



AREA RULE AND

PROOF

YOU NEED TO LEARN THE PROOF

AREA RULE

WE USE THIS RULE TO FIND THE AREA OF ANY KIND OF TRIANGLE.
TO 'SOLVE A TRIANGLE' MEANS YOU MUST CALCULATE THE UNKNOWN SIDES AND ANGLES.

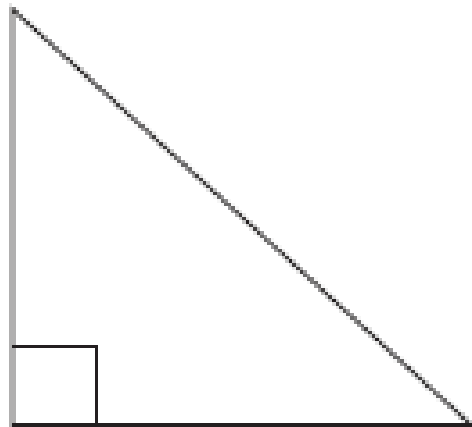
AREA RULE

LET US REVISE THE AREA OF A RIGHT ANGLED TRIANGLE FIRST

- **AREA OF A RIGHT-ANGLED TRIANGLE:**

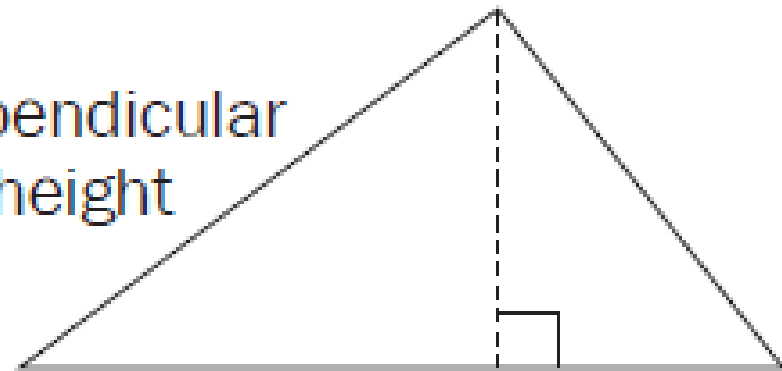
$$AREA \Delta = \frac{1}{2} BASE \times PERPENDICULAR HEIGHT$$

perpendicular
height



base

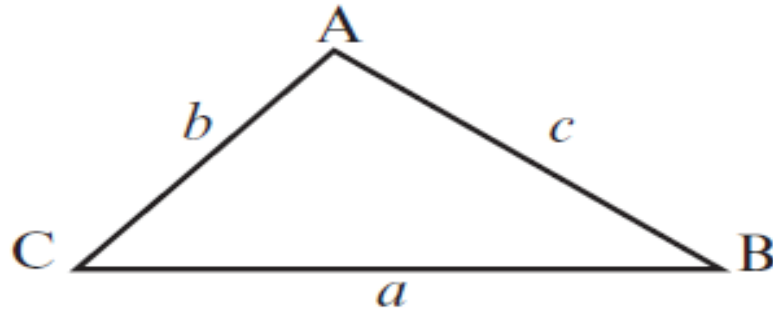
perpendicular
height



base

AREA RULE

The area of any $\triangle ABC$ is half the product of two sides and sine of the included angle



So if you choose to use angle A, then

$$\text{Area } \triangle ABC = \frac{1}{2} bc \sin A$$

If you choose to use angle B, then

$$\text{Area } \triangle ABC = \frac{1}{2} ac \sin B$$

If you choose to use angle C, then

$$\text{Area } \triangle ABC = \frac{1}{2} ab \sin C$$

Learn one form of the formula – you can work out the others from that.

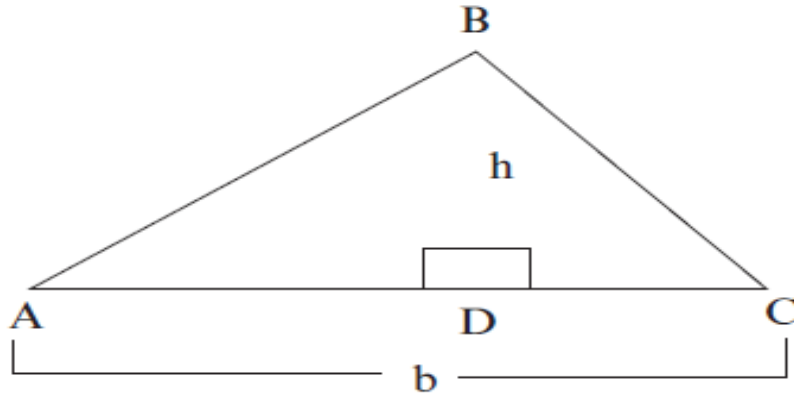
To find the area of any triangle, you need to know the lengths of two sides and the size of the angle between the two sides.

AREA RULE

PROOF

THE PROOF IS NEEDED TO BE LEARNT FOR EXAM PURPOSES.

If \hat{A} is acute



$$\text{Area of } \triangle ABC = \frac{1}{2} bh \dots \dots \dots (1)$$

$$\text{But } \sin A = \frac{h}{c} \therefore h = c \sin A$$

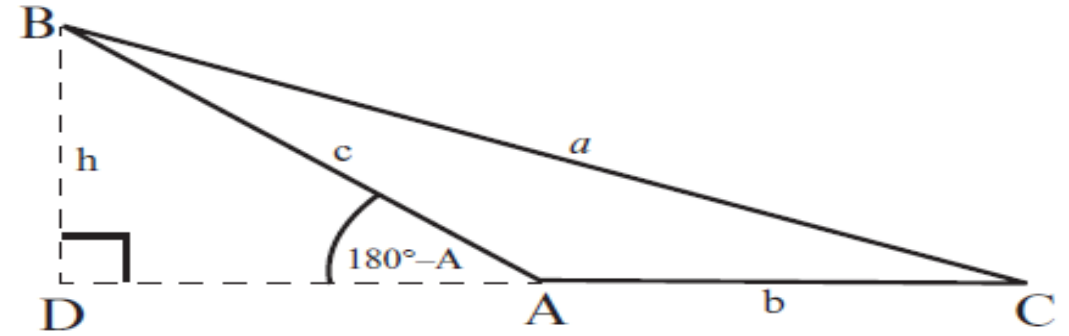
Substituting into (1)

$$\text{Area of } \triangle ABC = \frac{1}{2} bc \sin A$$

Similarly it can be shown that

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned}$$

If \hat{A} is obtuse



$$\text{Area of } \triangle ABC = \frac{1}{2} bh \dots \dots \dots (1)$$

$$\text{But } \sin (180^\circ - A) = \frac{h}{c} \therefore h = c \sin A$$

Substituting into (1)

$$\text{Area of } \triangle ABC = \frac{1}{2} bc \sin A$$

Similarly it can be shown that

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned}$$