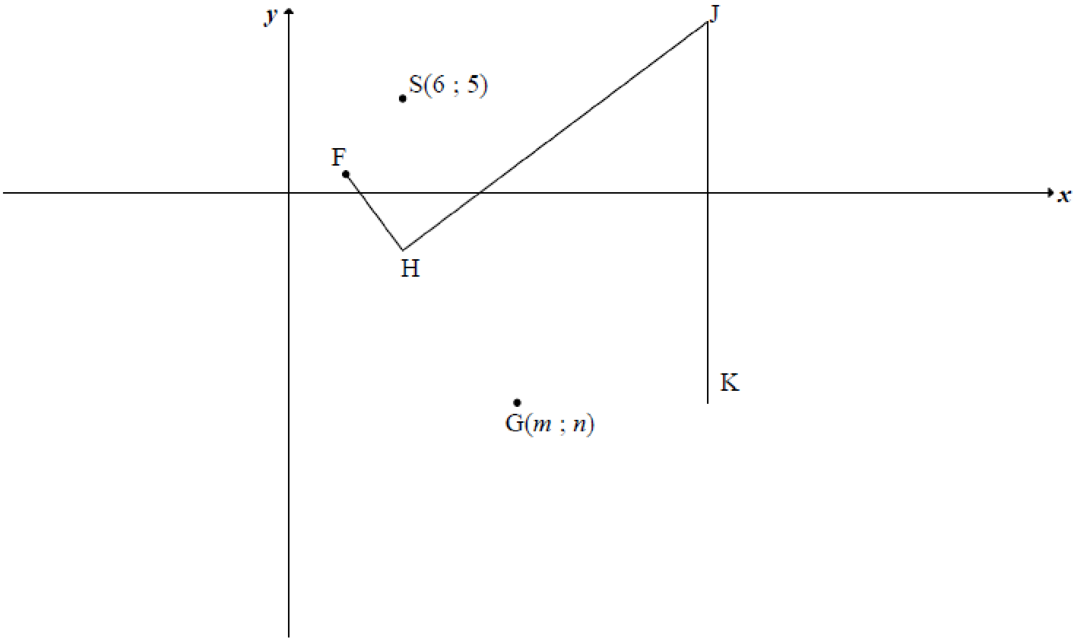
3.1.1	$m_{KN}=\frac{y_2-y_1}{x_2-x_1}$ $m_{KN}=\frac{2-(-1)}{-1-1}$ $=-\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta=m_{KN}=-\frac{3}{2}$ $\theta=180^\circ-56{,}31^\circ$ $\theta=123{,}69^\circ$ Answer only: Full marks	✓ $\tan\theta=m_{KN}=-\frac{3}{2}$ ✓ answer (2)
3.2	Inclination KL = 123,69° – 78,69° = 45° [ext ∠Δ] $\tan 45^\circ = m_{KL} = 1$	✓ S ✓ $\tan 45^\circ = m_{KL} = 1$ (2)
3.3	$y=x+c$ $2=-1+c$ $c=3$ $y=x+3$ **OR/*OF*** $y-y_1=1(x-x_1)$ $y-2=1(x-(-1))$ $y=x+3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)

3.4	$KN=\sqrt{(1+1)^2+(-1-2)^2}$ $KN=\sqrt{13}$ or 3,61 Answer only: Full marks	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ ...(2) (2) in (1):	✓ equation (1)
	$(x+3)^2+(x+3+5)^2=13$ $x^2+6x+9+x^2+16x+64=13$	✓ substituting eq (2)
	$2x^2+22x+60=0$ $x^2+11x+30=0$ $(x+5)(x+6)=0$	✓ standard form
	$x=-5$ or $x=-6$ $y=-2$ or $y=-3$ L$(-5\,;-2)$ or $(-6\,;-3)$	✓ x-values ✓ y-values (5)
	OR/OF	
	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ $\therefore x=y-3$ ...(2) (2) in (1):	✓ equation (1)
	$(y-3+3)^2+(y+5)^2=13$ $y^2+y^2+10y+25=13$ $2y^2+10y+12=0$	✓ substituting eq (2)
	$y^2+5y+6=0$ $(y+2)(y+3)=0$	✓ standard form
	$y=-2$ or $y=-3$ $x=-5$ or $x=-6$ L$(-5\,;-2)$ or $(-6\,;-3)$	✓ y-values (both) ✓ x-values (both) (5)
3.5.2	Midpoint of KM: $(-2\,;-1{,}5)$	✓ midpoint of KM
	$\therefore \frac{x_L+1}{2}=-2$ and $\frac{y_L-1}{2}=-\frac{3}{2}$ $\therefore$ L$(-5\,;-2)$	✓x value ✓ y value (3)
	OR/OF	
	$m_{KN}=m_{LM}$ $\frac{y-(-5)}{x-(-3)}=-\frac{3}{2}$ $2(x+3+5)=-3(x+3)$	✓ $m_{LM}=m_{KN}$
	$2x+16=-3x-9$ $5x=-25$ $x=-5$ $\therefore$ L$(-5\,;-2)$ Answer only: Full marks	✓x value ✓ y value (3)

	OR/*OF* N→M: $(x;y)\to(x-4;y-4)$ $\therefore$ L$(-1-4;2-4)$ **OR/OF** $\therefore$ L$(-5;-2)$ N→K: $(x;y)\to(x-2;y+3)$ $\therefore$ L$(-3-2;-5+3)$ $\therefore$ L$(-5;-2)$	
	OR/*OF* N→M: $(x;y)\to(x-4;y-4)$ $\therefore$ L$(-1-4;2-4)$ **OR/OF** $\therefore$ L$(-5;-2)$ N→K: $(x;y)\to(x-2;y+3)$ $\therefore$ L$(-3-2;-5+3)$ $\therefore$ L$(-5;-2)$	
	OR/*OF* N→M: $(x;y)\to(x-4;y-4)$ $\therefore$ L$(-1-4;2-4)$ **OR/OF** $\therefore$ L$(-5;-2)$ N→K: $(x;y)\to(x-2;y+3)$ $\therefore$ L$(-3-2;-5+3)$ $\therefore$ L$(-5;-2)$	
	OR/*OF* N→M: $(x;y)\to(x-4;y-4)$ $\therefore$ L$(-1-4;2-4)$ **OR/OF** $\therefore$ L$(-5;-2)$ N→K: $(x;y)\to(x-2;y+3)$ $\therefore$ L$(-3-2;-5+3)$ $\therefore$ L$(-5;-2)$	
	OR/*OF* N→M: $(x;y)\to(x-4;y-4)$ $\therefore$ L$(-1-4;2-4)$ **OR/OF** $\therefore$ L$(-5;-2)$ N→K: $(x;y)\to(x-2;y+3)$ $\therefore$ L$(-3-2;-5+3)$ $\therefore$ L$(-5;-2)$	✓ transformation ✓ x value ✓ y value (3)
3.6	T(–6 ; –3) (from Question 3.5.1) $KT=\sqrt{(-1-(-6))^2+(2-(-3))^2}$ $=\sqrt{50}$ $KN=\sqrt{13}$ (CA from 3.4) Area of $\Delta KTN=\frac{1}{2}KT.KN\sin L\hat{K}N$ $=\frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ = 12,50 square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)

	OR/*OF* In ΔKLM: $\frac{\text{TL}}{\sin 22{,}62°}=\frac{\sqrt{13}}{\sin 78{,}69°}$ $\text{TL}=1{,}414..$ $\text{KL}=\sqrt{(-1-(-5))^2+(2-(-2))^2}$ $=\sqrt{32}$ $\therefore\ \text{KT}=7{,}0708\ldots$ Area of $\Delta\text{KTN}=\frac{1}{2}\text{KT.KN}\sin\text{L}\hat{\text{K}}\text{N}$ $=\frac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$ $=12{,}50$ square units	✓ length of TL ✓ length of KT ✓ substitution into area rule ✓ answer (4)
		[22]

QUESTION/*VRAAG* 4



4.1	F(3;1)	✓x value ✓ y value (2)
4.2	$FS=\sqrt{(6-3)^2+(5-1)^2}$ FS = 5	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 ∴ HG = 2 FH = 10	✓ HG = 10 (1)
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J=90°$ [tan ⊥ radius / *rkl ⊥ radius*] $FJ^2=20^2+5^2$ [Pyth theorem/*stelling*] $FJ=\sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2+(y-n)^2=100$	✓ answer (1)

4.5.3	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii]	✓ K(22; n)
	$m = 22 - 10$ $m = 12$	✓ value of m
	F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig [HJ is 'n gemeemskaplike raaklyn]*	
	$FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$	✓ subst. of F and G in distance formula ✓ FG = 15
	$(n-1)^2 = 144$ $n - 1 = \pm 12$ **OR/*OF*** $n \neq 13$ or $n = -11$ $\therefore$ G(12; −11) $n^2 - 2n - 143 = 0$ $(n+11)(n-13) = 0$ $n = -11$ or $n \neq 13$	✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7)
	OR/*OF*	
	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii]	✓ K(22; n)
	$m = 22 - 10$ $m = 12$	✓ value of m
	Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$	✓ subst. of F and J in distance formula ✓ FJ = $\sqrt{425}$
	$0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$	✓ standard form
	$\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore$ G(12; −11)	✓ value of n ✓ coordinates of G (7)
		[18]

TRIGONOMETRY: SOLUTIONS

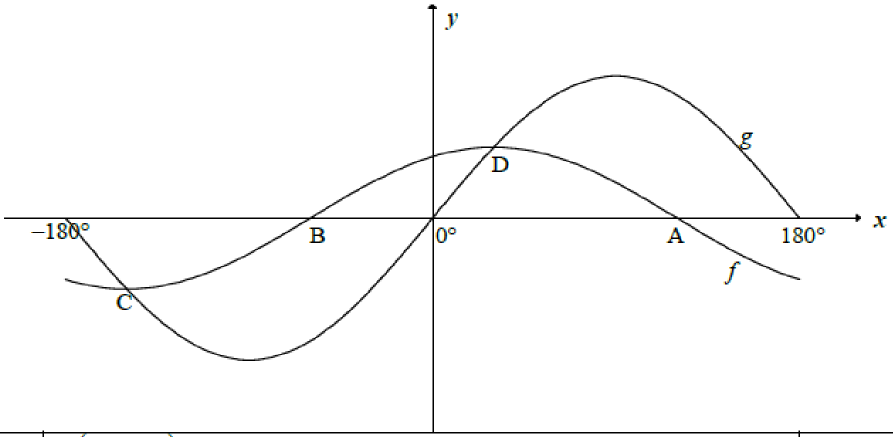
MAY/JUNE 2019

QUESTION/*VRAAG* 5

5.1.1	$\sin 191°$ $=-\sin 11°$	✓ $-\sin 11°$ (1)
5.1.2	$\cos 22°$ $=\cos(2\times 11°)$ $=1-2\sin^2 11°$	✓ answer (1)
5.2	$\cos(x-180°)+\sqrt{2}\sin(x+45°)$ $=-\cos x+\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)$ $=-\cos x+\sqrt{2}\left(\sin x\left(\frac{1}{\sqrt{2}}\right)+\cos x\left(\frac{1}{\sqrt{2}}\right)\right)$ $=-\cos x+\sin x+\cos x$ $=\sin x$ **OR** $\cos(x-180°)+\sqrt{2}\sin(x+45°)$ $=-\cos x+\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)$ $=-\cos x+\sqrt{2}\left(\sin x\left(\frac{\sqrt{2}}{2}\right)+\cos x\left(\frac{\sqrt{2}}{2}\right)\right)$ $=-\cos x+\sin x+\cos x$ $=\sin x$	✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5) ✓ $-\cos x$ ✓ expansion ✓ special angle ratios ✓simplification of last 2 terms ✓answer (5)
5.3	$\sin \text{P}+\sin \text{Q}=\sin \text{P}+\cos \text{P}$ $(\sin \text{P}+\cos \text{P})^2=\left(\frac{7}{5}\right)^2$ $\sin^2 \text{P}+2\sin \text{P}\cos \text{P}+\cos^2 \text{P}=\frac{49}{25}$ $2\sin \text{P}\cos \text{P}=\frac{49}{25}-1$ $\sin 2\text{P}=\left(\frac{49}{25}-\frac{25}{25}\right)$ $=\frac{24}{25}$	✓ $\sin \text{Q}=\cos \text{P}$ ✓ squaring ✓ expansion ✓ $\sin^2 \text{P}+\cos^2 \text{P}=1$ ✓answer (5)
		[12]

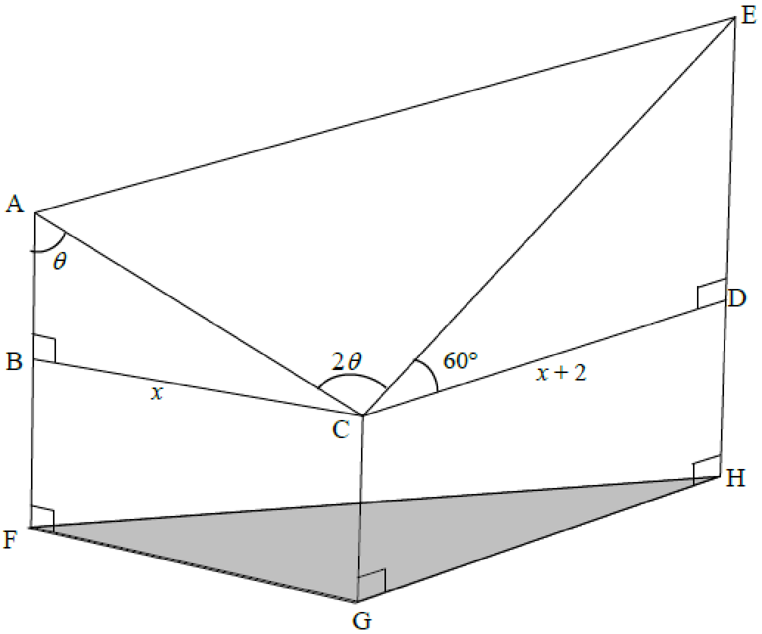
QUESTION/*VRAAG* 6

6.1	$\cos(x-30^\circ) = 2\sin x$ $\cos x\cos 30^\circ + \sin x\sin 30^\circ = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\sin x$ $\frac{\sqrt{3}}{2}\cos x = \frac{3}{2}\sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ + k.180^\circ;\quad k \in Z$ **OR** $x = 30^\circ + k.360^\circ$ or $x = 210^\circ + k.360^\circ;\quad k \in Z$	✓ expansion ✓ special ∠s ✓ simplification ✓ equation in tan ✓ 30° ✓ $k.180^\circ; k \in Z$ **OR** ✓ 30° and 210° ✓ $k.360^\circ;\quad k \in Z$ (6)



