ANNUITIES

- **Regular monthly payments**
- **Two types of annuities:**

  1) Future Value Annuities
  - used for investments, savings and sinking funds

  KNOW THESE FORMULAE AND WHEN TO USE THEM AS INDICATED

$$F_v = \frac{x\left[(1+i)^n - 1\right]}{i}$$

- F_v = Future Value of annuity
- x = Regular payments according to Time Period
- i = Interest rate
- n = Time period

  2) Present Value Annuities
  - used for bonds and loans

$$P_v = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

- P_v = Present Value of annuity
- x = Regular payments according to Time Period
- i = Interest rate
- n = Time period

FINANCIAL MATHS REVISION

Simple Interest:

$$A = P(1 + ni)$$

Compound Interest:

$$A = P(1 + i)^n$$

A = total amount (End Amount)
P = principle amount (Beginning Amount)
n = number of time periods
i = interest rate

TO WORK OUT n:
Substitute for A, P and i
- Simplify
- Write in logarithmic form
- Use the log keys on the calculator
- Round off the answer to the nearest year

FUTURE VALUE ANNUITY

Example 1
Josephine invests R500 per month into a long-term savings account, at an interest rate of 8% p.a. compounded monthly for ten years, in order to make provision for her grandchildren's education. Determine the value of Josephine's education savings fund after 10 years.

FUTURE VALUE ANNUITY FORMULA

Answer
$F_v = ?$
x = R500
$\therefore i = \frac{0,08}{12}$ (8% p.a. compounded monthly)
$\therefore n = 10 \times 12 = 120$ (10 years compounded monthly)

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$
$$= \frac{500\left[\left(1 + \frac{0,08}{12}\right)^{120} - 1\right]}{\frac{0,08}{12}}$$
$$= R91\ 473,02$$

FUTURE VALUE ANNUITY

Example 2 :
Pedro wants to save R600 000 by the end of 10 years. If the bank offers an interest rate of 10% p.a. compounded monthly, determine how much Pedro must invest every month.

FUTURE VALUE ANNUITY FORMULA

Answer (Working out x- the monthly payments)

$F_v = 600\ 000$
$x = ?$
$\therefore i = \frac{0,10}{12}$ (10% p.a. compounded monthly)
$\therefore n = 10 \times 12 = 120$ (10 years compounded monthly)

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$
$$600000 = \frac{x\left[\left(1 + \frac{0,10}{12}\right)^{120} - 1\right]}{\frac{0,10}{12}}$$
$$5000 = x\left[\left(1 + \frac{0,10}{12}\right)^{120} - 1\right]$$
$$x = R2929,04$$

FUTURE VALUE ANNUITY

Example 3 :
How long will it take Judy to save R550 000, if she invests R1 500 each month at an interest rate of 9,7% p.a. compounded monthly?

FUTURE VALUE ANNUITY FORMULA

Answer (Working out Time Period)

$F_v = R550\ 000$
$x = R1500$
$\therefore i = \frac{0,097}{12}$ (9.7% p.a. compounded monthly)
$\therefore n = ?$

Use Log to make n subject of formula

Round up Time period

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$
$$550\ 000 = \frac{1\ 500\left[\left(1 + \frac{0,097}{12}\right)^n - 1\right]}{\frac{0,097}{12}}$$
$$550\ 000 = \frac{1\ 500\left[\left(1 + \frac{0,097}{12}\right)^n - 1\right]}{\frac{0,097}{12}}$$
$$4\ 445,83 \ldots = 1\ 500\left[\left(1 + \frac{0,097}{12}\right)^n - 1\right]$$
$$\frac{1067}{360} = \left(1 + \frac{0,097}{12}\right)^n - 1$$
$$\frac{1427}{360} = \left(1 + \frac{0,097}{12}\right)^n$$
$$n = \log_{\left(\frac{12,097}{12}\right)} \frac{1427}{360}$$
$$n = 172\ months$$

PRESENT VALUE ANNUITY

Example 6 :
Anna would like to borrow R120 000 for an overseas holiday. Determine her monthly repayments, if the bank charges 9,7% p.a. compounded monthly and she pays off her loan at the end of 2 years.

