Gr 12

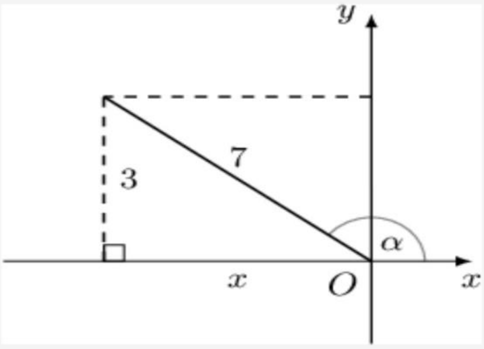
Trigonometry Exercises

**3.** Given $7\sin\alpha = 3$ for $\alpha > 90^\circ$.

Determine the following (leave answers in surd form):

**a)** $\cos 2\alpha$

Draw a sketch.




**b)** $\tan 2\alpha$

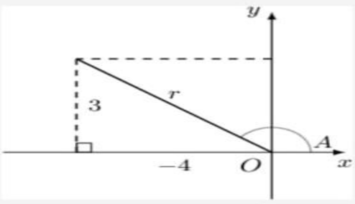
$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$$
$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\left(\frac{3}{7}\right)\left(-\frac{\sqrt{40}}{7}\right) = -\frac{6\sqrt{40}}{49}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{-\frac{6\sqrt{40}}{49}}{\frac{31}{49}} = -\frac{6\sqrt{40}}{49} \times \frac{49}{31} = -\frac{6\sqrt{40}}{31}$$

**4.** If $4\tan A + 3 = 0$ for $A < 270^\circ$, determine, without the use of a calculator:

$$\left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$$

Draw a sketch.




We are given that $A < 270^\circ$, therefore $A$ must lie in the second quadrant for the tangent function to be negative.

$$r^2 = 3^2 + (-4)^2 = 25$$
$$\therefore r = 5$$

$$\left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$$
$$= \sin^2\frac{A}{2} - \cos^2\frac{A}{2}$$
$$= -\left(\cos^2\frac{A}{2} - \sin^2\frac{A}{2}\right)$$
$$= -\cos 2\left(\frac{A}{2}\right)$$
$$= -\cos A$$
$$= -\left(-\frac{4}{5}\right)$$

**c)**

$\cos x \cos 10^\circ + \sin x \cos 100^\circ = 1 - 2\sin^2 x$

$$\begin{aligned}
\cos x \cos 10^\circ + \sin x \cos 100^\circ &= 1 - 2\sin^2 x \\
\cos x \cos 10^\circ + \sin x \cos(90^\circ + 10^\circ) &= \cos 2x \\
\cos x \cos 10^\circ - \sin x \sin 10^\circ &= \cos 2x \\
\cos(x + 10^\circ) &= \cos 2x \\
\text{First quadrant:} \quad x + 10^\circ &= 2x + k \cdot 360^\circ \\
x &= 10^\circ + k \cdot 360^\circ \\
\\
\text{Fourth quadrant:} \quad x + 10^\circ &= (360^\circ - 2x) + k \cdot \\
3x &= 350^\circ + k \cdot 360^\circ \\
\therefore x &= 116{,}7^\circ + k \cdot 120^\circ \\
\\
\text{Final answer:} \quad x &= 10^\circ + k \cdot 360^\circ \\
x &= 116{,}7^\circ + k \cdot 120^\circ
\end{aligned}$$

**d)** $6\sin^2 \alpha + 2\sin 2\alpha - 1 = 0$

$$\begin{aligned}
6\sin^2 \alpha + 2\sin 2\alpha - 1 &= 0 \\
6\sin^2 \alpha + 2(2\sin\alpha\cos\alpha) - 1 &= 0 \\
6\sin^2 \alpha + 4\sin\alpha\cos\alpha - (\sin^2\alpha + \cos^2\alpha) &= 0 \\
6\sin^2 \alpha + 4\sin\alpha\cos\alpha - \sin^2\alpha - \cos^2\alpha &= 0 \\
5\sin^2 \alpha + 4\sin\alpha\cos\alpha - \cos^2\alpha &= 0 \\
(5\sin\alpha - \cos\alpha)(\sin\alpha + \cos\alpha) &= 0 \\
\text{If } 5\sin\alpha - \cos\alpha &= 0 \\
5\sin\alpha &= \cos\alpha \\
\therefore \tan\alpha &= \frac{1}{5} \\
\therefore \alpha &= 11{,}3^\circ + k \cdot
\end{aligned}$$