6.2.1(a)	$A(120^\circ\ ;\ 0)$	✓ answer (1)
6.2.1(b)	$C(-150^\circ\ ;\ -1)$	✓ x value ✓ y value (2)
6.2.2(a)	$x \in (-90^\circ\ ;\ 30^\circ)$ OR $-90^\circ < x < 30^\circ$	✓ endpoints ✓ correct interval (2)
6.2.2(b)	$x \in (-160^\circ\ ;20^\circ)$ OR $-160^\circ < x < 20^\circ$	✓endpoints ✓correct interval (2)
6.2.3	$y = 2^{2\sin x+3}$ Range of $y = 2\sin x$: $y \in [-2\ ;\ 2]$ **OR** $-2 \le y \le 2$ Range of $y = 2\sin x + 3$: $y \in [1\ ;\ 5]$ **OR** $1 \le y \le 5$ Range: $y = 2^{2\sin x+3}$: $y \in [2\ \ ;\ 32]$ **OR** $2 \le y \le 32$ Answer only: full marks	✓ 1 ✓ 5 ✓ 2 ✓ 32 ✓ correct interval (5)
		[18]

QUESTION/*VRAAG* 7

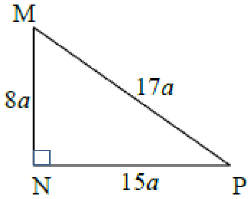


7.1.1	$\sin\theta=\frac{x}{AC}$ $AC=\frac{x}{\sin\theta}$ **OR** $\frac{\sin\theta}{x}=\frac{\sin 90°}{AC}$ $AC=\frac{x}{\sin\theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60°=\frac{x+2}{CE}$ $CE=\frac{x+2}{\cos 60°}$ $=\frac{x+2}{\frac{1}{2}}=2(x+2)$ **OR** $\frac{\sin 30}{x+2}=\frac{\sin 90°}{CE}$ $CE=\frac{x+2}{\sin 30°}$ $=2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	Area $\Delta ACE=\frac{1}{2}AC.EC.\sin A\hat{C}E$ $=\frac{1}{2}\left(\frac{x}{\sin\theta}\right)(2(x+2))\sin 2\theta$ $=\frac{x(x+2)\times 2\sin\theta\cos\theta}{\sin\theta}$ $=2x(x+2)\cos\theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin\theta}(2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

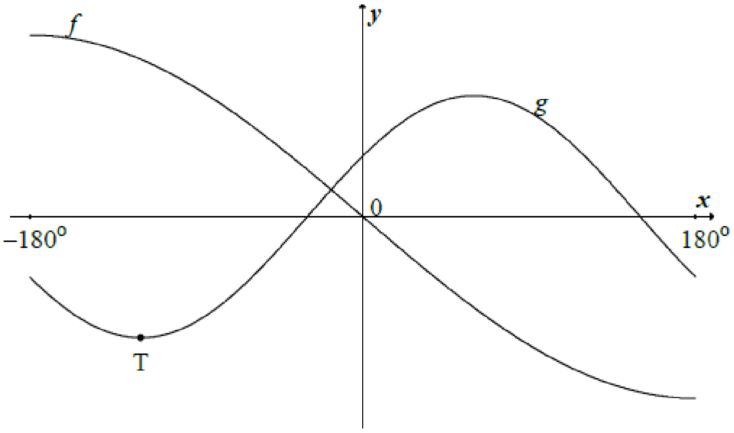
7.3	EC = 2(12 + 2) = 28 $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos A\hat{C}E$ $= \left(\frac{12}{\sin 55°}\right)^2 + 28^2 - 2\left(\frac{12}{\sin 55°}\right)(28)\cos 110°$ AE = 35,77*m*	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
		[11]

MAY/ JUNE 2018

QUESTION/*VRAAG* 5

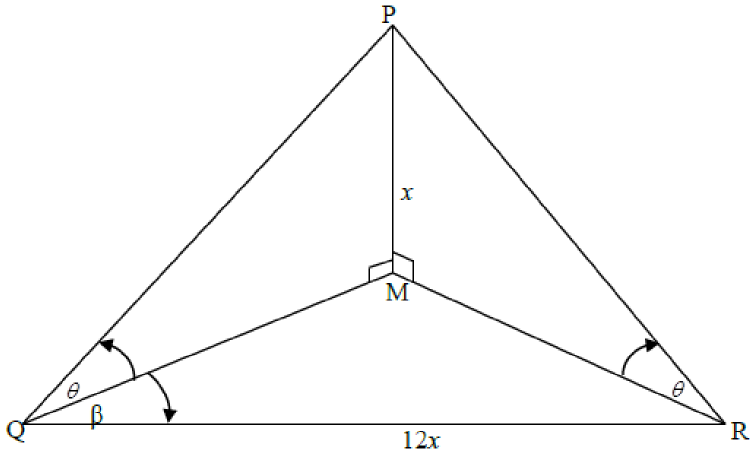
5.1.1	Given : $\sin M = \frac{15}{17}$ $MN^2 = 17^2 - 15^2$ $= 64$ $MN = 8$ OR  $\therefore \tan M = \frac{15}{8}$	✓ sketch or Pyth ✓ MN = 8 ✓answer (3)
5.1.2	$\sin M = \frac{NP}{MP}$ $\frac{NP}{51} = \frac{15a}{17a}$ $\therefore NP = 45$	✓ equating trig ratios ✓ answer (2)
5.2	$\cos(x - 360^\circ).\sin(90^\circ + x) + \cos^2(-x) - 1$ $= \cos x \ . \cos x + \cos^2 x - 1$ $= \cos^2 x + \cos^2 x - 1$ $= 2\cos^2 x - 1$ $= \cos 2x$	✓ $\cos x$ ✓ $\cos x$ ✓ $\cos^2 x$ ✓ identity (4)
5.3.1	$\sin(2x + 40^\circ)\cos(x + 30^\circ) - \cos(2x + 40^\circ)\sin(x + 30^\circ)$ $= \sin[(2x + 40^\circ) - (x + 30^\circ)]$ $= \sin(x + 10^\circ)$	✓ reduction ✓ answer (2)
5.3.2	$\sin(2x + 40^\circ)\cos(x + 30^\circ) - \cos(2x + 40^\circ)\sin(x + 30^\circ) = \cos(2x - 20^\circ)$ $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\cos(2x - 20^\circ) = \cos[90^\circ - (x + 10^\circ)]$ $2x - 20^\circ = 80^\circ - x + k.360^\circ$ or $2x - 20^\circ = 360^\circ - (80^\circ - x) + k.360^\circ$ $3x = 100^\circ + k.360^\circ$ or $2x - 20^\circ = 280^\circ + x + k.360^\circ$ $x = 33{,}33^\circ + k.120^\circ$ or $x = 300^\circ + k.360^\circ \ ; k \in Z$ **OR/*OF*** $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\sin[90^\circ - (2x - 20^\circ)] = \sin(x + 10^\circ)$ $110^\circ - 2x = x + 10^\circ + k.360^\circ$ or $110^\circ - 2x = 180^\circ - (x + 10^\circ) + k.360^\circ$ $3x = 100^\circ - k.360^\circ$ or $110^\circ - 2x = 170^\circ - x + k.360^\circ$ $x = 33{,}33^\circ - k.120^\circ$ or $x = -60^\circ - k.360^\circ \ ; k \in Z$	✓ equating ✓ co ratio ✓ $80^\circ - x$ ✓$280^\circ + x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33^\circ + k.120^\circ$ ✓ $x = 300^\circ + k.360^\circ$; $k \in Z$ (7) ✓ equating ✓ co ratio ✓ $x + 10^\circ$ ✓$170^\circ - x$ ✓ simplification/*vereenv* ✓ $x = 33{,}33^\circ - k.120^\circ$ ✓ $x = -60^\circ - k.360^\circ$; $k \in Z$ (7)
		[18]

QUESTION/*VRAAG* 6



6.1	Period = 720°	✓ answer (1)
6.2	$y \in [-2\ ;\ 2]$ **OR/*OF*** $-2 \le y \le 2$	✓✓ answer (2) ✓✓ answer (2)
6.3	$f(-120°) - g(-120°)$ $= -3\sin\left(-\frac{120°}{2}\right) - 2\cos(-120° - 60°)$ $= \frac{4+3\sqrt{3}}{2}$ or 4,60 (4,5980...)	✓ $x = -120°$ ✓ substitution ✓ answer (3)
6.4.1	x-intercepts of g at –90° + 60° = –30° and 90° + 60° = 150° $\therefore x \in (-30°\ ;\ 150°)$ **OR/*OF*** x-intercepts of g at –90° + 60° = –30° and 90° + 60° = 150° $-30° < x < 150°$	✓ value ✓ value ✓ answer (3) ✓ value ✓ value ✓ answer (3)
6.4.2	$x \in [-180°\ ;\ -120°) \cup (-30°\ ; 60°) \cup (150°\ ; 180°]$ **OR/*OF*** $-180° \le x < -120°$ or $-30° < x < 60°$ or $150° < x \le 180°$	✓ [–180° ; –120°) ✓ (–30° ; 60°) ✓ (150° ; 180°] ✓ notation for inclusive in the first/last interval (4) ✓ $-180° \le x < -120°$ ✓ $-30° < x < 60°$ ✓ $150° < x \le 180°$ 1 mark: each interval ✓ notation for inclusive in the first/last interval (4)
		[13]

QUESTION/*VRAAG* 7

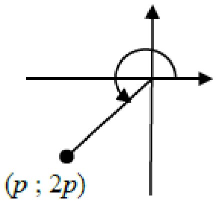


7.1	In PMQ : $\tan\theta = \frac{x}{\text{QM}}$ $\therefore \text{QM} = \frac{x}{\tan\theta}$ **OR/*OF*** $\frac{x}{\sin\theta} = \frac{\text{MQ}}{\sin P}$ $MQ = \frac{x\sin P}{\sin\theta}$ $= \frac{x\cos\theta}{\sin\theta}$ $= \frac{x}{\tan\theta}$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	In PMR : $\tan\theta = \frac{x}{\text{MR}}$ *OR* PMQ ≡ PMR [AAS/HHS] $\therefore \text{MR} = \frac{x}{\tan\theta} = \text{QM}$ $\text{Q}\hat{\text{M}}\text{R} = 180° - 2\beta$ $\frac{\sin\beta}{\text{MR}} = \frac{\sin \text{Q}\hat{\text{M}}\text{R}}{12x}$ $\sin\beta \times \frac{\tan\theta}{x} = \frac{\sin(180° - 2\beta)}{12x}$ $\tan\theta = \frac{\sin 2\beta}{12x} \times \frac{x}{\sin\beta}$ $\tan\theta = \frac{2\sin\beta\cos\beta}{12x} \times \frac{x}{\sin\beta}$ $\tan\theta = \frac{\cos\beta}{6}$ **OR**	✓ MR = QM ✓ correct substitution into the sine rule in ΔQMR ✓ reduction ✓ double angle (4)

	In PMR : $\tan\theta = \frac{x}{\text{MR}}$ *OR* PMQ $\equiv$ PMR [AAS/HHS] $\text{MR}^2 = \text{QM}^2 + \text{QR}^2 - 2\text{QM.QR}\cos\beta$ $\text{MR}^2 = \left(\frac{x}{\tan\theta}\right)^2 + (12x)^2 - 2\left(\frac{x}{\tan\theta}\right)(12x)(\cos\beta)$ $\frac{x^2}{\tan^2\theta} = \frac{x^2}{\tan^2\theta} + 144x^2 - 24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta)$ $24\left(\frac{x^2}{\tan\theta}\right)(\cos\beta) = 144x^2$ $\cos\beta = 6\tan\theta$ $\tan\theta = \frac{\cos\beta}{6}$	✓ correct substitution into the cosine rule in ΔQMR ✓ substitution ✓ MR = QM ✓ simplification (4)
7.3	$\frac{x}{\text{QM}} = \frac{\cos\beta}{6}$ [both equal $\tan\theta$] $x = \frac{60\cos 40}{6}$ $x = 7{,}66$ The height of the lighthouse is 8 metres	✓ equating ✓ subst. QM = 60 and $\beta = 40°$ ✓ answer (3)
		[9]

MAY/JUNE 2017

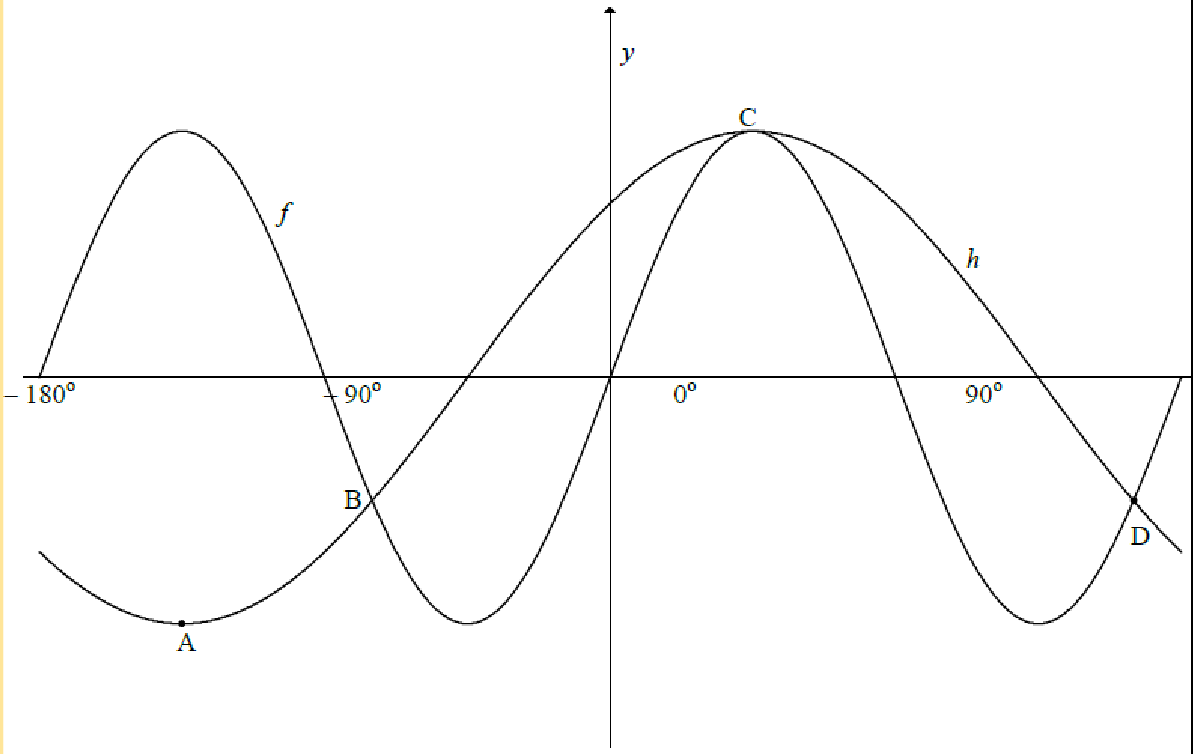
QUESTION/VRAAG 5

5.1.1	$\tan A = \frac{\sin A}{\cos A}$ $= \frac{2p}{p}$ $= 2$ **OR/*OF*** $\tan A = \frac{2p}{p}$ $= 2$ 	✓identity ✓value of tan A (2) ✓ $\frac{y}{x}$ ✓value of tan A (2)
5.1.2	$\sin^2 A + \cos^2 A = 1$ $(2p)^2 + p^2 = 1$ $4p^2 + p^2 = 1$ $5p^2 = 1$ $p^2 = \frac{1}{5}$ $\therefore p = -\frac{1}{\sqrt{5}}$	✓ $(2p)^2 + p^2 = 1$ ✓ simplification of LHS ✓answer (3)
5.2	$2\sin^2 x - 5\sin x + 2 = 0$ $(2\sin x - 1)(\sin x - 2) = 0$ $\sin x = \frac{1}{2}$ or $\sin x = 2$(no solution) ref $\angle = 30°$ $\therefore x = 30° + k.360°$ or $x = 150° + k.360°\,; k \in Z$	✓ factors or formula ✓ both equations ✓ no solution/*geen opl* ✓ $30° + k.360°$ ✓ $150° + k.360°$; ✓ $k \in Z$ (6)
5.3.1	$\sin(x + 300°) = \sin x \cos 300° + \cos x \sin 300°$	✓ expansion/*uitbreiding* (1)
5.3.2	$\sin(x + 300°) - \cos(x - 150°)$ $= \sin x \cos 300° + \cos x \sin 300° - (\cos x \cos 150° + \sin x \sin 150°)$ $= \sin x \cos 60° - \cos x \sin 60° - (-\cos x \cos 30° + \sin x \sin 30°)$ $= \sin x \cos 60° - \cos x \sin 60° + \cos x \cos 30° - \sin x \sin 30°$ $= \frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ $= 0$ **OR/*OF***	✓2nd expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ special angle values/ *spesiale hoekwaardes* ✓ answer (5)