PRESENT VALUE ANNUITY FORMULA

ANSWER WORK OUT THE x VALUE- PAYMENT PER MONTH

$P_v = 120\ 000$
$x = ?$
$\therefore i = \frac{0,097}{12}$ (9,7% p.a. compounded monthly)
$\therefore n = 2 \times 12 = 24$ (2 years compounded monthly)

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$$
$$120\ 000 = \frac{x\left[1 - \left(1 + \frac{0,097}{12}\right)^{-24}\right]}{\frac{0,097}{12}}$$
$$970 = x\left[1 - \left(1 + \frac{0,097}{12}\right)^{-24}\right]$$
$$x = R5520,79$$

PRESENT VALUE ANNUITY

Example 5 :
Callan would like to buy a new car. He is able to afford monthly payments of R1 200 per month. Determine the value of the car loan if the bank will charge 11% p.a. compounded monthly for a loan that will be paid back at the end of 5 years.

PRESENT VALUE ANNUITY FORMULA

ANSWER
$P_v = ?$
$x = 1\ 200$
$\therefore i = \frac{0,11}{12}$ (11% p.a. compounded monthly)
$\therefore n = 5 \times 12 = 60$ (5 years compounded monthly)

$$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$$
$$= \frac{1200\left[1 - \left(1 + \frac{0,11}{12}\right)^{-60}\right]}{\frac{0,11}{12}}$$
$$= R55\ 191,64$$

FINANCIAL MATHS REVISION

Simple Decay:

$$A = P(1 - ni)$$

Compound Decay:

$$A = P(1 - i)^n$$

A = total amount (End Amount)
P = principle amount (Beginning Amount)
n = number of time periods
i = interest rate

TO WORK OUT n:
Substitute for A, P and i
- Simplify
- Write in logarithmic form
- Use the log keys on the calculator
- Round off the answer to the nearest year

FUTURE VALUE ANNUITY

Example 4 :
A bus company would like to set up a sinking fund in order to make provision for replacing buses in 8 years time. A bus currently costs R890 000 with inflation at 7% p.a. However, depreciation is at 6,5% p.a. Determine the bus company's monthly investments into a sinking fund, if it accrues interest at 12% p.a. compounded monthly.

COMPOUND INTEREST AND DECAY FORMULA

FUTURE VALUE ANNUITY FORMULA

Answer
Inflation:
$A = ?$
$P = 890\,000$
$i = 0,07$
$n = 8$

$A = 890000(1 + 0,07)^8$
$= R1\,529\,185,70$
∴ The value of the bus in 8 years time

Depreciation:
$A = ?$
$P = 890\,000$
$i = 0,065$
$n = 8$

$A = 890000(1 - 0,065)^8$
$= R519\,855,80$
∴ The trade-in value of the bus in 8 years time

FIRST WORK OUT THE VALUE OF THE BUS IN 8 YEARS TIME DUE TO INFLATION

NEXT WORK OUT THE VALUE OF THE BUS IN 8 YEARS TIME DUE TO DEPRECIATION.

Sinking Fund = Inflation – Depreciation value
∴ $F_v = R1\,529\,185,70 - R519\,855,80$
$= R1\,009\,329,90$
THIS WILL BE THE FUTURE VALUE OF THE BUS (SINKING FUND)

THE FUTURE VALUE ANNUITY FORMULA WILL NOW BE USED TO WORK OUT HOW MUCH MUST BE INVESTED MONTHLY TO BE ABLE TO BUY THE BUS.

Example 4 continued:

$$F_v = \frac{x[(1 + i)^n - 1]}{i}$$

$$1009329,90 = \frac{x\left[\left(1 + \frac{0,12}{12}\right)^{8\times12} - 1\right]}{\frac{0,12}{12}}$$

$$10\,093,299 = x\left[\left(1 + \frac{0,12}{12}\right)^{96} - 1\right]$$

$$x = R6\,311,18$$

PRESENT VALUE ANNUITY

Example 7 :
Determine how long it would take Candice to pay back a loan of R50 000, if she is able to repay R2 500 per month; at an interest rate of 10% p.a. compounded monthly.

