Gr 11 Mathematics
Trig General Solution
Revise of Trig Functions

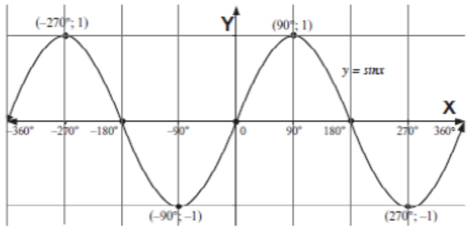
Example 1

Sketch the graph of $y = \sin x$ for x

- We can make use of a table or a calculator to determine the critical points on the graph.
- The endpoints of the domain must be included i.e. $x = -360°$ and $x = 360°$
- All intercepts with the x and y axis must be indicated as well as all minimum and maximum points (turning points)

Solution

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	0	1	0	-1	0	1	0	-1	0



Domain: all the possible x values on the graph
Range: all the possible y-values on the graph
Amplitude: the maximum distance from the equilibrium position(in the above graph the equilibrium position is the x-axis.
Period: number of degrees to complete a wave or a cycle.

Example 2

Use the graph $y = \sin x$ above to answer these questions:

1. What are the maximum and minimum values of $y = \sin x$? (2)
2. Write down the domain and the range of $y = \sin x$. (4)
3. Write down the x-intercepts of $y = \sin x$. (2)
4. What is the amplitude of the graph of $y = \sin x$? (1)
5. What is the period of the graph of $y = \sin x$? (1)

[10]

Solutions

		$y = \sin x$	
1	Maximum Values	1 ✓ , at $x = -270°$ and $90°$	
	Minimum Values	-1 ✓, at $x = -90°$ and $270°$	(2)
2	Domain	$x \in [-360°; 360°]$, $x \in \mathbb{R}$ ✓✓	
	Range	$[-1; 1]$ $y \in \mathbb{R}$ ✓✓	(4)
3	x-intercepts	$-360°$, $-180°$, $0°$, $180°$ and $360°$ ✓✓	(2)
4	Amplitude	1 ✓	(1)
5	Period	$360°$ ✓	(1)
			[10]

Example 1 (Try Yourself – using identities)

Prove the following identities:

1. $\sin x \cdot \tan x + \cos x = \frac{1}{\cos x}$ (4)
2. $(\sin x + \tan x)\left(\frac{\sin x}{1 + \cos x}\right) = \sin x . \tan x$ (7)
3. $\frac{1}{\cos x} = \frac{\cos x}{1 + \sin x} + \tan x$ (6)
4. $\frac{1}{\tan x} + \tan x = \frac{\tan x}{\sin^2 x}$ (5)

[22]

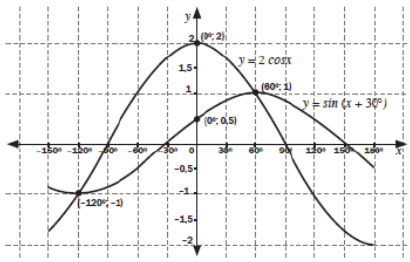
1. Given $f(x) = 2\cos x$ and $g(x) = \sin(x + 30°)$

a) Sketch the graphs of f and g on the same set of axes for $x \in [-150°; 180°]$
Clearly show all intercepts with the axes and the coordinates of turning points. (7)

Use your graph to answer the following questions:

b) Write down the period of f. (1)
c) For which values of x is $f(x) = g(x)$? (2)
d) For which values of x is $f(x) > 0$? (2)
e) For which values of x is $g(x)$ increasing? (2)
f) Determine one value of x for which $f(x) - g(x) = 1,5$. (1)
g) If the curve of f is moved down one unit, write down the new equation of f. (2)
h) If the curve of g is moved 45° to the left, write down the new equation of g. (2)

1. a) ✓✓✓ for $g(x) = 2\cos x$ and ✓✓✓✓ for $f(x) = \sin(x + 30°)$



(7)

b) period $= 360°$ ✓ (1)
c) $x = -120°$ or $60°$ ✓✓ (2)
d) for $f(x) > 0$; $x \in (-90°; 90°)$ ✓✓ (2)
e) $g(x)$ increasing when $x \in (-120°; 60°)$ ✓✓ (2)
f) $x = 0°$ ✓ (1)

g) New $f(x) = 2\cos x - 1$ ✓✓ (2)
h) Original equation: $g(x) = \sin(x + 30°)$, with 45° shift to the left:
$g(x) = \sin(x + 30° + 45°)$ so $g(x) = \sin(x + 75°)$ ✓✓ (2)

- Choose either the **left-hand side** or the **right-hand side** and simplify it to look like the other side.
- If both sides look difficult, you can try to simplify on both sides until you reach a point where both sides are the same.
- It is usually helpful to write $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$.
- Sometimes you need to simplify $\frac{\sin\theta}{\cos\theta}$ to $\tan\theta$.
- If you have $\sin^2 x$ or $\cos^2 x$ with $+1$ or -1, use the squares identities ($\sin^2\theta + \cos^2\theta = 1$).
- Find a common denominator when fractions are added or subtracted.
- Factorise if necessary

Answers

1. LHS: $\sin x . \tan x + \cos x$

$= \sin x . \frac{\sin x}{\cos x} + \cos x$ ✓ $+ \cos x$

$= \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1}$

$= \frac{\sin^2 x + \cos^2 x \checkmark}{\cos x \checkmark} = \frac{1}{\cos x}$ ✓ = RHS (4)

$\therefore \sin x . \tan x + \cos x = \frac{1}{\cos x}$ (4)