	$\sin(x+300°)-\cos(x-150°)$ $=\sin x\cos 300°+\cos x\sin 300°-(\cos x\cos 150°+\sin x\sin 150°)$ $=\sin x\cos 60°-\cos x\sin 60°-(-\cos x\cos 30°+\sin x\sin 30°)$ $=\sin x\cos 60°-\cos x\sin 60°+\cos x\cos 30°-\sin x\sin 30°$ $=\sin x\sin 30°-\cos x\sin 60°+\cos x\sin 60°-\sin x\sin 30°$ $=0$	✓2^{nd} expansion/ *2de uitbreiding* ✓✓ reduction/*reduksie* ✓ co-ratios / *ko-verh* ✓ answer (5)
5.4	Consider: $\frac{\tan x+1}{\sin x\tan x+\cos x}=\sin x+\cos x$ $\text{LHS}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\sin x.\frac{\sin x}{\cos x}+\cos x\right)}=\frac{\left(\frac{\sin x+\cos x}{\cos x}\right)}{\left(\frac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\frac{\frac{\sin x+\cos x}{\cos x}}{\frac{1}{\cos x}}$ $=\frac{\sin x+\cos x}{\cos x}\times\frac{\cos x}{1}$ $=\sin x+\cos x$ $=\text{RHS}$ **OR**/***OF*** $\text{LHS}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\sin x.\frac{\sin x}{\cos x}+\cos x\right)}=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\left(\frac{\sin^2 x+\cos^2 x}{\cos x}\right)}$ $=\frac{\left(\frac{\sin x}{\cos x}+1\right)}{\frac{1}{\cos x}}$ $=\left(\frac{\sin x}{\cos x}+1\right)\times\frac{\cos x}{1}$ $=\sin x+\cos x$ $=\text{RHS}$	✓ identity of $\tan x$ ✓ $\frac{\sin x+\cos x}{\cos x}$ ✓ $\frac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify (5) ✓ identity of $\tan x$ ✓ $\frac{\sin^2 x+\cos^2 x}{\cos x}$ ✓ $\sin^2 x+\cos^2 x=1$ ✓ simplify ✓ multiplication (5)
5.5.1	$\left(\sqrt{1+k}\right)^2=(\sin x+\cos x)^2$ $1+k=\sin^2 x+2\sin x\cos x+\cos^2 x$ $1+k=1+\sin 2x$ $k=\sin 2x$	✓ square both sides ✓ $\sin^2 x+\cos^2 x=1$ ✓ $\sin 2x$ (3)

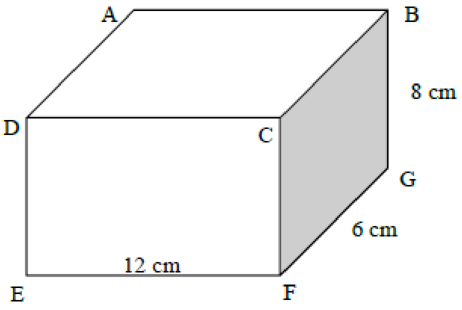
5.5.2	From 5.5.1 $\sin x + \cos x = \sqrt{1+\sin 2x}$ $\therefore$ max value: $\sin x + \cos x = \sqrt{1+1}$ $= \sqrt{2}$	✓ max of $\sin 2x = 1$ ✓ answer (2)
	OR/***OF*** Maximum value of $1 + \sin 2x = 1 + 1$ $= 2$ $\therefore$ maximum value of $\sin x + \cos x = \sqrt{2}$	✓ max of $\sin 2x = 1$ ✓ answer (2)
	OR/***OF*** $(\sin x + \cos x)^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x$ $= 1 + \sin 2x$ $\therefore$ max value $(\sin x + \cos x)^2 = 1 + 1 = 2$ $\therefore$ max value $\sin x + \cos x = \sqrt{2}$	✓ max of $\sin 2x = 1$ ✓ answer (2)
		[27]

QUESTION/VRAAG 6



6.1	Period = 180°	✓ answer (1)
6.2	$-75°$	✓ answer (1)
6.3	$\sin 2x \leq \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$ $\sin 2x \leq \cos 45°.\cos x + \sin 45°.\sin x$ $\sin 2x \leq \cos(x-45°)$ $x \in [-75° \ ;165°]$	✓ $\cos 45°.\cos x + \sin 45°.\sin x$ ✓ $\cos(x-45°)$ ✓ ✓ answer (4)
		[6]

QUESTION/VRAAG 7



Figure/*Figuur* (i)

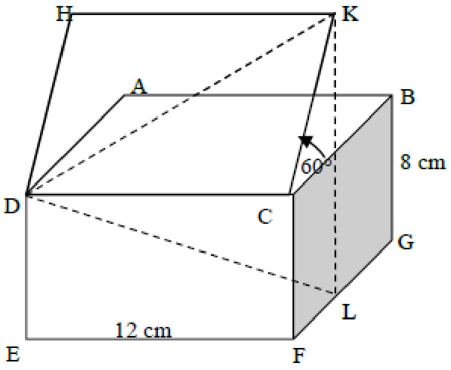
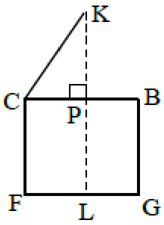


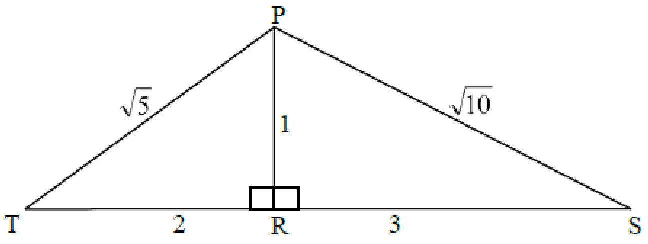
Figure / *Figuur* (ii)

7.1	KC = 6 cm	✓ answer (1)
7.2	Let P be the point of intersection of KL and CB $\frac{KP}{KC} = \sin 60°$ $KP = 6\sin 60°$ $KP = 3\sqrt{3}$ or 5,20 $\therefore KL = 8 + 3\sqrt{3}$ or 13,20 cm 	✓ trig ratio ✓ length of KP ✓ answer (3)
7.3	$DK^2 = 6^2 + 12^2$ $DK = \sqrt{180}$ or $6\sqrt{5}$ or 13,42 cm $\frac{\sin K\hat{D}L}{KL} = \frac{\sin D\hat{L}K}{DK}$ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ $= \frac{8+3\sqrt{3}}{6\sqrt{5}}$ or $\frac{13{,}20}{13{,}42}$ or 0,98	✓ $DK = 6\sqrt{5}$ ✓ use of sine rule ✓ $\frac{\sin K\hat{D}L}{\sin D\hat{L}K} = \frac{KL}{DK}$ ✓ answer (4) **[8]**

MAY / JUNE 2016

QUESTION/*VRAAG* 5

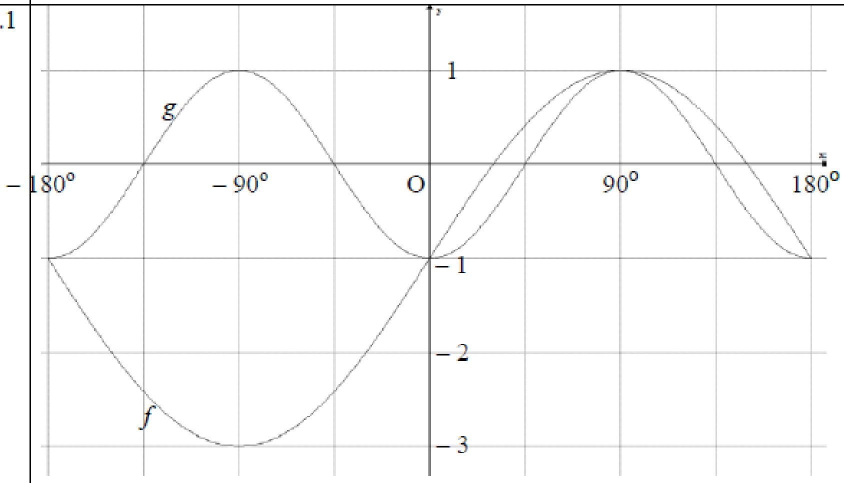
5.1



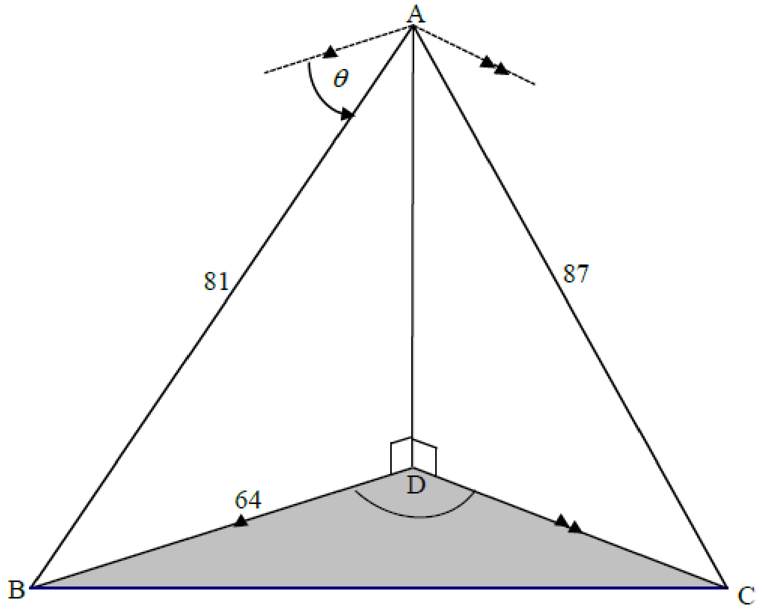
5.1.1(a)	$\sin \mathrm{T} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} = 0{,}45$	✓value (1)
5.1.1(b)	$\cos \mathrm{S} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0{,}95$	✓value (1)
5.1.2	$\cos(\mathrm{T}+\mathrm{S}) = \cos \mathrm{T} \cos \mathrm{S} - \sin \mathrm{T} \sin \mathrm{S}$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$ $= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}$ $= \frac{5}{\sqrt{50}}$ *or* $\frac{1}{\sqrt{2}}$ *or* $\frac{\sqrt{2}}{2}$	✓expansion ✓ $\frac{2}{\sqrt{5}}$ ✓ $\frac{1}{\sqrt{10}}$ ✓ simplification ✓ answer (5)
5.2	$\frac{1}{\cos(360° - \theta)\sin(90° - \theta)} - \tan^2(180° + \theta)$ $= \frac{1}{(\cos\theta)(\cos\theta)} - \tan^2\theta$ $= \frac{1}{\cos^2\theta} - \left(\frac{\sin^2\theta}{\cos^2\theta}\right)$ $= \frac{1-\sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta}$ *OR* $\frac{1-\sin^2\theta}{1-\sin^2\theta}$ $= 1$	✓ $\cos\theta$ ✓ $\cos\theta$ ✓ $\tan^2\theta$ ✓ $\frac{\sin^2\theta}{\cos^2\theta}$ ✓identity ✓ answer (6)

5.3	$(\sin x - \cos x)^2 = \left(\frac{3}{4}\right)^2$ $\sin^2 x - 2\sin x\cos x + \cos^2 x = \frac{9}{16}$ $1 - 2\sin x\cos x = \frac{9}{16}$ $2\sin x\cos x = \frac{7}{16}$ $\therefore \sin 2x = \frac{7}{16}$	✓ squaring both sides ✓expanding LHS ✓ using identity ✓ simplifying ✓answer (5) **[18]**

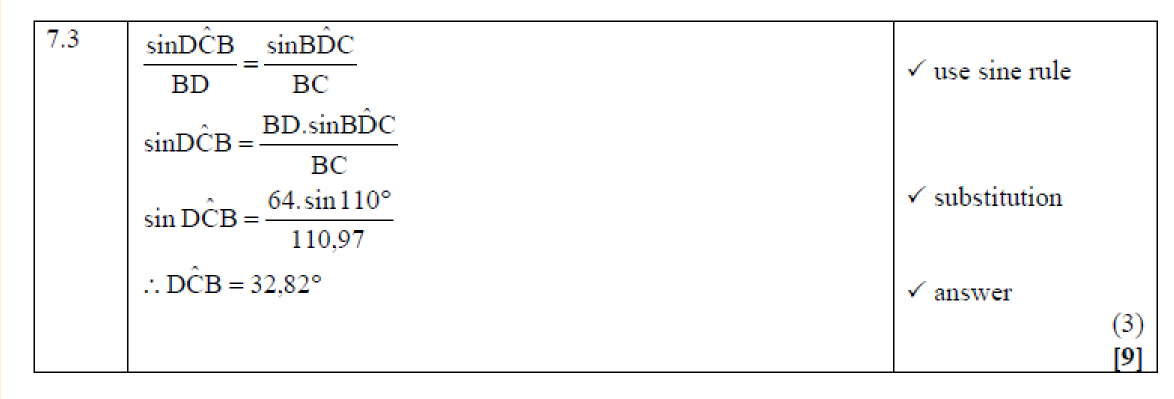
QUESTION/*VRAAG* 6

6.1	$4\sin x + 2\cos 2x = 2$ $2\sin x + \cos 2x - 1 = 0$ $2\sin x + (1 - 2\sin^2 x) - 1 = 0$ $2\sin^2 x - 2\sin x = 0$ $2\sin x(\sin x - 1) = 0$ $2\sin x = 0$ or $\sin x - 1 = 0$ $\sin x = 0$ $\quad \sin x = 1$ $x = k.180°$ or $x = 90° + k.360, k \in Z$	✓using identity ✓ standard form ✓ factors ✓ $\sin x = 0$ or $\sin x = 1$ ✓ $k.180°$ ✓ $90° + k.360, k \in Z$ (6)
6.2.1		✓ turning point (–90° ; –3) ✓ turning point (90° ; 1) ✓ (–180° ; –1) & (0° ; –1) (3)
6.2.2	$(-90°; 0°)$ **OR/*OF*** $-90° < x < 0°$	✓ ✓ answer (2) ✓ ✓ answer (2)
6.2.3	$f(x) = g(x)$ $\therefore -180°; 0°; 90°; 180°$ $f(x + 30°) = g(x + 30°)$ $\therefore x = -30°; 60°; 150°$	✓ any ONE correct ✓ other 2 correct (2) **[13]**