PRESENT VALUE ANNUITY FORMULA

ANSWER WORK OUT THE TIME PERIOD
$P_v = 50\,000$
$x = 2\,500$
∴ $i = \frac{0,10}{12}$ (10% p.a. compounded monthly)
∴ $n = ?$

USE LOGS TO MAKE n SUBJECT OF FORMULA
REMINDER:
If $8 = 2^x$
THEN IN LOG FORM
$x = \log_2 8$

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$50\,000 = \frac{2\,500\left[1 - \left(1 + \frac{0,10}{12}\right)^{-n}\right]}{\frac{0,10}{12}}$$

$$\frac{1250}{3} = 2\,500\left[1 - \left(1 + \frac{0,1}{12}\right)^{-n}\right]$$

$$\frac{1250}{3} = 2\,500\left[1 - \left(1 + \frac{0,1}{12}\right)^{-n}\right]$$

$$\frac{1}{6} = 1 - \left(1 + \frac{0,1}{12}\right)^{-n}$$

$$-\frac{5}{6} = -\left(1 + \frac{0,1}{12}\right)^{-n}$$

$$\frac{5}{6} = \left(1 + \frac{0,1}{12}\right)^{-n}$$

$$\frac{5}{6} = \left(\frac{12}{12} + \frac{0,1}{12}\right)^{-n} = \left(\frac{12,1}{12}\right)^{-n}$$

$$-n = \log_{\frac{12,1}{12}}\frac{5}{6}$$

$$n = -\log_{\frac{12,1}{12}}\frac{5}{6}$$

$$n = -(-21.96)$$

$$n = 22\,months$$

PRESENT VALUE ANNUITY

PRESENT VALUE ANNUITY FORMULA

Example 8 :
Byron takes out a loan of R300 000 to buy a piece of land, which he intends to pay back over 20 years, at an interest rate of 10,5% p.a. compounded monthly. His monthly instalments are R2995,14. If Byron inherits money and would like to settle the outstanding balance on the property after 5 years, determine how much he will have to still pay.

ANSWER
$P_v = ?$
$x = 2995,14$
∴ $i = \frac{0,105}{12}$ (10,5% p.a. compounded monthly)
∴ $n = 15 \times 12 = 180$ (15 years remaining on loan)

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$= \frac{2995,14\left[1-\left(1+\frac{0,105}{12}\right)^{-180}\right]}{\frac{0,105}{12}}$$

$$= R270\,955,57$$

5 years were paid off of the 20 therefore there are 15 years left. You need to work out the value of the loan for 15 years- which is still left. Compounded monthly 15 x 12 = 180 months

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FINANCIAL MATHS- SIMPLE DECAY

EXAMPLE 1

A car worth R120 000 depreciates at a rate of 12% (simple interest) p.a. How much will the car be worth after 5 years?

$A = ?$
$P = R120000$
$i = 12\% = \frac{12}{100} = 0.12$
$n = 5\,years$

$A = P(1 - i.n)$
$A = 120000(1 - 0.12 \times 5)$
$A = 48000$
∴ The car is worth R48000 after 5 years

The same formula as Simple Interest except there is a minus

FINANCIAL MATHS- COMPOUND DECAY

EXAMPLE 2

A car worth R120 000 depreciates at a rate of 12% p.a. (on a reducing balance). How much will the car be worth after 5 years?

$A = ?$
$P = R120000$
$i = 12\% = \frac{12}{100} = 0.12$
$n = 5\,years$

$A = P(1 - i)^n$
$A = 120000(1 - 0.12)^5$
$A = 63327.83002 \ldots$
∴ The car is worth R63327.83 after 5 years

The same formula as Compound Interest except there is a minus

FINANCIAL MATHS- COMPOUND AND SIMPLE DECAY

EXAMPLE 3

The value of a piece of machinery depreciates from R10 000 to R5 000 in 4 years. What is the rate of depreciation, correct to two decimal places, if calculated on the:

a) Straight line method (i.e. simple depreciation)

$A = R5000$
$P = R10000$
$i = ?$
$n = 4\,years$

$A = P(1 - i.n)$
$5000 = 10000(1 - i.4)$
$\frac{5000}{10000} = 1 - i.4$
$\frac{1}{2} - 1 = -i.4$
$-\frac{1}{2} = -i.4$
$\frac{1}{4} = i$
$0.125 = i$
∴ The interest rate is 12.5%

b) Reducing balance (i.e. compound depreciation)
$A = R5000$
$P = R10000$
$i = ?$
$n = 4\,years$

$A = P(1 - i)^n$
$5000 = 10000(1 - i)^4$
$\frac{5000}{10000} = (1 - i)^4$
$\frac{1}{2} = (1 - i)^4$
$\sqrt[4]{\frac{1}{2}} = 1 - i$
$\sqrt[4]{\frac{1}{2}} - 1 = -i$
$0.1591035 = i$
∴ The interest rate is 15.9%