Grade 10 Mathematics
Analytical Geometry
SUMMARY

MIDPOINT BETWEEN TWO COORDINATES $(x_1; y_1)$ and $(x_2; y_2)$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

DISTANCE BETWEEN TWO COORDINATES $(x_1; y_1)$ and $(x_2; y_2)$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

GRADIENT BETWEEN TWO COORDINATES $(x_1; y_1)$ and $(x_2; y_2)$

$$Gradient = \frac{difference\ in\ y}{difference\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

Determine the midpoint M between the points A (-2; 1) and B (1; -3,5)

Example 2

Determine the length of the line segment (Distance between two points) between the following points:

P (-3; 5) and Q (-1; -5)

Example 3

Determine the gradient of the line segment between the following points:

A (-5; -9) and B (3; 2)

Answer

Example 1

Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$.

$x_1 = -2 \quad y_1 = 1 \quad x_2 = 1 \quad y_2 = -3,5$

Substitute values into the mid-point formula:

$$M(x; y) = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x = \frac{x_1 + x_2}{2} = \frac{-2 + 1}{2} = -0,5$$

$$y = \frac{y_1 + y_2}{2} = \frac{1 + (-3,5)}{2} = -1,25$$

The mid-point is at $M(-0,5; -1,25)$.

Example 2

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 + 3)^2 + (-5 - 5)^2} \\ &= \sqrt{(2)^2 + (-10)^2} \\ &= \sqrt{4 + 100} \\ &= \sqrt{104} \end{aligned}$$

Example 3

Solution:
Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$

$x_1 = -5 \quad y_1 = -9 \quad x_2 = 3 \quad y_2 = 2$

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-9)}{3 - (-5)} \\ &= \frac{11}{8} \end{aligned}$$

REMEMBER

- Parallel lines have equal Gradients
- Perpendicular Lines $m_1 \times m_2 = -1$

Example 4:

a. Determine the gradient of the line **parallel** to y=3x+4
b. Determine the gradient of the line **perpendicular** to y=3x+4

Answer to Example 4

a. Parallel lines have equal gradients so m = 3 of other line.
b. $m_1 \times m_2 = -1$

$$3 \times m_2 = -1$$
$$3m_2 = -1$$
$$m_2 = \frac{-1}{3} = -\frac{1}{3}$$