QUESTION/*VRAAG* 7



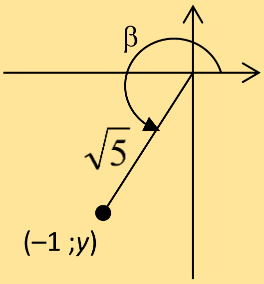
7.1	$\hat{ABD} = \theta$ [alternate ∠ s; || lines] $\cos\theta = \frac{BD}{AB} = \frac{64}{81}$ $\theta = 38°$ OR/***OF*** $\sin B\hat{A}D = \frac{64}{81}$ $B\hat{A}D = 52{,}18°$ $\theta = 38°$	✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3) ✓correct trig ratio ✓substitution into correct ratio ✓ answer (to the nearest degree) (3)
7.2	$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos B\hat{A}C$ $= 81^2 + 87^2 - 2(81)(87)\cos 82{,}6°$ $= 12314{,}754...$ $BC = 110{,}97\text{ m}$	✓use cosine rule ✓correct substitution into cosine rule ✓answer (3)



7.3	$\frac{\sin D\hat{C}B}{BD}=\frac{\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B=\frac{BD.\sin B\hat{D}C}{BC}$ $\sin D\hat{C}B=\frac{64.\sin 110°}{110,97}$ $\therefore D\hat{C}B=32,82°$	✓ use sine rule ✓ substitution ✓ answer (3) **[9]**

MAY/JUNE 2015

QUESTION/*VRAAG* 5

5.1	$\cos\beta = -\frac{1}{\sqrt{5}}$ and/*en* $180° < \beta < 360°$ $(-1)^2 + y^2 = (\sqrt{5})^2$ $1 + y^2 = 5$ $y^2 = 4$ $y = -2$  $\therefore \sin\beta = -\frac{2}{\sqrt{5}}$	<u>sketch/*skets*:</u> ✓ correct quad/ *korrekte kwadr* ✓ $x = -1$ ✓subst into Pyth/ *subst in Pyth* ✓ value of/*waarde van y* ✓ value of/*waarde van* sin β (5)
5.2	$\frac{(-\tan x).(-\sin(90° - x))}{4\sin x}$ $\frac{(-\tan x).(-\cos x)}{4\sin x}$	✓– tan x ✓ –sin(90° – x) ✓– cosx ✓ sin x

	$= \dfrac{(-\frac{\sin x}{\cos x}).(-\cos x)}{4\sin x}$ $= \dfrac{1}{4}$	✓ $\dfrac{\sin x}{\cos x}$ ✓ answer/*antw* (6)
5.3.1	tan A = $\dfrac{\sin \mathrm{A}}{\cos \mathrm{A}} = \dfrac{p}{q}$	✓ answer/*antw* (1)
5.3.2	$p^4 - q^4 = (p^2 + q^2)(p^2 - q^2)$ $= (\sin^2 \mathrm{A} + \cos^2 \mathrm{A})(\sin^2 \mathrm{A} - \cos^2 \mathrm{A})$ $= (1)(\sin^2 \mathrm{A} - \cos^2 \mathrm{A})$ $= -1(\cos^2 \mathrm{A} - \sin^2 \mathrm{A})$ $= -\cos 2\mathrm{A}$	✓ factors/*faktore* ✓identity/*identiteit* ✓ −1 as CF/*GF* ✓ answer/*antw* (4)
5.4.1	$\mathrm{LHS}/LK = \dfrac{\cos^2\theta - \cos 2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\cos^2\theta - (2\cos^2\theta - 1)}{\sin\theta.\cos\theta}$ $= \dfrac{1 - \cos^2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\sin^2\theta}{\sin\theta.\cos\theta}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta = \mathrm{RHS}/RK$	✓ writing as single term/*skryf as enkelterm* ✓ expansion/*uitbreiding* ✓ simplify/*vereenv* ✓ identity/*identiteit* ✓ simplify/*vereenv* (5)

QUESTION/*VRAAG* 6

6.1	Period of/*Periode van* $f = 120°$	✓ 120° (1)
6.2	$b = 3$	✓$b = 3$ (1)
6.3	$x = -45°$ or/*of* $x = -22{,}5°$ or/*of* $x = 67{,}5°$	✓$x = -45°$ ✓$x = -22{,}5°$ ✓$x = 67{,}5°$ (3)
6.4	$x \in (-45° ; -22{,}5°) \cup (67{,}5° ; 90°]$ **OR/*OF*** $-45° < x < -22{,}5°$ or/*of* $67{,}5° < x \leq 90°$	✓ critical values ✓ notation ✓ critical values ✓ notation (4) ✓*kritieke waardes* ✓*notasie* ✓*kritieke waardes* ✓*notasie* (4) **[9]**

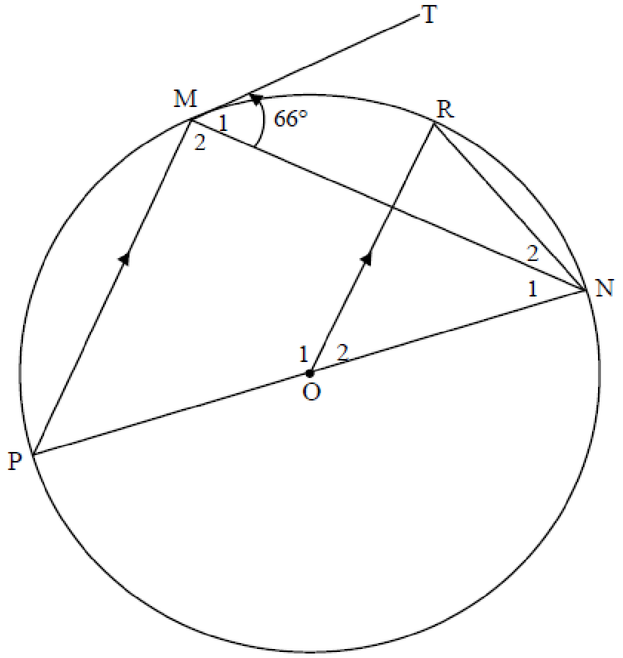
QUESTION/*VRAAG 7*

7.1	$\mathrm{QR}^2 = \mathrm{PQ}^2 + \mathrm{RP}^2 - 2.\mathrm{PQ}.\mathrm{RP}.\cos\hat{\mathrm{P}}$ $(\sqrt{3}x)^2 = x^2 + x^2 - 2.x.x.\cos\hat{\mathrm{P}}$ $\cos\hat{\mathrm{P}} = \frac{x^2 + x^2 - (\sqrt{3}x)^2}{2x.x}$ $\cos\hat{\mathrm{P}} = \frac{-x^2}{2x^2}$ $\cos\hat{\mathrm{P}} = -\frac{1}{2}$ $\hat{\mathrm{P}} = 120°$	✓ correct subst into cosine rule/*korrek subst in cos-reël* ✓ $\cos\hat{\mathrm{P}}$ as subj/ *onderw* ✓simplify/*vereenv* ✓ answer/*antw* (4)
7.2	$\mathrm{P\hat{R}Q}$ = $\mathrm{P\hat{Q}R}$ = 30° (∠s opp equal sides/∠*e teenoor gelyke sye*) $\mathrm{Q\hat{R}S} = 150°$ (∠s on a str line/∠*e op reguitlyn*) Area of/*Opp van* Δ QRS = $\frac{1}{2}(\mathrm{QR})(\mathrm{RS})(\sin \mathrm{Q\hat{R}S})$ = $\frac{1}{2}(\sqrt{3}x)(\frac{3}{2}x)(\sin 150°)$ = $(\frac{3\sqrt{3}}{4}x^2)(\frac{1}{2})$ = $\frac{3\sqrt{3}}{8}x^2$	✓S ✓ S ✓ correct subst into area rule/*korrek subst in opp-reël* ✓simplify/*vereenv* ✓ answer/*antw* (5) **[9]**

SOLUTIONS EUCLIDEAN GEOMETRY

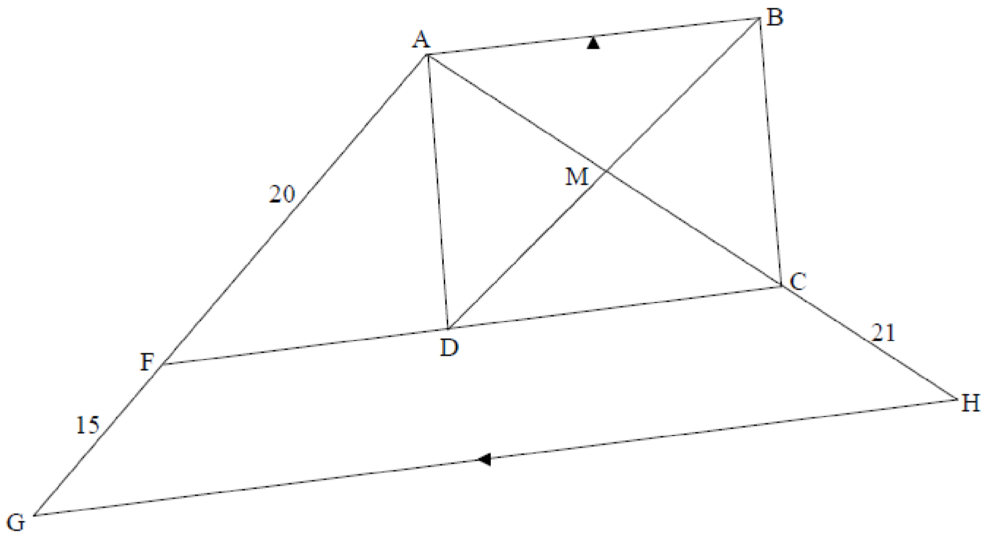
FEB/MAR 2018

QUESTION/*VRAAG* 8



8.1.1	$\hat{P} = \hat{M}_1 = 66°$ [tan chord theorem/*raaklyn koordst*]	✓S ✓R (2)
8.1.2	$\hat{M}_2 = 90°$ [∠ in semi circle/∠ *in halfsirkel*]	✓S ✓R (2)
8.1.3	$\hat{N}_1 = 180° - (90° + 66°)$ [sum of ∠s of /*som van* ∠*e* ΔMNP] $= 24°$	✓S (1)
8.1.4	$\hat{O}_2 = \hat{P} = 66°$ [corres. ∠s;/*ooreenk* ∠*e,* PM ∥ OR]	✓S ✓R (2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$ [sum of ∠s of/*som van* ∠*e* ΔRNO] $= 114°$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$ [∠s opposite = radii/∠*e teenoor = radii*] $\therefore \hat{N}_2 = 33°$ **OR/*OF*** $P\hat{O}R = 114°$ [∠s on straight line/∠*e op reguitlyn*] $P\hat{N}R = 57°$ [∠ at centre = twice ∠ at circumference/ *midpts*∠ = 2 × *omtreks*∠] $\therefore \hat{N}_2 = 33°$	✓S ✓S/R ✓S (3) ✓S ✓S/R ✓S (3)

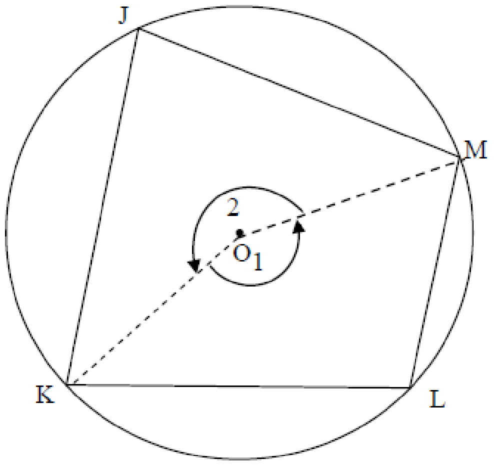
8.2



8.2.1	FC || AB || GH [opp sides of **rectangle**/*teenoorst sye v reghoek*]	✓ R (1)
8.2.2	$\frac{AC}{CH}=\frac{AF}{FG}$ [line || one side of Δ] **OR** [prop theorem; FC || GH] *[lyn || een sy van Δ]* ***OF*** *[eweredighst; FC || GH]* $\frac{AC}{21}=\frac{20}{15}$ $AC=\frac{20\times 21}{15}$ $=28$ DB = AC = 28 [diags of rectangle =/*hoeklyne v reghoek* =] $DM=\frac{1}{2}DB=14$ [diags of rectangle bisect/*hoekl v reghoek halveer*]	✓S ✓R ✓ AC ✓ S ✓ S (5)
		[16]

