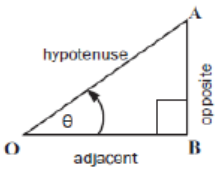
Grade 11 Mathematics

Trig Identities

The trigonometric ratios

Using θ as the **reference angle** in ΔABO

- The side opposite the 90 ° is the hypotenuse side, therefore side AO is the hypotenuse side.
- The side opposite θ is the opposite side, therefore AB is the opposite side.
- The side adjacent to θ is called the adjacent side, therefore OB is the adjacent side.



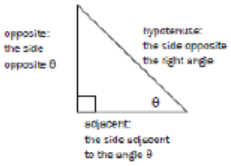
We work with the ratios of the sides of the triangle:

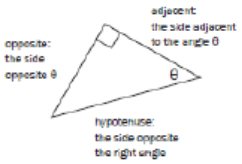
- The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ is called **sine** θ (abbreviated to **sin θ**)
- The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is called **cosine θ** (abbreviated to **cos θ**)
- The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is called **tangent θ** (abbreviated to **tan θ**)

Therefore $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = AB/AO$

$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = OB/AO$

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = AB/OB$



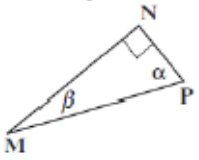


LEARN THESE!!!!!!!

$\sin\theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}$

Example 1

1. ΔMNP is a right-angled triangle. Write down the trig ratios for:
   a) sin α b) sin β (4)
   c) tan β d) cos α (3)
2. If MP = 13 and NP = 5, calculate cos β.



[7]

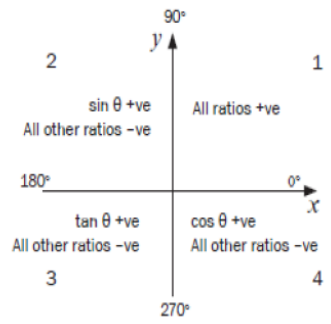
Answer

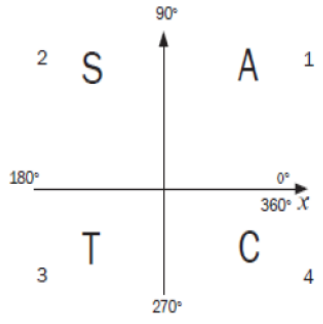
1. a) $\sin\alpha = \frac{MN}{MP}$ ✓ (1) b) $\sin\beta = \frac{NP}{MP}$ ✓ (1)
   c) $\tan\beta = \frac{NP}{MN}$ ✓ (1) d) $\cos\alpha = \frac{NP}{MP}$ ✓ (1) (4)

2. MP = 13 and NP = 5, so we can find MP,
   $MP^2 = MN^2 + NP^2$ .....Pythagoras ✓
   $13^2 = MN^2 + 5^2$
   $169 = MN^2 + 25$
   $MN^2 = 169 - 25$
   $MN^2 = 144$ ✓
   $\therefore MN = 12$
   $\cos\beta = \frac{MN}{MP} = \frac{12}{13}$ ✓ (3)

[7]

Trig Ratios in each quadrant of Cartesian Plane





Identities

QUOTIENT IDENTITY

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

SQUARE IDENTITY

$\sin^2\theta + \cos^2\theta = 1$

From the above we can derive the following:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Example 2

1. If $\sin\theta$ is negative and $\cos\theta$ is positive, then which statement is true?

   A. $0° < \theta < 90°$ B. $90° < \theta < 180°$

   C. $180° < \theta < 270°$ D. $270° < \theta < 360°$ (1)

2. If $\tan\theta < 0$ and $\cos\theta < 0$, then which statement is true?

   A. $0° < \theta < 90°$ B. $90° < \theta < 180°$

   C. $180° < \theta < 270°$ D. $270° < \theta < 360°$ (1)

3. Will the following trig ratios be positive or negative?

   a) $\sin 315°$

   b) $\cos(-215°)$

   c) $\tan 215°$

   d) $\cos 390°$ (4)

   [6]

Answer

1. $\sin\theta$ is negative in 3rd and 4th quadrants; $\cos\theta$ is positive in 1st and 4th quadrants.

   So θ is in the 4th quadrant. D. $270° < \theta < 360°$ ✓ (1)

2. $\tan\theta < 0$ in 2nd and 4th quadrants; $\cos\theta < 0$ in 2nd and 3rd quadrants.

   So θ is in the 2nd quadrant. B. $90° < \theta < 180°$ ✓ (1)

3. a) $\sin 315°$ is in 4th quadrant so it is negative. ✓ (1)

   b) $\cos(-215°)$ is in 2nd quadrant so it is negative. ✓ (1)

   c) $\tan 215°$ is in 3rd quadrant, so it is positive. ✓ (1)

   d) $\cos 390°$ is the same as $\cos 30°$ in the 1st quadrant, so it is positive. ✓ (1)

   [6]