Gr 11

Identities and Reduction Formula Exercises

Exercise 1

Without using a calculator, determine the value of:

1. $\cos 150°$ **2.** $\sin(-45°)$ **3.** $\tan 480°$

[7]

Solutions

1. $\cos 150°$ — rewrite as (180 – ?)

$= \cos(180° - 30°)$ — quadrant II, cos θ negative

$= -\cos 30°$ ✓ — special ratios

$= -\frac{\sqrt{3}}{2}$ ✓ (2)

2. $\sin(-45°)$ — $\sin(-\theta) = -\sin\theta$; quadrant IV, sin θ negative

$= -\sin 45°$ ✓ — special ratios

$= -\frac{1}{\sqrt{2}}$ ✓ (2)

3. $\tan 480°$ — write as an angle in the first rotation of 360°

$= \tan(480° - 360°)$

$= \tan 120°$ ✓ — quadrant II, rewrite as (180 – ?)

$= \tan(180° - 60°)$ — tan θ negative

$= -\tan 60°$ ✓ — special ratios

$= -\sqrt{3}$ ✓ (3)

[7]

Exercise 2

Write the trig ratios as the trig ratios of their co-functions:

1. $\sin 50°$ **2.** $\cos 70°$ **3.** $\sin 100°$ **4.** $\cos 140°$

[4]

Solutions

1. $\sin 50° = \sin(90° - 40°) = \cos 40°$ ✓
2. $\cos 70° = \cos(90° - 20°) = \sin 20°$ ✓
3. $\sin 100° = \sin(90° + 10) = \cos 10°$ ✓
4. $\cos 140° = \cos(90° + 50°) = -\sin 50°$ ✓

[4]

NOT ALWAYS SPECIAL ANGLES

SUMMARY

Any angle (obtuse or reflex) can be reduced to an acute angle by using:

- Convert negative angles to positive angles
- Reduce angles greater than 360°
- Use reduction formulae
- Use co-functions

Example 6 (Try Yourself)

Simplify without using a calculator:

1. $\frac{\sin(180° + x).\cos 330°.\tan 150°}{\sin x}$ (4)
2. $\frac{\cos 750°.\tan 315°.\cos(-\theta)}{\cos(360° - \theta).\sin 300°.\sin(180° - \theta)}$ (8)
3. $\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$ (9)
4. $\frac{\cos 260°.\cos 170°}{\sin 10°.\sin 190°.\cos 350°}$ (7)

[28]

Co-Functions

The functions change from cos to sin or sin to cos if we use 90°+ or 90°- to reduce.
The signs of whether the function is positive or negative may also change.

$\sin(90° - \theta) = \cos\theta$ (quadrant I)

$\sin(90° + \theta) = \cos\theta$ (sin θ positive in quadrant II)

$\cos(90° - \theta) = \sin\theta$ (quadrant I)

$\cos(90° + \theta) = -\sin\theta$ (cos θ negative in quadrant II)

SOME OTHER POSSIBILITIES THAT COULD COME UP

IT NEEDS TO BE 90°+ or 90°- in order to use the co-function reduction formula

$\sin(\theta - 90°) = \sin[-(90° - \theta)]$ (common factor of –1)

$= -\sin(90° - \theta)$ (sin θ negative in quadrant IV)

$= -\cos\theta$

$\cos(\theta - 90°) = \cos[-(90° - \theta)]$ (common factor of –1)

$= +\cos(90° - \theta)$ (cos θ positive in quadrant IV)

$= +\sin\theta$

Answers

1. $\frac{\sin(180^\circ + x).\cos 330^\circ.\tan 150^\circ}{\sin x}$ reduction formulae in numerator

✓ ✓ ✓

$= \frac{(-\sin x)(+\cos 30^\circ)(-\tan 30^\circ)}{\sin x}$ (use brackets to separate ratios)

$= \frac{+\sin x.\frac{\sqrt{3}}{2} ✓.\frac{\sqrt{3}}{3}}{\sin x}$ special angles

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{3}$

$= \frac{3}{6} = \frac{1}{2}$ (4)

2. $\frac{\cos 750^\circ.\tan 315^\circ.\cos(-\theta)}{\cos(360^\circ - \theta).\sin 300^\circ.\sin(180^\circ - \theta)}$ use reduction formulae

$= \frac{\cos 30^\circ ✓.(-\tan 45^\circ) ✓.\cos\theta ✓}{\cos\theta ✓.(-\sin 60^\circ) ✓.\sin\theta ✓}$ use special angles

$= \frac{\frac{\sqrt{3}}{2}.(-1)\cos\theta}{\cos\theta.\left(-\frac{\sqrt{3}}{2}\right)\sin\theta}$ ✓

$= \frac{-1}{-\sin\theta} = \frac{1}{\sin\theta}$ ✓ (8)

3. $\frac{\tan 480^\circ.\sin 300^\circ.\cos 14^\circ.\sin(-135^\circ)}{\sin 104^\circ.\cos 225^\circ}$

$= \frac{\tan 120^\circ.(-\sin 60) ✓.\cos 14^\circ.\sin 225^\circ}{\sin 76^\circ ✓.(-\cos 45^\circ) ✓}$

$= \frac{\cos(180^\circ + 80^\circ).\cos(180^\circ - 10^\circ)}{\sin 10^\circ.\sin(180^\circ + 10^\circ).\cos(360^\circ - 10^\circ)}$

$= \frac{(-\tan 60^\circ) ✓.(-\sin 60^\circ).\sin 76^\circ ✓.(-\sin 45^\circ) ✓}{\sin 76^\circ.(-\cos 45^\circ)}$

$= \frac{(-\sqrt{3}).\left(\frac{-\sqrt{3}}{2}\right).\sin 76.\left(\frac{-\sqrt{2}}{2}\right)}{\sin 76^\circ.\left(\frac{-\sqrt{2}}{2}\right)}$ ✓✓

$= \frac{3}{2}$ ✓ (9)

4. $\frac{\cos 260^\circ.\cos 170^\circ}{\sin 10^\circ.\sin 190^\circ.\cos 350^\circ}$

$= \frac{-\cos 80^\circ ✓.(-\cos 10^\circ)}{\sin 10^\circ.(-\sin 10^\circ) ✓.\cos 10^\circ ✓}$

$= \frac{(-\sqrt{3}).\left(\frac{-\sqrt{3}}{2}\right).\sin 76.\left(\frac{-\sqrt{2}}{2}\right)}{\sin 76^\circ.\left(\frac{-\sqrt{2}}{2}\right)}$ ✓✓

$= \frac{-\sin 10^\circ ✓.(-\cos 10^\circ)}{\sin 10^\circ.(-\sin 10^\circ).\cos 10^\circ}$

$= \frac{-1}{\sin 10^\circ}$ ✓ (7)

[28]