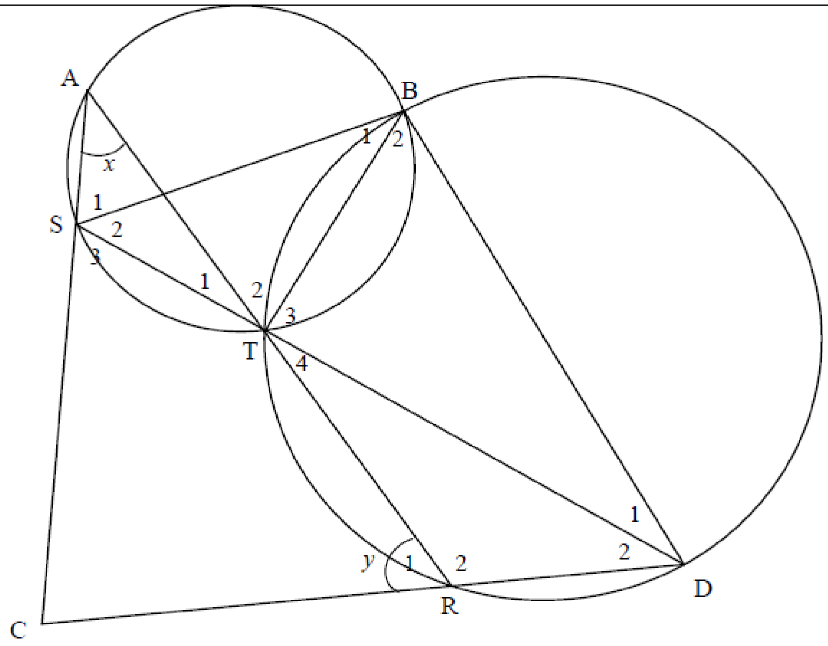
QUESTION/*VRAAG* 9

9.1



9.1	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO*	✓ construction
	Proof:	
	$\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]*	✓ S/R
	$\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference]	✓ S
	$\hat{O}_1 + \hat{O}_2 = 360°$ [∠s around a point / *∠e om 'n punt*]	✓ S/R
	$\therefore 2\hat{J} + 2\hat{L} = 360°$	✓ S
	$\therefore 2(\hat{J} + \hat{L}) = 360°$	
	$\therefore \hat{J} + \hat{L} = 180°$	(5)
	OR/*OF*	
	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO*	✓ construction
	Proof:	
	Let $\hat{J} = x$	
	$\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]*	✓ S ✓ R
	$\hat{O}_2 = 360° - 2x$ [∠s around a point / *∠e om 'n punt*]	✓ S/R
	$\therefore \hat{L} = 180° - x$ [∠ at centre = twice ∠ at circumference]	✓ S
	$\therefore \hat{J} + \hat{L} = 180°$	(5)

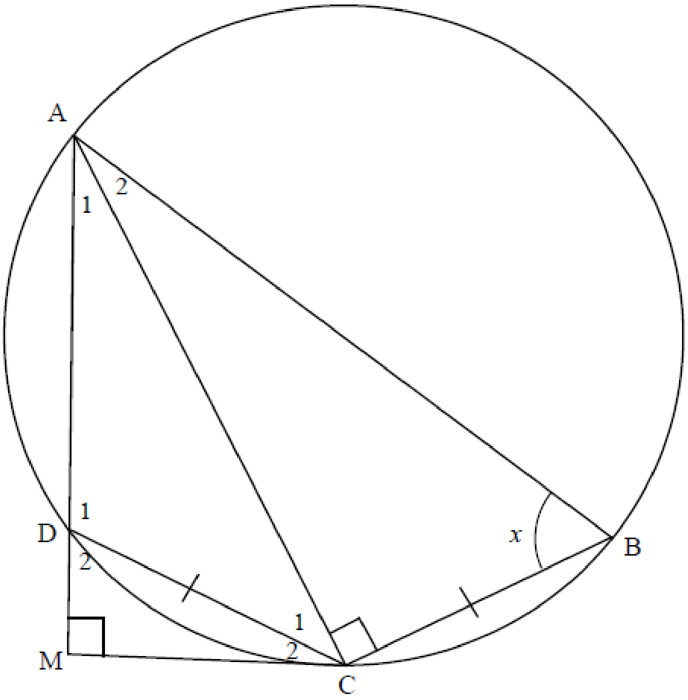
9.2



9.2.1(a)	$\hat{B}_1 = x$ [$\angle$s in same seg/ *$\angle$e in dieselfde segm*]	✓ S ✓ R (2)
9.2.1(b)	$\hat{B}_2 = y$ [ext $\angle$ of cyclic quad/*buite$\angle$ koordevh*]	✓ S ✓ R (2)
9.2.2	$\hat{C} = 180° - (x + y)$ [sum of $\angle$s of/*som v $\angle$e,* Δ ACR] $S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$ $S\hat{B}D + \hat{C} = 180°$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst $\angle$e koordevh*] **OR/*OF*** $\hat{S}_1 = \hat{T}_2$ [$\angle$s in same segment/*$\angle$e in dies. segment*] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = B\hat{D}R$ [ext $\angle$ of cyc quad/*buite$\angle$ koordevh*] $\therefore\ \hat{S}_1 = B\hat{D}R$ $\therefore$ SCDB is cyc quad [ext $\angle$ of quad = opp $\angle$/*buite$\angle$ = tos $\angle$*]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

9.2.3	$\hat{T}_4 = y - 30°$ [ext ∠ of/*buite* ∠ Δ TDR]	✓ S
	$\hat{T}_1 = y - 30°$ [vert opp ∠s =/*regoorst* ∠*e* =]	✓ S
	$y - 30° + x + 100° = 180°$ [sum of ∠s of/*som v* ∠*e*, Δ AST]	
	$\therefore x + y = 110°$	
	$S\hat{B}D = 110°$	
	∴ SD not diameter [line does not subtend 90° ∠]	✓ S
	SD nie 'n middellyn *[lyn onderspan nie 90°∠]*	✓ R
		(4)
	OR/*OF*	
	$A\hat{S}T = \hat{C} + \hat{D}_2$ [ext ∠ of/*buite* ∠ Δ SCD]	✓ S
	$\hat{C} = 100° - 30° = 70°$	✓ S
	$S\hat{B}D = 180° - 70°$ [opp ∠s cyclic quad/ teenoorst ∠e kdvh]	
	$= 110°$	✓ S
	∴ SD not diameter [line does not subtend 90° ∠]	
	SD nie 'n middellyn *[lyn onderspan nie 90°∠]*	✓ R
		(4)
		[16]

QUESTION/*VRAAG* 10



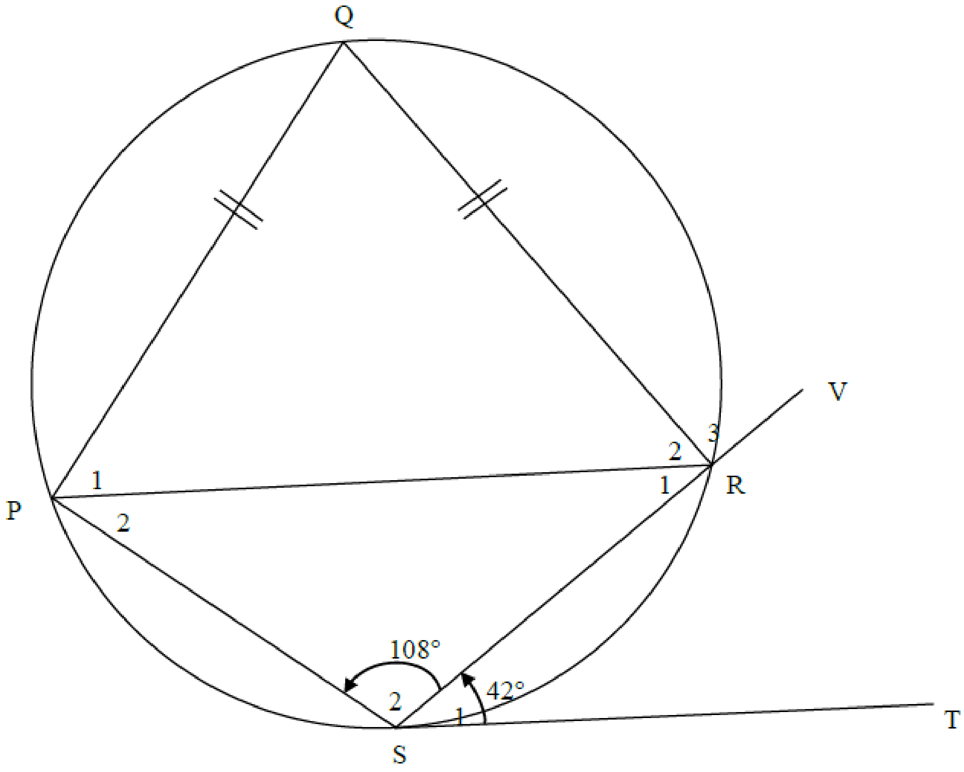
10.1.1	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s = *kde onderspan* = ∠*e*]	✓ S ✓R
	$\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite∠ koordevh.*]	✓ S/R
	$\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v* ∠*e*, ΔDCM]	✓ $\hat{C}_2 = 90° - x$
	$\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$	
	∴ MC is a tangent to the circle at C [converse: tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF*	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* = ∠*e*]	✓ S ✓ R
	$\hat{C}_1 + \hat{C}_2 = x$ [sum of ∠s of/*som v* ∠*e*, ΔACM]	✓✓ $\hat{C}_1 + \hat{C}_2 = x$
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	
	∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF* In ΔAMC and ΔACB:	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* = ∠*e*]	✓ S ✓ R
	$A\hat{M}C = A\hat{C}B = 90°$ [given]	
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	✓✓ $\hat{C}_1 + \hat{C}_2 = x$

	$\therefore$ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad.] *[bewys **OF** buite ∠v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of ∠s in Δ] *[Bewys **OF** som v ∠e in Δ]* ΔACB ||| ΔCMD [∠, ∠, ∠] **OR**/***OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad.] *[bewys **OF** buite ∠v koordevh]* $A\hat{C}B = A\hat{M}C = 90°$ [given/*gegee*] ΔACB ||| ΔCMD [∠, ∠, ∠] **OR**/***OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior ∠ of cyclic quad] *[bewys **OF** buite ∠v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of ∠s in Δ] *[Bewys **OF** som v ∠e in Δ]* $A\hat{C}B = A\hat{M}C = 90°$ [given **OR** sum of ∠s in Δ] *[gegee **OF** som v ∠e in Δ]* ΔACB ||| ΔCMD	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.1	$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD] $\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC] $\therefore DC^2 = AB \times MD$ In ΔAMC and/*en* ΔCMD $\hat{M}$ is common/*gemeen* $\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*] **OR**/***OF*** $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /*raaklyn koordst* **OR**/***OF*** exterior ∠ of cyclic quad/ *buite ∠v kdvh*] ΔAMC ||| ΔCMD [∠, ∠, ∠] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$	✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)

	OR/*OF*	
	$\frac{AC}{MC}=\frac{AB}{DC}$ [ΔACB ||| ΔCMD]	✓ $\frac{AC}{MC}=\frac{AB}{DC}$
	$\therefore CM\times AB = AC\times DC$	
	In ΔAMC and/*en* ΔACB	✓ S
	$\hat{C}=\hat{M}=90^\circ$ [given]	
	$\hat{A}_1=\hat{A}_2$ [proven]	✓ S
	OR/*OF*	
	$A\hat{C}M=\hat{B}=x$ [proven]	
	ΔAMC ||| ΔACB [∠, ∠, ∠]	
	$\frac{AC}{AM}=\frac{BC}{MC}$	✓ AC.MC = AM.BC
	$\therefore AC\times MC = AM\times BC$	
	$\therefore AC=\frac{BC.AM}{MC}$	
	$CM\times AB=\frac{BC.AM}{MC}\times DC$	✓ equating
	$CM^2=\frac{DC.AM}{AB}\times DC$ [BC = DC]	✓ S
	$\frac{CM^2}{DC^2}=\frac{AM}{AB}$	(6)
10.2.2	In ΔDMC:	
	$\frac{CM}{DC}=\sin x$	✓ trig ratio
	$\frac{CM^2}{DC^2}=\sin^2 x\ \frac{AC}{AB}=\frac{CM}{DC}$	✓ square both sides
	$\therefore \frac{AM}{AB}=\sin^2 x$	(2)
	OR/*OF*	
	In ΔABC:	
	$\sin x=\frac{AC}{AB}$	
	In ΔAMC:	
	$\sin x=\frac{AM}{AC}$	✓ 2 equations for $\sin x$
	$\sin x.\sin x=\frac{AC}{AB}\times\frac{AM}{AC}=\frac{AM}{AB}$	✓ product (2)
		[16]

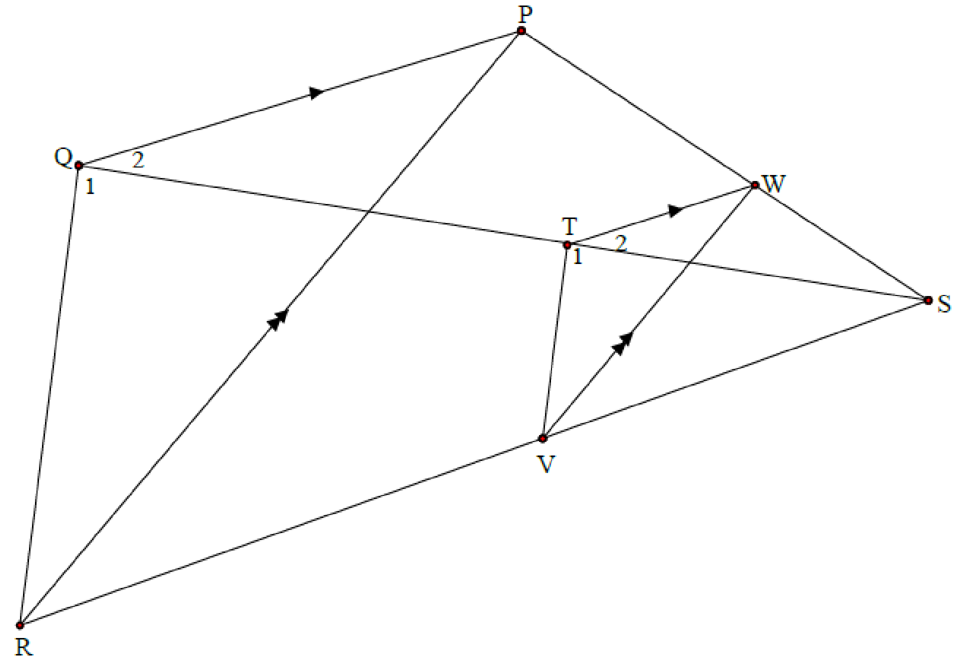
FEB/MAR 2017

QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72^\circ$ [opp ∠s of cyclic quad/*teenoorst ∠e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{R}_2 = \frac{180^\circ - 72^\circ}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*] $= 54^\circ$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42^\circ$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= 54^\circ + 42^\circ$ $= 96^\circ$ **OR/*OF*** $\hat{R}_1 = 180^\circ - 108^\circ - 42^\circ = 30^\circ$ [sum of/*som van*∠s/e in Δ] $\hat{R}_3 = 180^\circ - \hat{R}_1 - \hat{R}_2$ [∠s on str line/*∠e op reguitlyn*] $= 180^\circ - 30^\circ - 54^\circ$ [sum of/*som van*∠s/e in Δ] $= 96^\circ$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30^\circ$ ✓ S (2) **[8]**