2. LHS: $(\sin x + \tan x)\left(\frac{\sin x}{1+\cos x}\right)$ RHS: $\sin x . \tan x$

$= \left(\sin x + \frac{\sin x}{\cos x} \checkmark\right)\left(\frac{\sin x}{1+\cos x}\right)$ $= \sin x . \frac{\sin x}{\cos x}$ ✓

$= \left(\frac{\sin x \cos x + \sin x \checkmark}{\cos x \checkmark}\right)\left(\frac{\sin x}{1+\cos x}\right)$ $= \frac{\sin^2 x}{\cos x}$ ✓

$= \left(\frac{\sin x(\cos x + 1) \checkmark}{\cos x}\right)\left(\frac{\sin x}{1+\cos x}\right)$

$= \frac{\sin^2 x}{\cos x}$ ✓ (7)

$\therefore$ LHS = RHS (7)

3. RHS: $\frac{\cos x}{1+\sin x} + \tan x$

$= \frac{\cos x}{1+\sin x} + \frac{\sin x}{\cos x}$ ✓

$= \frac{\cos^2 x + \sin x(1+\sin x) \checkmark}{\cos x(1+\sin x) \checkmark}$

$= \frac{\cos^2 x + \sin x + \sin^2 x \checkmark}{\cos x(1+\sin x)}$ trig identity: $\cos^2 x + \sin^2 x = 1$

$= \frac{1+\sin x \checkmark}{\cos x(1+\sin x)}$

$= \frac{1}{\cos x}$ ✓ = LHS

$\therefore \frac{1}{\cos x} = \frac{\cos x}{1+\sin x} + \tan x$ (6)

4. $\frac{1}{\tan x} + \tan x = \frac{\tan x}{\sin^2 x}$

LHS: $\frac{1}{\tan x} + \tan x$ RHS: $\frac{\tan x}{\sin^2 x}$

$= \frac{1}{\frac{\sin x}{\cos x}} + \frac{\sin x}{\cos x}$ ✓ $= \frac{\sin x}{\cos x}$ ✓ $. \frac{1}{\sin^2 x}$

$= \frac{\cos x}{\sin x}$ ✓ $+ \frac{\sin x}{\cos x}$ $= \frac{1}{\sin x . \cos x}$

$= \frac{\cos^2 x + \sin^2 x \checkmark}{\sin x . \cos x \checkmark}$

$= \frac{1}{\sin x . \cos x}$

$\therefore$ LHS = RHS (5)

[22]

Example 2

2. Solve for x: $\sin x = -0{,}7$

This time, place the reference angle in quadrants III and IV ($\sin x$ is negative)

$x = 180° + 44{,}42.....° + k360°$ or $x = 360° - 44{,}42.....° + k360°$ $k \in \mathbb{Z}$

$x = 224{,}42° + k360°$ or $x = 315{,}57° + k360$; $k \in \mathbb{Z}$

(Correct to two decimal place)

Example 3

3. Solve for x: $\cos x = -0{,}7$ Reference angle = 134,427....°

$\cos x$ is negative in quadrants II and III.

$x = 360° - 134{,}43° = 225{,}57°$

$x = 134{,}43° + k360°$ or $x = 225{,}57° + k360°$; $k \in \mathbb{Z}$

(Correct to two decimal place)

Determine the general solution for x in the following equations:

a) $5 \sin x = \cos 320°$ (correct to 2 decimal places)

b) $3 \tan x + \sqrt{3} = 0$ (without using a calculator)

c) $\frac{\tan x - 1}{2} = -3$ (correct to one decimal place) (10)

Answers

a) $5 \sin x = \cos 320°$ ✓

$5 \sin x = 0{,}766044$

$\sin x = 0{,}15320...$ ✓

Ref angle = 8,81°

$x = 8{,}81° + k360°$ OR $x = 180° - 8{,}81° + k360°$ ✓

$x = 171{,}19° + k360°$ ✓ $k \in \mathbb{Z}$ (4)

Calculator keys:
cos 320 =
÷ 5 =
SHIFT sin ANS =

b) $3 \tan x + \sqrt{3} = 0$

$3 \tan x = -\sqrt{3}$

$\tan x = \frac{-\sqrt{3}}{3}$ ✓ [special angle: tan 30° $\tan 30° = \frac{\sqrt{3}}{3}$]

Ref angle = 30°

$x = 180° - 30° + k180°$ ✓

$x = 150° + k180°$ ✓ $k \in \mathbb{Z}$ (3)

c) $\frac{\tan x - 1}{2} = -3$ multiply both sides by 2

$\tan x - 1 = -6$

$\tan x = -5$ ✓ reference angle is 78,69...°

$\therefore x = 180° - 78{,}69...° + k180°$ ✓

$x = 101{,}31° + k180°$; $k \in \mathbb{Z}$ ✓ (3) (10)

Example 5

5. Solve for x: $\tan x = 0{,}7$

$\tan x$ is positive in quadrants I and III.

Reference angle = 34,99° (correct to 2 dec places)

$x = 34{,}99....°$ or $180° + 34{,}99......° = 214{,}99.......°$

Now the period of the tan graph is 180°, so the other points of intersection occur 180° to the right or left of the solutions.

$x = 34{,}99° + k180°$; $k \in \mathbb{Z}$

(Correct to two decimal place)

NB!!!

For Tan equations general solution, we use +k.180° because the period of a Tan function is 180°

Example 6

6. Solve for x: $\tan x = -0{,}7$

$\tan x$ is negative in quadrants II and IV.

The reference angle is $-34{,}99.....°$

$180° - 34{,}99.......° = 145{,}01.....°$

$x = 145{,}01° + k180°$; $k \in \mathbb{Z}$.