16 July 2020

Grade 12 Sketching Cubic Functions ANSWERS

The Standard Form of a cubic function is $f(x) = ax^3 + bx^2 + cx + d$

To Draw a Cubic function the following is needed:

1. SHAPE
2. Y-Intercept
3. X-Intercepts
4. Stationery points (Turning Points)
5. Point of Infliction (Where the concavity changes)

Example 2

Try yourself

1. $f(x) = -x^3 - x^2 + x + 10$

a) Write down the coordinates of the y-intercept of f

b) Show that (2; 0) is the only x-intercept.

c) Calculate the coordinates of the turning points of f

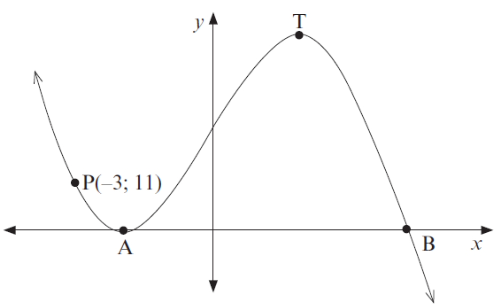
d) Sketch the graph of f. Show all intercepts with axes and all turning points.

e) Determine the point of inflection.

Example 3

Try yourself

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$. A and T are turning points of g. A and B are the x-intercepts of g. P(–3; 11) is a point on the graph.



a) Determine the length of AB.

b) Determine the x-coordinate of T.

c) Determine the equation of the tangent to g at P(–3; 11) in the form $y = \ldots$

d) Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots.

e) Determine the x-coordinate of the point of inflection. (14)

Example 2 ANSWERS

a) When $x = 0$, $y = 10$, therefore are $(0; 10)$ ✓ (1)

b) Assuming that $(2; 0)$ is the x-intercept, then $x - 2$ is a factor of $f(x)$

$f(x) = -x^3 - x^2 + x + 10 = (x - 2)(-x^2 - 3x - 5)$ ✓✓

$\therefore x - 2 = 0$ or $-x^2 - 3x - \ = 0$ ✓

$x = 2$ but $-x^2 - 3x - 5 = 0$ has no real solution. Hence $(x - 2)$ is the only x-intercept ✓✓ (5)

c) At the turning point $f'(x) = -3x^2 - 2x + 1 = 0$ ✓

$(-3x + 1)(x + 1) = 0$

$x = \frac{1}{3}$ or $x = -1$ ✓✓

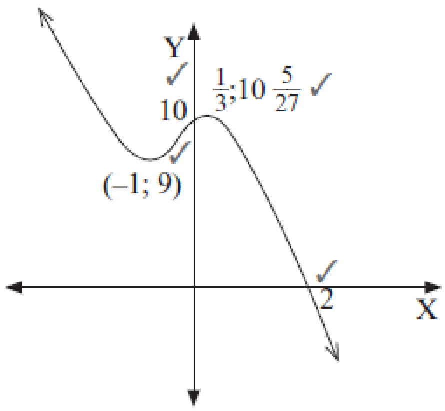
When $x = \frac{1}{3}$, $y = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 10 = \frac{270 - 3 + 9 - 1}{27} = \frac{275}{27} = 10\frac{5}{27}$

Therefore turning point is $\left(\frac{1}{3}; \frac{275}{27}\right) = \left(\frac{1}{3}; 10\frac{5}{27}\right)$ ✓

When $x = -1$, $y = 1 - 1 - 1 + 10 = 9$

Therefore turning point is $-(1; 9)$ ✓ (5)

d)



(4)

e) At the point of inflection $f''(x) = -6x - 2 = 0$ ✓

$\therefore$ at $x = -\frac{2}{6} = -\frac{1}{3}$ ✓ (2)

Example 3 Answers

a) Since A and B are the x-intercepts of g they are solutions of $-(2x - 5)(x + 2)^2 = 0$ ✓

i.e. $x = -2$ and $x = \frac{5}{2}$ The distance between -2 and $\frac{5}{2}$ is $\frac{5}{2} - (-2) = 4{,}5$ units ✓ (2)

b) T is a turning point. $g'(x) = -6x^2 - 6x + 12 = 0$. ✓

$-6(x^2 + x - 2) = 0$

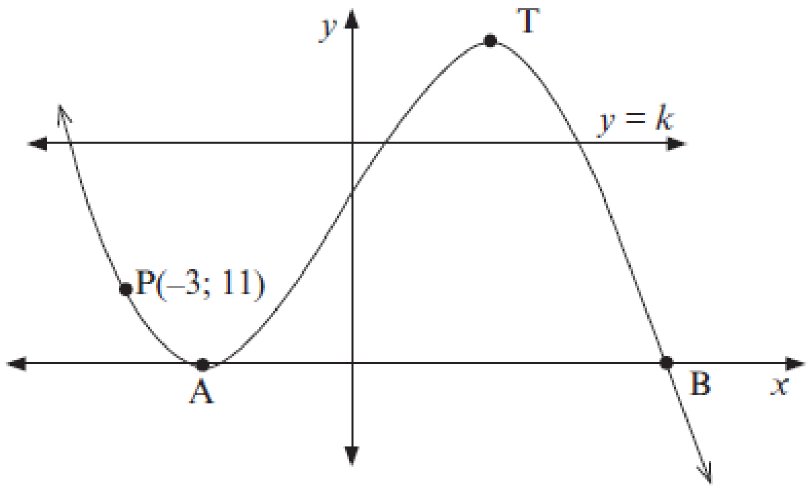
$-6(x + 2)(x - 1) = 0$

When $x = -2$ or $x = 1$. ✓✓

So the x-coordinate of T is 1. (3)

c) $g'(3) = -6(-3)^2 - 6(-3) + 12 = -24$ ✓
So the equation of the tangent line is $y - 11 = -24(x + 3)$ ✓
which simplifies to $y = -24x - 61$ ✓ (3)

d) The graph of $y = k$ is shown together with $g(x)$ below.
Using these graphs we can observe that, provided the line lies above the y-value of A and below that of T, the equation $-2x^3 - 3x^2 + 12x + 20 = k$ will have 3 distinct roots.
At T, $g(1) = -2 - 3 + 12 + 20 = 27$. So for $0 < k < 27$ the equation has 3 distinct roots. ✓✓✓✓ (4)



e) $g''(x) = -12x - 6$
$-12x - 6 = 0$ when $x = \frac{6}{-12} = -\frac{1}{2}$ ✓✓ (2)

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