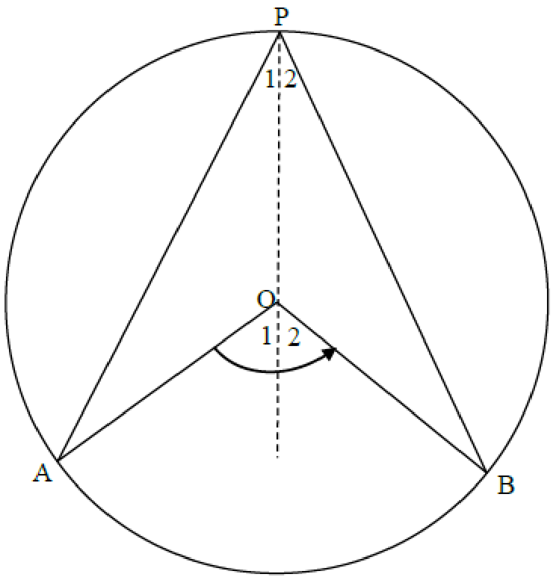
QUESTION/*VRAAG* 9



9.1.1	$\frac{ST}{TQ}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ∥ QP] $=\frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR}=\frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ∥ RP] $=\frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ}=\frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] $\therefore$ TV ∥ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1=\hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ∥ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| <u>ΔRPS</u>	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR}=\frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $=\frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR}=\frac{SV}{SR}$ [ΔVWS ||| Δ RPS] $=\frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

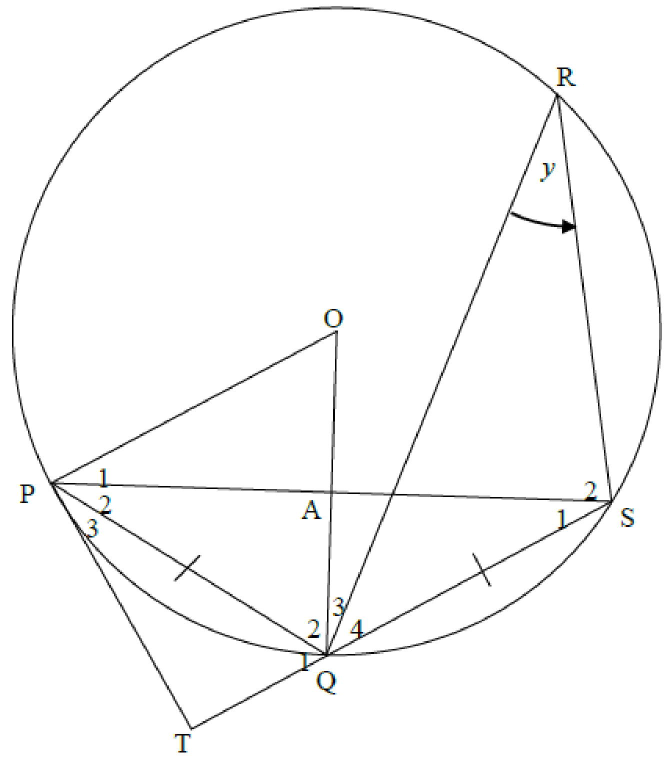
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : OP = OA [radii] $\therefore \hat{P}_1 = \hat{A}$ [∠s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext ∠ of Δ] $\therefore \hat{O}_1 = 2\,\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\,\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

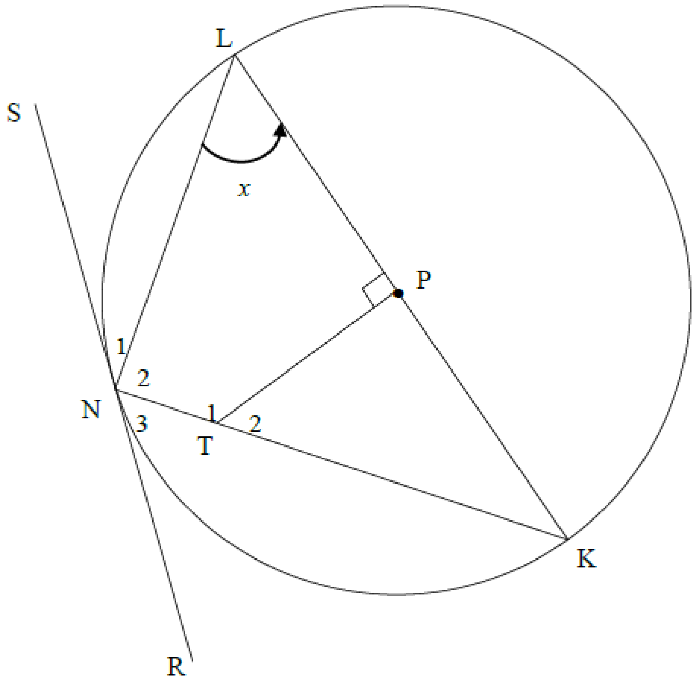
10.2



10.2.1	∠s in the same segment/ ∠*e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/∠*e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore\ \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre =2×∠at circ/*midpts∠= 2×omtreks∠*]	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore\ T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

10.2.5	$O\hat{P}Q + O\hat{Q}P = 180^\circ - 2y$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$\therefore\ O\hat{Q}P = 90^\circ - y$ [∠s opp equal sides/∠*e to = sye;* OP = OQ]	✓ S ✓ R
	In ΔPAQ:	
	$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180^\circ$	
	$90^\circ - y + y + Q\hat{A}P = 180^\circ$ [sum of/*sum v* ∠s/*e* in Δ]	✓ S
	$Q\hat{A}P = 90^\circ$	✓ S
	$\therefore\ O\hat{A}P = 90^\circ$ [∠s/*e* on straight line/*op reguitlyn*]	(5)
	OR/*OF*	
	$O\hat{P}T = 90^\circ$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$\therefore\ \hat{P}_1 = 90^\circ - 2y$	✓ S
	$\hat{P}_1 + \hat{O} + O\hat{A}P = 180^\circ$ [sum of/*sum v* ∠s/*e* in Δ]	
	$(90^\circ - 2y) + 2y + O\hat{A}P = 180^\circ$	✓ S
	$\therefore\ O\hat{A}P = 90^\circ$	✓ S
		(5)
	OR/*OF*	
	POSQ is a kite/'*n vlieër*	
	$\therefore$ OQ ⊥ PS [diag of a kite/*hoeklyne v vlieër*]	✓✓✓ S
	$\therefore\ O\hat{A}P = 90^\circ$	✓✓ R
		(5)
	OR/*OF*	
	In ΔOAP and ΔOAS	
	OP = OS (radii)	✓ S
	OA is common	✓ S
	$P\hat{O}A = 2y$	
	$= 2\hat{P}_2$	
	$= Q\hat{O}S$	✓ S
	ΔOAP ≡ ΔOAS (SAS)	✓ R
	$O\hat{A}P = O\hat{A}S$ (≡Δs)	
	$O\hat{A}P = O\hat{A}S = 90^\circ$ (∠s on str line)	✓ S
		(5) **[19]**

QUESTION/*VRAAG* 11



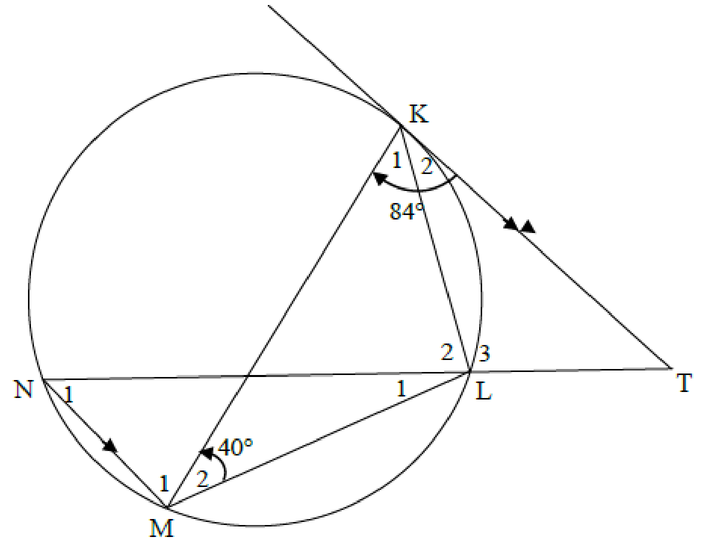
11.1	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*]	✓ S ✓ R
	∴ TPLN is a cyclic quad/ 'n *koordevh* [opp ∠s of quad is suppl/ *teenoor∠e v vh is suppl*]	✓ R (3)
	OR	
	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*]	✓ S ✓ R
	∴ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]	✓ R (3)
11.2	$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*]	✓ R
	$\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ]	
	$\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*]	✓ S✓ R (3)
	OR/*OF*	
	$\hat{K} = 90° - x$ [sum of/*som v* ∠s/e in Δ]	✓ R
	$\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*]	✓ S✓ R (3)
	OR/*OF*	
	$\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*]	✓ R
	$\hat{N}_2 = 90°$ [∠in semi circle/ *halfsirkel*]	✓ S
	$\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ S (3)

11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore \Delta KTP \parallel\mid \Delta KLN$ [∠∠∠] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of ∠s in Δ] $\therefore \Delta KTP \parallel\mid \Delta KLN$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [||| Δs] $\therefore KT \, . \, KN = KP \, . \, KL$ But $KL = 2KP$ [radii: $PK = LP$] $\therefore KT \, . \, KN = KP \, . \, 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

FEB/ MAR 2016

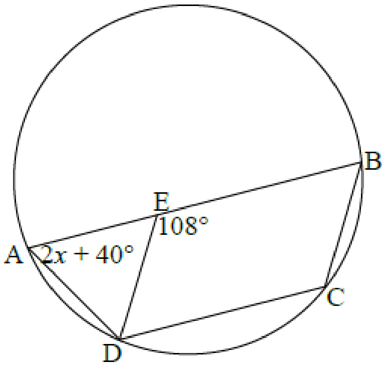
QUESTION/*VRAAG* 8

8.1



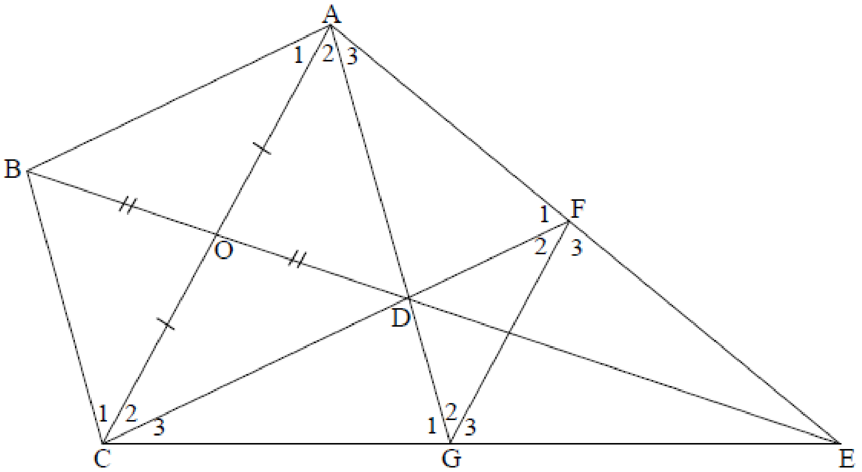
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40^\circ$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/*∠e in dies segm*] $\hat{K}_1 = 84^\circ - 40^\circ = 44^\circ$ $\therefore \hat{N}_1 = 44^\circ$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44^\circ$ [alt/*verw* ∠s/*e*; KT || NM]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite ∠v Δ*] $= 40^\circ + 44^\circ$ $= 84^\circ$	✓R ✓S (2)
8.1.5	In ΔKLM: $44^\circ + 84^\circ + 40^\circ + \hat{L}_1 = 180^\circ$ [∠s sum in Δ/*∠e som in Δ*] $\therefore \hat{L}_1 = 12^\circ$	✓S (1)

8.2



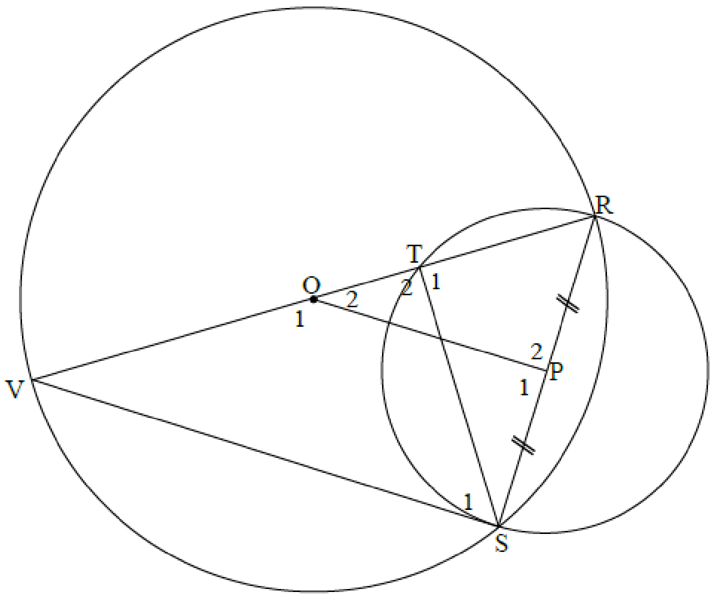
8.2	$\hat{C} = 108°$	[opp∠s of ∥m/*tos ∠e v ∥m*]	✓S ✓R
	$2x + 40° + 108° = 180°$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5)
		OR/*OF*	
	$\hat{C} = 180° - (2x + 40°)$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$180° - (2x + 40°) = 108°$	[opp∠s of ∥m/*tos ∠e v ∥m*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5) **[15]**

QUESTION/*VRAAG* 9



9.1	ABCD is a ||m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB}=\frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF || BA] $\frac{ED}{DB}=\frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG || BC]	✓S ✓ R ✓S ✓ R (4)
9.3	$\frac{FE}{AF}=\frac{GE}{CG}$ [proved/*bewys*] ∴AC || FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2=\hat{F}_2$ [alt/*verw* ∠s/*e*; AC || FG] $\hat{A}_1=\hat{C}_2$ [alt/*verw* ∠s/*e*; AB || CD] ∴ $\hat{A}_1=\hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1=\hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2=\hat{F}_2$ [$\hat{A}_1=\hat{F}_2$] ∴ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2=\hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2=\hat{G}_2$ [alt/*verw*-∠s/*e*; AC || FG] ∴ $\hat{C}_2=\hat{G}_2$ ∴ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

