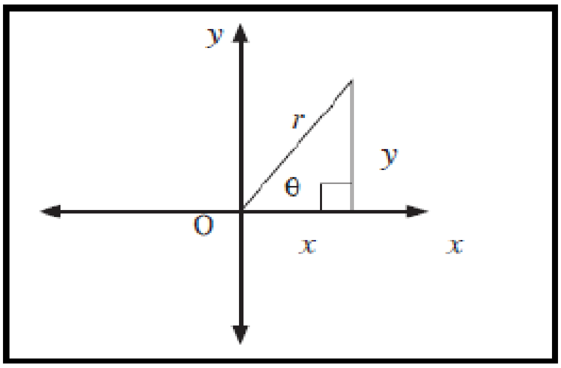
PROOF OF THE ABOVE IDENTITIES (NEED TO KNOW)

Write the proofs out a couple of times until you can do them without looking at the notes.



Proof of the identities are examinable with the RHS and break it down into its x, y and r values.

Proof: $\frac{\sin\theta}{\cos\theta}$

$= \frac{y}{r} \div \frac{x}{r}$

$= \frac{y}{r} \times \frac{r}{x}$

$= \frac{y}{x} = \tan\theta$

Proof: $\sin^2\theta + \cos^2\theta$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$ Use LCD r^2

$= \frac{x^2 + y^2}{r^2}$ $x^2 + y^2 = r^2$ (Pythagoras)

$= \frac{r^2}{r^2} = 1$

We can use the identities and the reduction formulae to help us simplify trig expressions

Special Angles

These will always be valid for 30⁰; 45⁰; 60⁰. You must learn and remember the special angles ratios. It is used with questions that say WITHOUT THE USE OF A CALCULATOR.

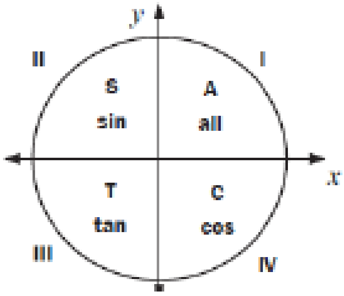
θ	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Reduction Formula

We use Reduction Formula to simplify the trig function or expression to obtain an acute angle.

a) Reduction formulae

Quadrant II: 180° − θ	Quadrant III: 180° + θ	Quadrant IV: 360° − θ
sin(180° − θ) = sin θ	sin(180° + θ) = −sin θ	sin(360° − θ) = −sin θ
cos(180° − θ) = −cos θ	cos(180° + θ) = −cos θ	cos(360° − θ) = cos θ
tan(180° − θ) = −tan θ	tan(180° + θ) = tan θ	tan(360° − θ) = −tan θ



b) **Angles greater than 360°**

We can add or subtract 360° (or multiples of 360°) and will always end up with an angle in the first revolution. For example, 390° can be written as (30° + 360°), so 390° has the same terminal arm as 30°.

c) **Negative angles:**

- (−θ) lies in quadrant IV and is the same as 360° − θ.

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

Example 3

$\sin(360^\circ + \theta) = \sin\theta$ $\cos(360^\circ + \theta) = \cos\theta$ $\tan(360^\circ + \theta) = \tan\theta$

Example 4

In this example the reduction formula is used to obtain acute angles.
We always try first to obtain an acute angle that is one of the special angles first if possible.

Without using a calculator, determine the value of:

1. $\cos 150^\circ$ 2. $\sin(-45^\circ)$ 3. $\tan 480^\circ$

[7]

Solutions

1. $\cos 150^\circ$	rewrite as (180 – ?)
$= \cos(180^\circ - 30^\circ)$	quadrant II, cos θ negative
$= -\cos 30^\circ$ ✓	special ratios
$= -\frac{\sqrt{3}}{2}$ ✓ (2)	
2. $\sin(-45^\circ)$	$\sin(-\theta) = -\sin\theta$; quadrant IV, sin θ negative
$= -\sin 45^\circ$ ✓	special ratios
$= -\frac{1}{\sqrt{2}}$ ✓ (2)	
3. $\tan 480^\circ$	write as an angle in the first rotation of 360°
$= \tan(480^\circ - 360^\circ)$	
$= \tan 120^\circ$ ✓	quadrant II, rewrite as (180 – ?)
$= \tan(180^\circ - 60^\circ)$	tan θ negative
$= -\tan 60^\circ$ ✓	special ratios
$= -\sqrt{3}$ ✓ (3)	[7]