COS RULE AND PROOF

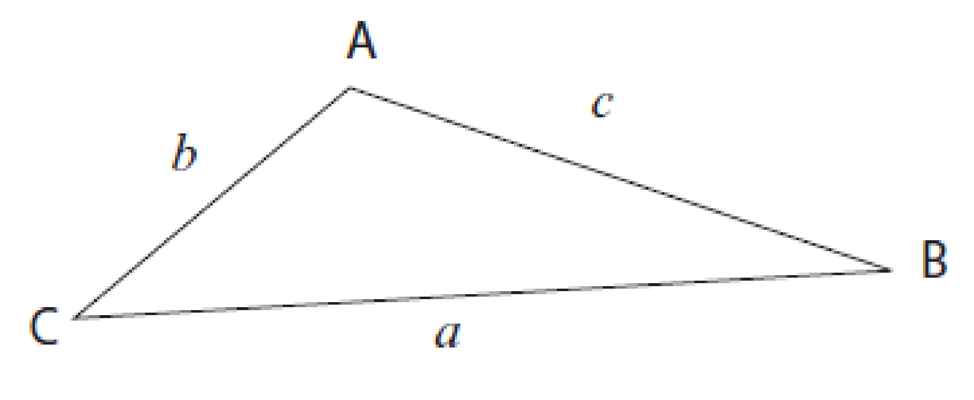
YOU NEED TO LEARN THE PROOF

COS RULE

WE USE THIS RULE TO FIND THE LENGTHS OF SIDES, SIZES OF ANGLES OF ANY KIND OF TRIANGLE. TO 'SOLVE A TRIANGLE' MEANS YOU MUST CALCULATE THE UNKNOWN SIDES AND ANGLES.

YOU APPLY THE COS RULE IF YOU ARE GIVEN THE VALUES OF:

- **TWO SIDES AND THE INCLUDED ANGLE** OR
- **THREE SIDES OF A TRIANGLE**



$IN\ \Delta ABC$:

$$a^2 = b^2 + c^2 - 2.b.c.CosA$$

OR

$$b^2 = a^2 + c^2 - 2.a.c.CosB$$

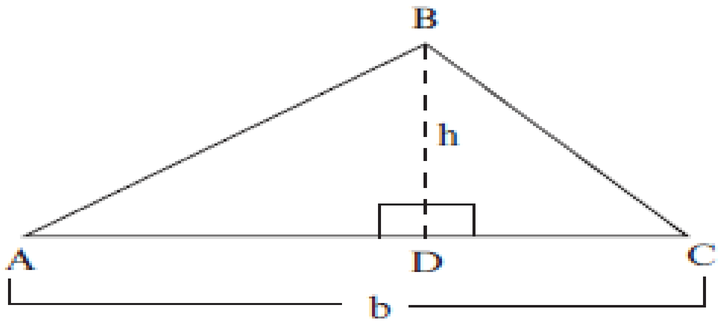
OR

$$c^2 = a^2 + b^2 - 2.a.b.CosC$$

COS RULE
PROOF

The proof is needed to be learnt for exam purposes.

If $\hat{A}$ is acute



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)

$= BD^2 + (b - AD)^2$

$= BD^2 + b^2 - 2bAD + AD^2$

But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 - 2bAD$ ....(1)

In Δ ABD: $\cos A = \frac{AD}{c}$ $\therefore$ $AD = c \cos A$ ....(2)

Substituting (2) into (1)

$\therefore a^2 = b^2 + c^2 - 2bc \cos A$

Similarly it can be shown that:

$b^2 = a^2 + c^2 - 2ac \cos B$ and

$c^2 = a^2 + b^2 - 2ab \cos C$

SOME EXPLAINATIONS

1. REMEMBER

BC = a

AC = b

AB = c

2. AC =AD + CD

$\therefore$ CD =AC – AD

$\therefore$ because AC = b (in DIAGRAM), **CD = b – AD**

3. USING PYTHAGORUS IN Δ BDC, $BC^2 = BD^2 + CD^2$

$BC = a$

$a^2 = BD^2 + CD^2$

$a^2 = BD^2 + (b - AD)^2$ ***FROM point 2 above***

$(b - AD)^2 = b^2 - 2b.AD + AD^2$ ***Using DISTRIBUTIVE LAW***

$\therefore a^2 = BD^2 + (b^2 - 2b.AD + AD^2)$

$a^2 = BD^2 + b^2 - 2b.AD + AD^2$

4. $AB^2 = AD^2 + BD^2$ in Δ ABD

AB = c

$\therefore c^2 = \mathbf{BD^2 + AD^2}$

$a^2 = \mathbf{BD^2} + b^2 - 2b.AD + \mathbf{AD^2}$

$a^2 = b^2 + \mathbf{c^2} - 2b.AD$

5. In Δ $ABD: \cos A = \frac{AD}{AB} = \frac{AD}{c}$

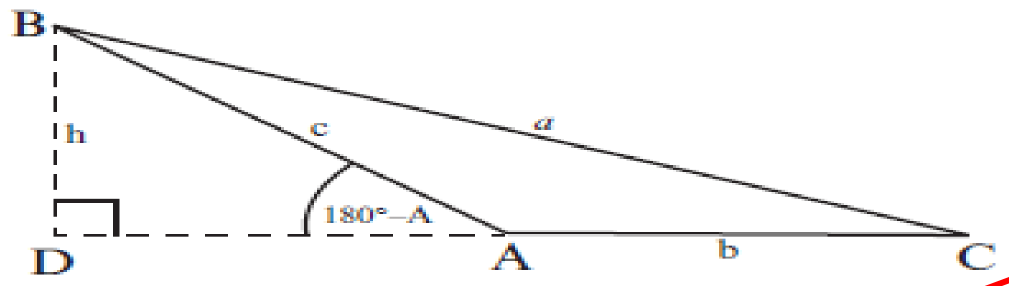
$\therefore AD = c.\cos A$

$a^2 = b^2 + \mathbf{c^2} - 2b.c.\cos A$

COS RULE
PROOF

THE PROOF IS NEEDED TO BE LEARNT FOR EXAM PURPOSES.

If A is obtuse



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)

$= BD^2 + (b + AD)^2$

$= BD^2 + b^2 + 2bAD + AD^2$

But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 + 2bAD$ ....(1)

InΔABD:.

$\cos(180° - A) = \frac{AD}{c}$ $\therefore$ $AD = -c\cos A$..........(2)

Substituting (2) into (1)

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$

Similarly it can be shown that:

$b^2 = a^2 + c^2 - 2ac\cos B$ and

$c^2 = a^2 + b^2 - 2ab\cos C$

SOME EXPLAINATIONS

1. REMEMBER

BC = a

AC = b

AB = c

2. CD = AC+AD

∴ CD =b + AD

∴ because AC = b (in DIAGRAM), **CD = b + AD**

3. USING PYTHAGORUS IN Δ BDC, $BC^2 = BD^2 + CD^2$

$BC = a$

$a^2 = BD^2 + CD^2$

$a^2 = BD^2 + (b + AD)^2$ ***FROM point 2 above***

$(b + AD)^2 = b^2 + 2b.AD + AD^2$ ***Using DISTRIBUTIVE LAW***

$\therefore a^2 = BD^2 + (b^2 + 2b.AD + AD^2)$

$a^2 = BD^2 + b^2 + 2b.AD + AD^2$

4. $AB^2 = AD^2 + BD^2$ in $\Delta\, ABD$

AB = c

$\therefore c^2 = \mathbf{BD^2 + AD^2}$

$a^2 = \mathbf{BD^2} + b^2 - 2b.AD + \mathbf{AD^2}$

$a^2 = b^2 + \mathbf{c^2} + 2b.AD$

5. In $\Delta\, ABD$: $\cos(180° - A) = \frac{AD}{AB} = \frac{AD}{c}$

$\therefore AD = -c.\cos A$

$a^2 = b^2 + \mathbf{c^2} - 2b.c.\cos A$