10.2

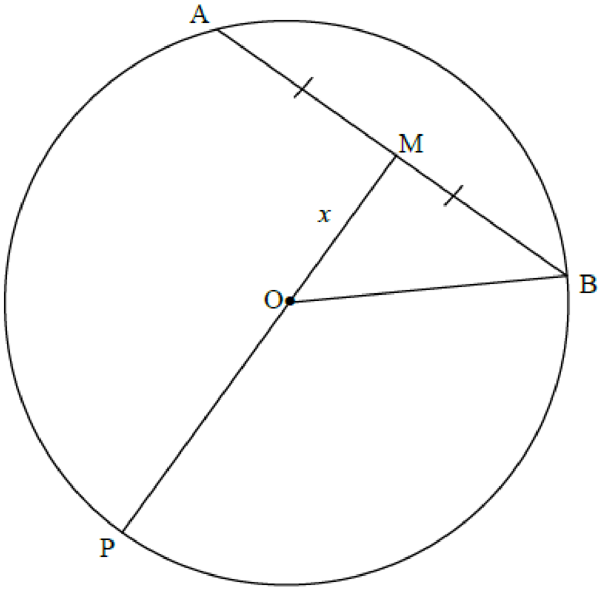


10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] $\therefore \Delta ROP \; \vert\vert\vert \; \Delta RVS$ [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] $\therefore \Delta ROP \; \vert\vert\vert \; \Delta RVS$ [∠,∠,∠]	✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ $\therefore \Delta RVS \mid\mid\mid \Delta RST$ [∠,∠,∠]	✓S ✓ R ✓S & ∠;∠;∠ **OR**/***OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [∠ s on straight line/∠*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ $\therefore \Delta RTS \mid\mid\mid \Delta STV$ [∠,∠,∠] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓$\Delta RTS \mid\mid\mid \Delta STV$ ✓ratio/*verh* (6)
		[21]

FEB/MAR 2015

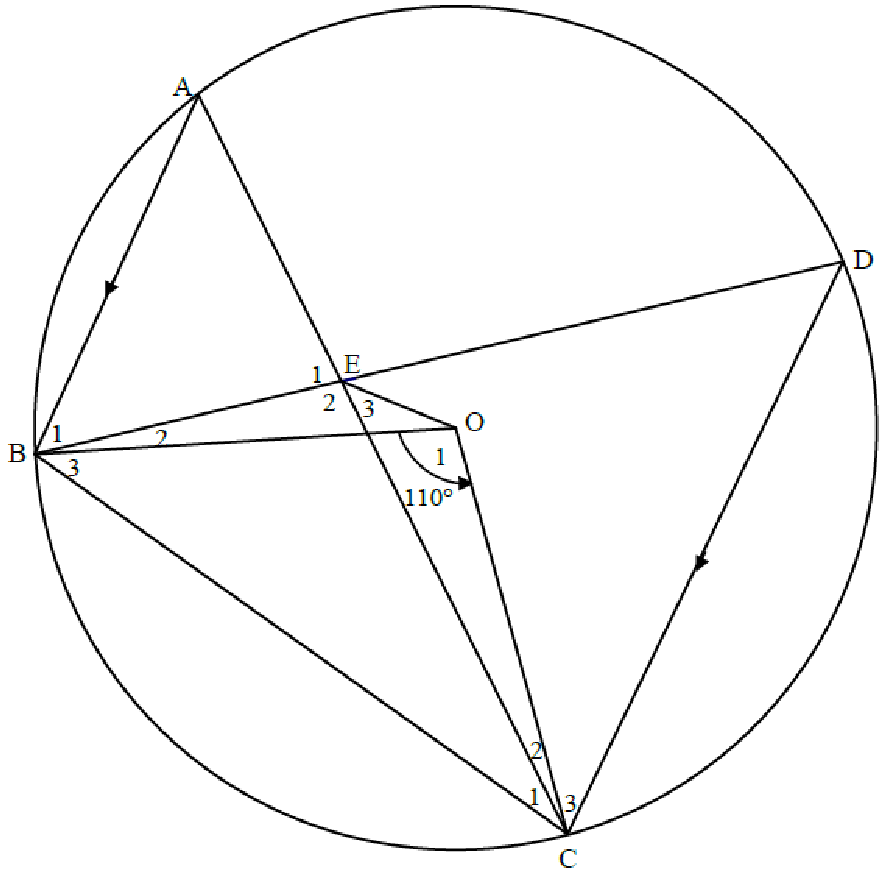
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/*antw* (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/*lyn vanaf midpt na midpt van koord is loodreg op koord* **OR/*OF*** line from centre bisects chord/*lyn vanaf midpt halveer koord*	✓ answer/*antw* (1) ✓ answer/*antw* (1)
7.3	$\frac{\text{MP}}{\text{OM}}=\frac{5}{2}$ $\frac{x+\text{OP}}{x}=\frac{5}{2}$ $2x+2\text{OP}=5x$ $\text{OP}=\frac{3x}{2}$ **OR/*OF*** $\frac{\text{OP}}{\text{OM}}=\frac{3}{2}$ $\text{OP}=\frac{3x}{2}$	✓ $\frac{x+\text{OP}}{x}=\frac{5}{2}$ ✓ $\text{OP}=\frac{3x}{2}$ (2) ✓ $\frac{\text{OP}}{\text{OM}}=\frac{3}{2}$ ✓ $\text{OP}=\frac{3x}{2}$ (2)

7.4	OM² + MB² = OB² $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8{,}94$ or $4\sqrt{5}$ or $\sqrt{80}$	✓ subst into/*subst* Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/*antw* (3) [7]

QUESTION/VRAAG 8

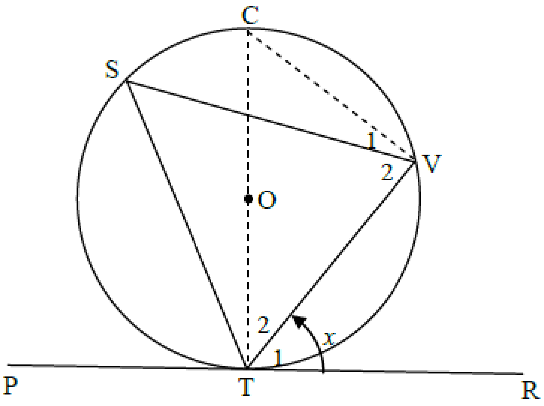


8.1.1	$\hat{D}=\frac{1}{2}\hat{O}_1=55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*)	✓S ✓R (2)
8.1.2	$\hat{A}=\frac{1}{2}\hat{O}_1=55°$ (∠ at centre=2×∠at circ/∠ *by midpt=2×∠by omt*) **OR/*OF*** $\hat{A}=\hat{D}=55°$ (∠s in same segment/∠*e in dieselfde segment*)	✓S ✓R (2) ✓S ✓R (2)
8.1.3	$\hat{B}_1=\hat{D}=55°$ (alternate ∠s/*verwiss∠e*; AB | | DC) $\hat{E}_2=\hat{B}_1+\hat{A}$ (ext∠ ofΔ = sum of opp∠^{s}/*buite∠ vΔ=som v tos∠e*) $=55°+55°$ $\hat{E}_2=110°$	✓S ✓R ✓R ✓ answer/*antw* (4)
8.2	$\hat{E}_2=\hat{O}_1=110°$ (proven in/*bewys in* 8.1.3) BEOC is a cyclic quadrilateral (equal ∠s subtended by line/ *gelyke ∠e onderspan deur lyn*)	✓S ✓R (2) **[10]**

QUESTION/VRAAG 9

9.1	the interior opposite angle/*die teenoorstaande binnehoek*.	✓ answer/*antw* (1)

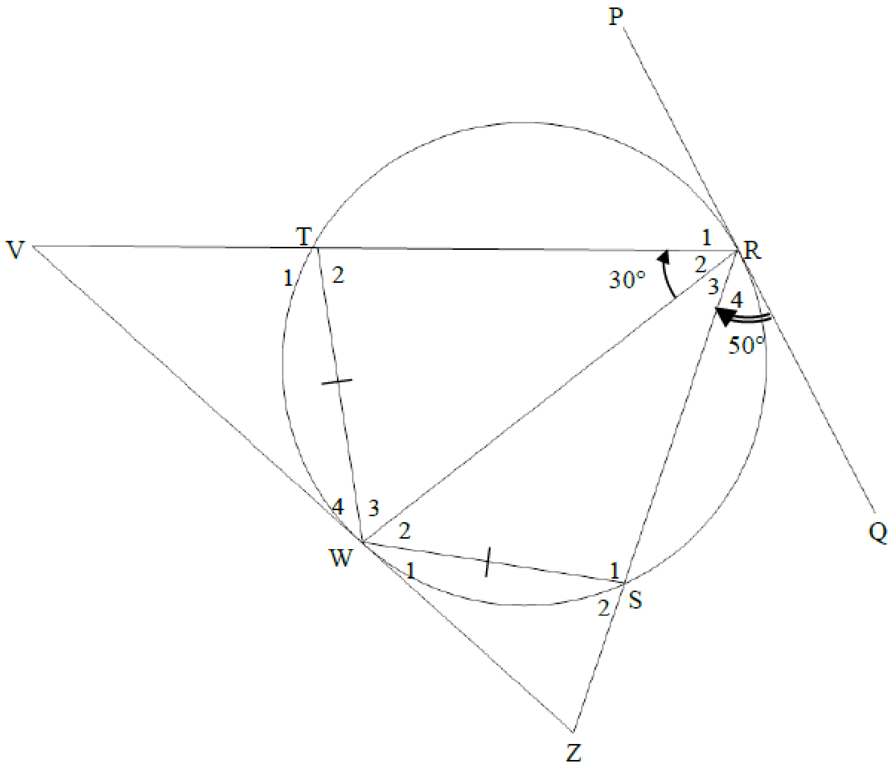
9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = 90°$	**∠ in semi-circle/∠*in halfsirkel***	✓ S ✓ R
$\hat{T}_2 = 90° - x$	**Tangent ⊥ diameter/radius/*raaklyn ⊥ middellyn/radius***	✓ R
$\therefore \hat{C} = x$	Sum of the angles of triangle/*Som van die hoeke van 'n driehoek*	✓ S
$\therefore \hat{S} = x$	**∠'s same segment/∠*e in dieselfde segment***	✓ R
$\therefore V\hat{T}R = \hat{S}$		(5)

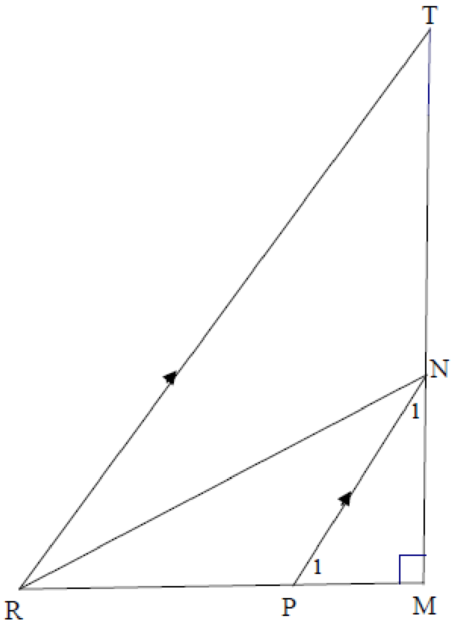
9.3



9.3.1	Equal chords subtend equal ∠s/*Gelyke koorde onderspan gelyke ∠e*	✓ R (1)
9.3.2	$\hat{W}_4 = 30°$ (tan chord theorem/*rkl-koordst*) $\hat{W}_1 = 30°$	✓ answer/*antw* ✓ reason/*rede* ✓ answer/*antw* (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext∠ of Δ/*buite ∠ v Δ*) $\therefore \hat{S}_2 = 80°$ **OR/*OF*** $\hat{R}_2 = \hat{R}_3 = 30°$ (= chords subtend =∠s /= *kde onderspan=∠e*) $\hat{R}_4 = \hat{W}_2 = 50°$ (tan chord theorem/*rkl-koordst*) $\therefore \hat{S}_2 = 80°$	✓ S ✓R ✓ S (3) ✓ S ✓R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80°$ (ext ∠ of cyclic quad/*buite∠ van koordevh*) $V + \hat{W}_4 = \hat{T}_2$ (ext∠ of Δ/*buite∠ van Δ*) $\therefore\ \hat{V} = 50°$	✓ S ✓ R ✓ S ✓ S (4)
9.3.4	In ΔRVW and/*en* ΔRWS: $\hat{R}_2 = \hat{R}_3 = 30°$ (proven/*bewys* in 9.3.1) $\hat{V} = \hat{W}_2 = 50°$ (proven/*bewys* in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd ∠ in Δ) $\therefore \Delta RVW \mid\mid\mid \Delta RWS$ (∠∠∠) $\therefore \frac{WR}{RV} = \frac{RS}{WR}$ (ΔRVW ||| ΔRWS) $\therefore\ WR^2 = RV.RS$	✓ using the correct Δs/ *gebruik korrekte Δe* ✓ S ✓ S ✓ R (3rd ∠ in Δ) or (∠∠∠) ✓ S (5) **[22]**

QUESTION/VRAAG 10



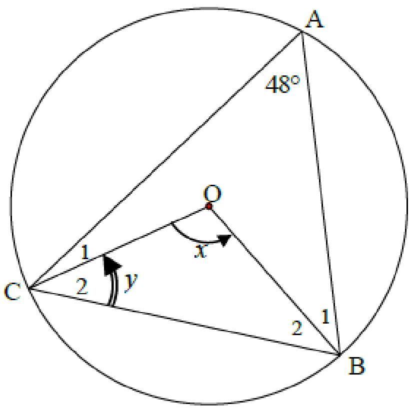
10.1.1	corresponding ∠s/*ooreenkomstige∠e*; PN | | RT	✓ answer/*antw* (1)
10.1.2	∠; ∠; ∠ **OR/*OF*** ∠; ∠	✓ answer/*antw* (1)
10.2	$\frac{PM}{RM}=\frac{PN}{RT}$ $(\Delta PNM \mid\mid\mid \Delta RTM)$ $=\frac{PN}{3PN}$ $=\frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM}=\frac{1}{3}$ $\therefore \frac{RP}{RM}=\frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $=\left(\frac{3}{2}RP\right)^2 - \left(\frac{1}{2}RP\right)^2$ $=\frac{9}{4}RP^2 - \frac{1}{4}RP^2$ $= 2RP^2$ **OR/*OF***	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = \frac{3}{2}RP$ ✓ $PM = \frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (3PM)^2 - PM^2$ $= 8PM^2$ $= 2(2PM)^2$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = RP + PM$ ✓ $(3PM)^2 - PM^2$ ✓ $RP = 2PM$ (4)
	OR/*OF*	
	$RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2)$ (Pyth) $= RM^2 - PM^2$ $= (RP + PM)^2 - PM^2$ $= RP^2 + 2RP.PM + PM^2 - PM^2$ $= RP^2 + 2RP.\frac{1}{2}RP$ $= 2RP^2$	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = RP + PM$ ✓ expansion/ *uitbreiding* ✓ $PM = \frac{1}{2}RP$ (4) **[8]**

