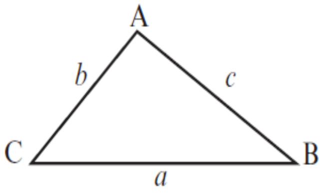
Grade 11

Trigonometry

COS, SIN and AREA Rules Summary

AREA RULE

The area of any ΔABC is half the product of two sides and sine of the included angle



So if you choose to use angle A, then

Area ΔABC = $\frac{1}{2}bc \sin A$

If you choose to use angle B, then

Area ΔABC = $\frac{1}{2}ac \sin B$

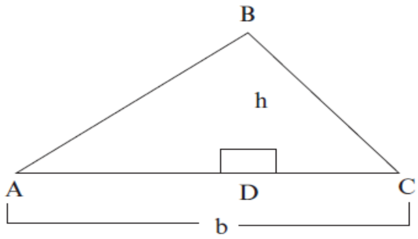
If you choose to use angle C, then

Area ΔABC = $\frac{1}{2}ab \sin C$

Learn one form of the formula – you can work out the others from that.

To find the area of any triangle, you need to know the lengths of two sides and the size of the angle between the two sides.

If $\hat{A}$ is acute



Area of ΔABC = $\frac{1}{2}bh$...............(1)

But sin A = $\frac{h}{c}$ ∴ $h = c \sin A$

Substituting into (1)

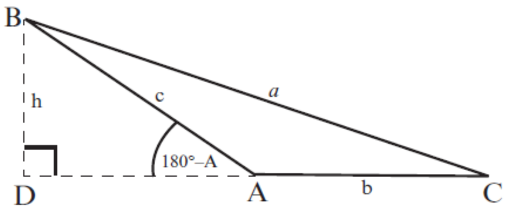
Area of ΔABC = $\frac{1}{2}bc \sin A$

Similarly it can be shown that

Area of ΔABC = $\frac{1}{2}ab \sin C$

= $\frac{1}{2}ac \sin B$

If $\hat{A}$ is obtuse



Area of ΔABC = $\frac{1}{2}bh$...............(1)

But sin (180° – A) = $\frac{h}{c}$ ∴ $h = c \sin A$

Substituting into (1)

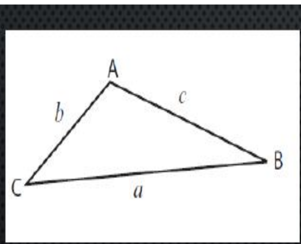
Area of ΔABC = $\frac{1}{2}bc \sin A$

Similarly it can be shown that

Area of ΔABC = $\frac{1}{2}ab \sin C$

= $\frac{1}{2}ac \sin B$

SINE RULE



IN Δ ABC :

- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

OR

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

PROOF

USING THE AREA RULE

$\frac{1}{2}b.c.\sin A = \frac{1}{2}a.b.\sin C = \frac{1}{2}a.c.\sin B$

DIVIDE EACH EXPRESSION BY $\frac{1}{2}a.b.c$

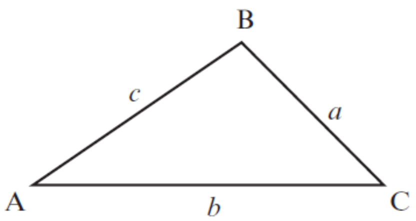
$\frac{\frac{1}{2}b.c.\sin A}{\frac{1}{2}a.b.c} = \frac{\frac{1}{2}a.b.\sin C}{\frac{1}{2}a.b.c} = \frac{\frac{1}{2}a.c.\sin B}{\frac{1}{2}a.b.c}$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

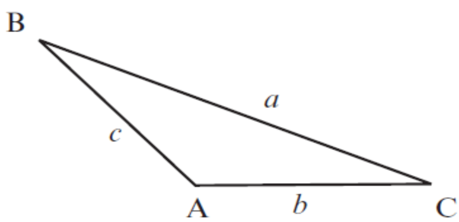
OR BY INERTING THE EXPRESSIONS

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

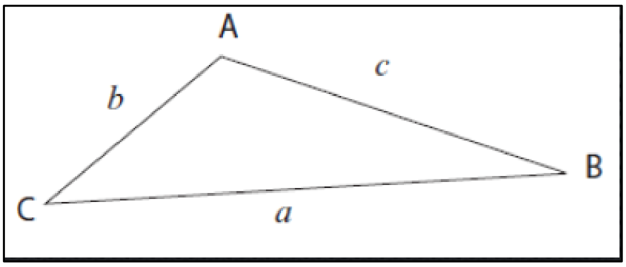
If $\hat{A}$ is acute



If $\hat{A}$ is obtuse



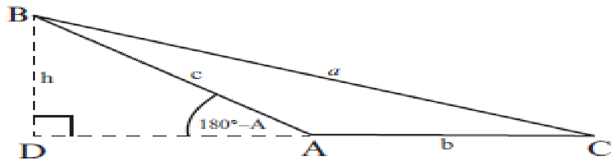
COS RULE



$$\text{IN } \Delta ABC:$$
$$a^2 = b^2 + c^2 - 2.b.c.CosA$$
OR
$$b^2 = a^2 + c^2 - 2.a.c.CosB$$
OR
$$c^2 = a^2 + b^2 - 2.a.b.CosC$$

PROOF

If Ä is obtuse



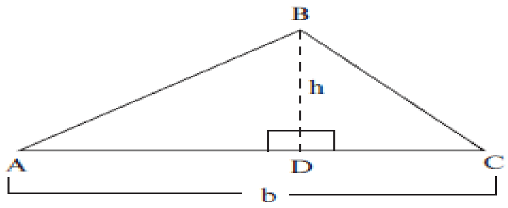
In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)
$= BD^2 + (b + AD)^2$
$= BD^2 + b^2 + 2bAD + AD^2$
But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)
Thus $a^2 = b^2 + c^2 + 2bAD$ ....(1)

InΔABD:.
$\cos(180° - A) = \frac{AD}{c}$ $\therefore$ $AD = -c \cos A$..........(2)

Substituting (2) into (1)
$\therefore a^2 = b^2 + c^2 - 2bc \cos A$

Similarly it can be shown that:
$b^2 = a^2 + c^2 - 2ac \cos B$ and
$c^2 = a^2 + b^2 - 2ab \cos C$

If Ä is acute



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)
$= BD^2 + (b - AD)^2$
$= BD^2 + b^2 - 2bAD + AD^2$
But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 - 2bAD$ ....(1)

In Δ ABD: $\cos A = \frac{AD}{c}$ $\therefore$ $AD = c \cos A$ ....(2)

Substituting (2) into (1)
$\therefore a^2 = b^2 + c^2 - 2bc \cos A$

Similarly it can be shown that:
$b^2 = a^2 + c^2 - 2ac \cos B$ and
$c^2 = a^2 + b^2 - 2ab \cos C$