GRADE 12
Calculus –Cubic functions.
WEBSITE NOTES

TOPIC:

- Cubic graphs
- Practical problems concerning optimisation, rate of change and motion.

Revise Cubic function interpretation

Example 1

If $f(x) = ax^3+bx^2+cx+d$ passes through the points (-1; 0), (2; 0) and (3; 0) and (0; 6).
Determine the values of a, b, c and d.

$x_1; x_2; x_3$ are the x-intercepts

$f(x) = a\,(x - x_1)\,(x - x_2)\,(x - x_3)$

is the x-intercept method that you use for this question type.

Answer

$f(x) = a\,(x - x_1)\,(x - x_2)\,(x - x_3)$

- **Substitute the x-intercepts into the equation**

  $f(x) = a\,(x - (-1))\,(x - 2)\,(x - 3)$

- **Substitute (0; 6) – the y intercept - into the equation**

  $6 = a\,(0 - (-1))\,(0 - 2)\,(0 - 3)$

  $6 = a\,(1)\,(-2)\,(-3)$

  $6 = a.6$

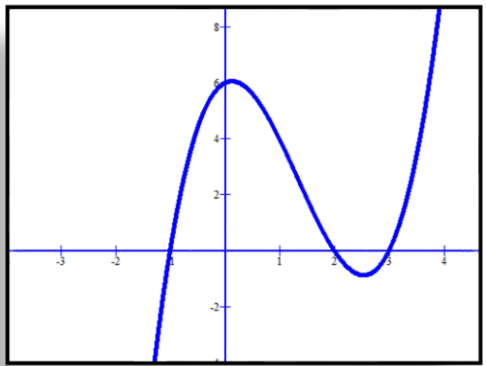
  $a = 1$

- **Substitute back into the equation**

  $f(x) = 1\,(x + 1)\,(x - 2)\,(x - 3)$

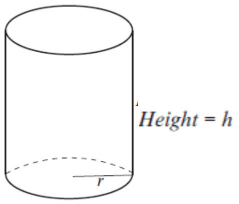
- **Work out the equation**
  $f(x) = (x^2-x-2)\,(x-3)$
  $f(x) = x^3-x^2-2x-3x^2+3x+6$
  $f(x) = x^3-4x^2+x+6$

- **Therefore a = 1, b = -4, c = 1 and d = 6**



Practical problems concerning optimisation, rate of change and motion
Example 1

A drinking glass, in the shape of a cylinder (shown here), must hold 200 ml of liquid when full. Find the value of *r* for which the total surface area of the glass is a minimum.



REMEMBER THAT A QUADRATIC FUNCTION WILL EITHER HAVE A MINIMUM OR MAXIMUM

Answer

Surface area of glass = area of base + area of curved surface

$S = \pi r^2 + 2\pi r h$

We know the glass holds 200 ml = 200 cm^3.
The volume of the glass is $\pi r^2 h$
So $\pi r^2 h = 200$ so $h = \frac{200}{\pi r^2}$

$s = \pi r^2 + 2.\pi.r.\frac{200}{\pi r^2}$
$s = \pi r^2 + 2.\frac{200}{r}$
$s = \pi r^2 + 400r^{-1}$
$s' = 2\pi r - 400r^{-2}$
For a minimum s' =0
$0 = 2\pi r - 400r^{-2}$
$2\pi r = 400r^{-2}$
$2\pi r = \frac{400}{r^2}$
$2\pi r^3 = 400$
$r^3 = \frac{200}{\pi}$
$r = \sqrt[3]{\frac{200}{\pi}} = 3.99cm^3$

Example 2
A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after *t* minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

1. How high is the car above sea level when it starts its journey on the mountainous pass?

2. Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass.

3. Interpret your answer to QUESTION 2.

4. How many minutes after the journey has started will the rate of change of height with respect to time be a minimum?

Answer

1. $s(t) = 5t^3 - 65t^2 + 200t + 100$
   $t = 0$ Therefore it is $5(0)^3 - 65(0)^2 + 200(0) + 100 = 100$ metres

2. RATE OF CHANGE IS THE DERIVATIVE
   $s'(0) = 15t^2 - 130t + 200$
   $s'(4) = 15(4)^2 - 130(4) + 200$
   $= -80$ metres per minute

3. The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change.

4. $s'(t) = 15t^2 - 130t + 200$
   $s''(t) = 30t - 130$
   $30t = 130$
   $\therefore t = \frac{130}{3}$
   $t = 4.\dot{3}$

It is asking when the rate of change will be a minimum.
So, you have worked out the rate of change in QUESTION 2
$s'(t) = 15t^2 - 130t + 200$
Now to work out the Derivative of the Derivative.