NOV 2014

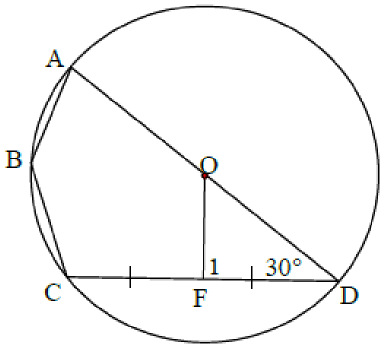
QUESTION/VRAAG 8

8.1



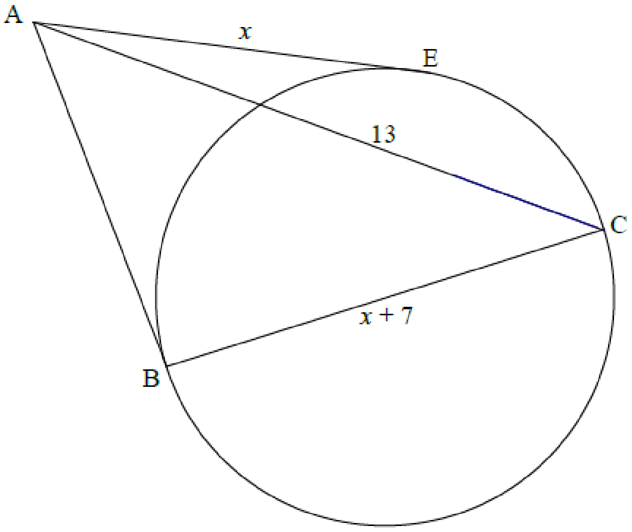
8.1.1	$x = 96°$ (∠ at centre = 2∠ at circumference/ *∠ by midpt = 2∠ by omtrek*)	✓ S ✓ R (2)
8.1.2	$\hat{C}_2 + \hat{B}_2 = 180° - 96° = 84°$ (sum of ∠s in Δ/ *som v∠e in Δ*) $y = \hat{B}_2 = 42°$ (∠s opp = sides/*∠e teenoor = sye*)	✓ S ✓ S (2)

8.2



8.2.1	$\hat{F}_1 = 90°$ (line from centre to midpt chord/ *lyn vanaf midpt na midpt kd)*	✓ S ✓ R (2)
8.2.2	$A\hat{B}C = 150°$ (opposite ∠s of cyclic quad/ *tos ∠e v koordevh*)	✓ S ✓ R (2)

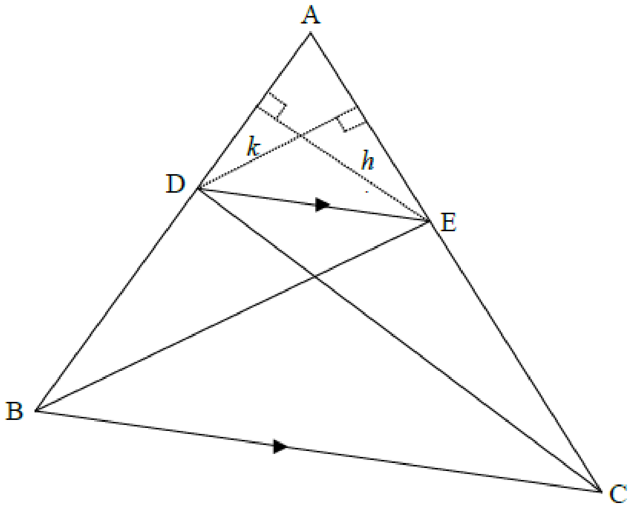
8.3



8.3.1 (a)	tangent ⊥ radius/diameter / *raaklyn ⊥ radius/middellyn*	✓ R (1)
8.3.1 (b)	tangents from common pt **OR** tangents from same pt / *raaklyne v gemeensk pt* ***OF*** *raaklyne vanaf dies pt*	✓ R (1)
8.3.2	$AB^2 + BC^2 = AC^2$ $x^2 + (x+7)^2 = 13^2$ (Theorem of/*Stelling van*Pythagoras) $x^2 + x^2 + 14x + 49 = 169$ $2x^2 + 14x - 120 = 0$ $x^2 + 7x - 60 = 0$ $(x-5)(x+12) = 0$ $x = 5 \quad (x \neq -12)$	✓ $AB^2 + BC^2 = AC^2$ ✓ $x^2 + (x+7)^2 = 13^2$ ✓ standard form ✓ answer (4) **[14]**

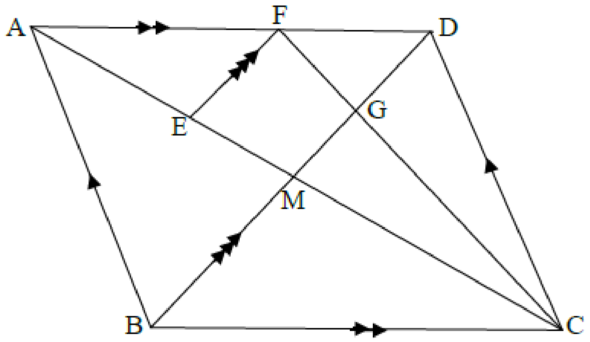
QUESTION/VRAAG 9

9.1



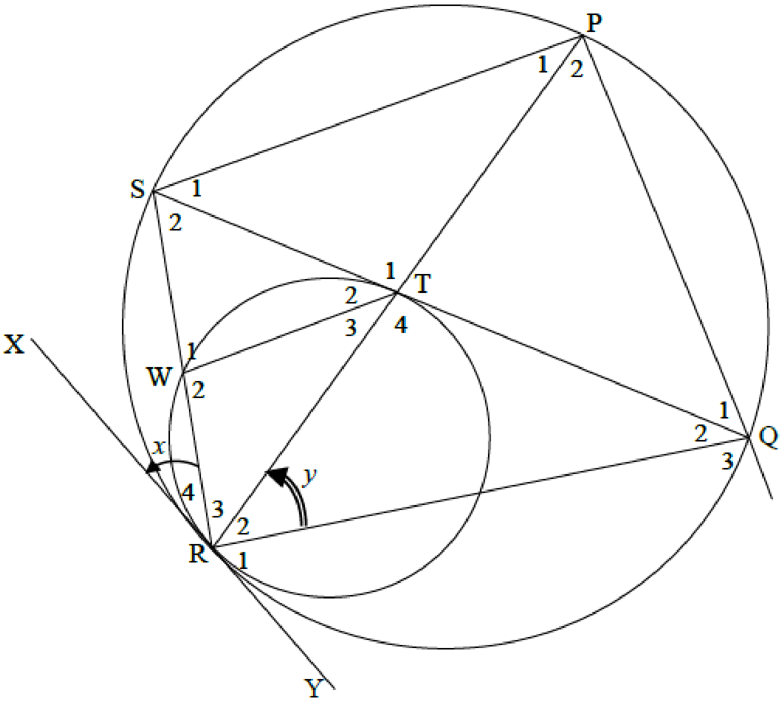
9.1.1	Same base (DE) and same height (between parallel lines) *Dieselfde basis (DE) en dieselfde hoogte (tussen ewewydige lyne)*	✓ same base/*dies basis* between | | lines/ *tussen | | lyne* (1)
9.1.2	$\frac{\text{AD}}{\text{DB}}$ $\frac{\frac{1}{2}\text{AE}\times k}{\frac{1}{2}\text{EC}\times k}$ But/*Maar* area ΔDEB = area ΔDEC (Same base and same height/*dieselfde basis en dieselfde hoogte*) $\therefore \frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEB}} = \frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$	✓ S ✓ S ✓ S ✓ R ✓ S (5)

9.2

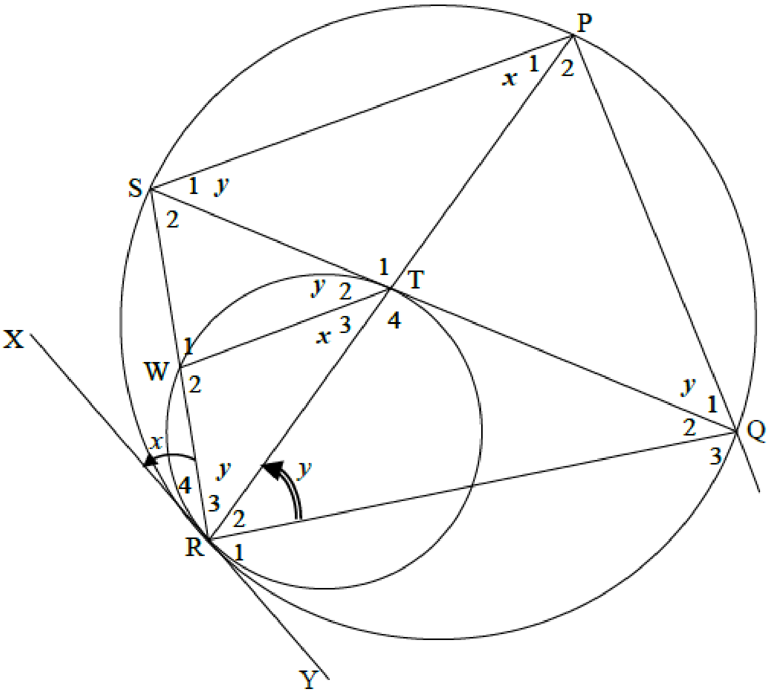


9.2.1	$\frac{EM}{AM}=\frac{FD}{AD}$ (Line parallel one side of Δ **OR** prop th; EF | | BD) *(Lyn ewewydig aan sy v Δ* ***OF*** *eweredigst; EF ||BD)* $\frac{EM}{AM}=\frac{3}{7}$	✓ S ✓R ✓ answer/*antw* (3)
9.2.2	CM = AM (diags of parm bisect/*hoekl parm halv*) $\frac{CM}{ME}=\frac{AM}{ME}=\frac{7}{3}$ (from 9.2.1/*vanaf 9.2.1*)	✓ S ✓R ✓ answer/*antw* (3)
9.2.3	h of ΔFDC = h of ΔBDC (AD | | BC) $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{\frac{1}{2}FD.h}{\frac{1}{2}BC.h}$ $=\frac{FD}{AD}$ (opp sides of parm =) *(tos sye v parm =)* $=\frac{3}{7}$ **OR/*OF*** $\frac{\text{area }\Delta FDC}{\text{area }\Delta ADC}=\frac{FD}{AD}=\frac{3}{7}$ (same heights) *(dieselfde hoogtes)* But Area ΔADC = Area ΔBDC (diags of parm bisect area) *(hoekl v parm halv opp)* $\frac{\text{area }\Delta FDC}{\text{area }\Delta BDC}=\frac{3}{7}$	✓ AD | | BC ✓ subst into area form/ *subst in opp formule* ✓ S ✓ answer/*antw* (4) ✓ S ✓ R ✓ S ✓ answer/*antw* (4) **[16]**

QUESTION/VRAAG 10



10.1.1	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.2	Tangent chord theorem/*Raaklyn-koordstelling*	✓ R (1)
10.1.3	Corresponding angles equal/*Ooreenkomstige ∠e gelyk*	✓ R (1)
10.1.4	∠s subtended by chord PQ **OR** ∠s in same segment *∠e onderspan deur dieselfde koord* ***OF*** *∠e in dieselfde segment*	✓ R (1)
10.1.5	alternate ∠s/*verwisselende ∠e* ; WT $\parallel$ SP	✓ R (1)
10.2	$\frac{RW}{RS}=\frac{RT}{RP}$ (Line parallel one side of Δ **OR** prop th; WT $\parallel$ SP) (*Lyn ewewydig aan sy v Δ* ***OF*** *eweredighst: WT $\parallel$ SP)* $\therefore RT=\frac{WR.RP}{RS}$ **OR/*OF*** $\Delta RTW \ \vert\vert\vert\ \Delta RPS$ (∠; ∠; ∠) $\therefore \frac{RW}{RS}=\frac{RT}{RP}$ ($\Delta RTW \ \vert\vert\vert\ \Delta RPS$) $\therefore RT=\frac{RW.RP}{RS}$	✓ S ✓ R (2) ✓ S ✓ S (2)
10.3	$y=\hat{T}_2=\hat{R}_3$ (tan chord theorem/*Rkl-koordst*) $y=\hat{R}_3=\hat{Q}_1$ (∠s in same segment/*∠e in dieselfde segment*)	✓ S ✓ R ✓ S ✓ R (4)



10.4	$\hat{Q}_3 = P\hat{S}R$ (ext ∠ of cyc quad/*buite∠ v kdvh*) $P\hat{S}R = \hat{W}_2$ (corresp∠s/*ooreenk ∠e* ; WT || SP) $\therefore \hat{Q}_3 = \hat{W}_2$ **OR/OF** $\hat{Q}_2 = x$ (∠s in same segment/*∠e in dies segment*) $\hat{Q}_3 = 180° - (x + y)$ (∠s on straight line/*∠e op reguitlyn*) $\hat{W}_2 = 180° - (x + y)$ (∠s of ΔWRT/*∠e v ΔWRT*) $\therefore \hat{Q}_3 = \hat{W}_2$	✓ S ✓ R ✓ S (3) ✓ R ✓ S ✓ S (3)
10.5	In ΔRTS and ΔRQP: $\hat{R}_3 = \hat{R}_2 = y$ (proven above/*hierbo bewys*) $\hat{S}_2 = \hat{P}_2$ (∠s in same segment/*∠e in dies segment*) $R\hat{T}S = R\hat{Q}P$ (3rd angle of Δ) $\therefore$ΔRTS ||| ΔRQP (∠; ∠; ∠)	✓ S ✓ S/R ✓ S **OR/OF** (∠; ∠; ∠) (3)

10.6	$\frac{RT}{RQ}=\frac{RS}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	$\frac{RS}{RP}\times\frac{RS}{RP}=\frac{RT}{RQ}\times\frac{RS}{RP}$		✓ $\times\frac{RS}{RP}$ on both sides
	$\left(\frac{RS}{RP}\right)^2=\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$		
	$=\left(\frac{RW}{RS}\right)\left(\frac{RS}{RQ}\right)$	(proven in 10.2/*bewys in 10.2*)	✓ $\left(\frac{RT}{RP}\right)\left(\frac{RS}{RQ}\right)$ (3)
	$=\frac{RW}{RQ}$		
	OR/OF		
	$\frac{RT}{RQ}=\frac{RS}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	But $RT=\frac{WR.RP}{RS}$	(proven in 10.2/*bewys in 10.2*)	✓ $RT=\frac{WR.RP}{RS}$
	$\therefore \frac{RT}{RQ}=\frac{WR.RP}{RQ.RS}=\frac{RS}{RP}$		
	$WR.RP^2=RQ.RS^2$		✓multiplication/ *vermenigvuldig*
	$\therefore \frac{WR}{RQ}=\frac{RS^2}{RP^2}$		(3)
	OR/OF		
	$\frac{RT}{RS}=\frac{RQ}{RP}$	($\Delta RTS \mid\mid\mid \Delta RQP$)	✓ S
	$RQ=\frac{RT.RP}{RS}$		
	and $WR=\frac{RT.RS}{RP}$	(proven in 10.2/*bewys in 10.2*)	✓ $WR=\frac{RT.RS}{RP}$
	$\frac{WR}{RQ}=\frac{\frac{RT.RS}{RP}}{\frac{RT.RP}{RS}}$		
	$=\frac{RT.RS}{RP}\times\frac{RS}{RT.RP}$		✓ simplification/ *vereenvoudiging*
	$=\frac{RS^2}{RP^2}$		(3) **[20]**