16 July 2020

Grade 12 Sketching Cubic Functions

The Standard Form of a cubic function is $f(x) = ax^3 + bx^2 + cx + d$

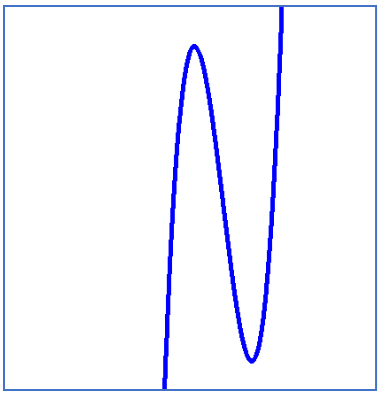
To Draw a Cubic function the following is needed:

1. SHAPE
2. Y-Intercept
3. X-Intercepts
4. Stationery points (Turning Points)
5. Point of Infliction (Where the concavity changes)

Let us look at an example

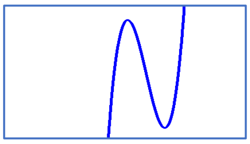
$$f(x) = x^3 - 4x^2 - 11x + 30$$

1. **SHAPE**
   The value of a - (Coefficient of x^3) is 1. Therefore a>0 and the shape will be

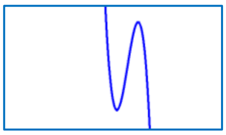


NOTE THE FOLLOWING:

If **a >0** the shape will be



If **a<0** the shape will be



2. **Y-Intercept (when x=0)**
   Substitute x=0 into f(x)

$$f(0) = (0)^3 - 4(0)^2 - 11.(0) + 30$$
$$f(0) = 30$$

Therefore, the y-intercept is **(0; 30)**

3. **X-Intercepts (When y=0)**
   Using Factor Theorem and Synthetic or Long Division or any other method to factorise the cubic function to get the x-intercepts. (Cut x-axis)

$$f(x) = x^3 - 4x^2 - 11x + 30$$
$$f(x) = (x-2)(x-5)(x+3)$$
$$0 = (x-2)(x-5)(x+3)$$
$$(x-2) = 0 \; or \; (x-5) = 0 \; or \; (x+3) = 0$$
$$\therefore x = 2 \; or \; x = 5 \; or \; x = -3$$

Therefore, the x-intercepts are **(2;0), (5;0), (-3;0)**

4. **Stationary Points (Turning Points) – When f'(x) = 0**

$$f(x) = x^3 - 4x^2 - 11x + 30$$

WORK OUT THE DERIVATIVE

$$f'(x) = 3x^2 - 8x - 11$$

EQUATE DERIVATIVE TO 0 AND FACTORISE TO OBTAIN THE X-COORDINATE OF TURNING POINT

$$0 = 3x^2 - 8x - 11$$

$$0 = (3x - 11)(x + 1)$$

$$(3x - 11) = 0 \ or \ (x + 1) = 0$$

$$x = \frac{11}{3} \ or \ x = -1$$

SUBSTITUTE THE X VALUES INTO THE ORIGINAL CUBIC FUNCTION TO OBTAIN THE Y-COORDINATES

$$f(x) = x^3 - 4x^2 - 11x + 30$$

TURNING POINT 1

$$f\left(\frac{11}{3}\right) = \left(\frac{11}{3}\right)^3 - 4\left(\frac{11}{3}\right)^2 - 11\left(\frac{11}{3}\right) + 30$$

$$f\left(\frac{11}{3}\right) = -14.81$$

THEREFORE, TURNING POINT 1 WILL BE $\left(\frac{\mathbf{11}}{\mathbf{3}};\ \mathbf{-14.81}\right)$

TURNING POINT 2

$$f(-1) = (-1)^3 - 4(-1)^2 - 11(-1) + 30$$

$$f(-1) = 36$$

THEREFORE, TURNING POINT 2 WILL BE $(\mathbf{-1};\ \mathbf{36})$

5. **Point of infliction (Work out second Derivative – DERIVATIVE OF THE DERIVATIVE)**

$$f(x) = x^3 - 4x^2 - 11x + 30$$

$$f'(x) = 3x^2 - 8x - 11$$

$$f''(x) = 6x - 8$$

MAKE THE SECOND DERIVATIVE = 0 TO OBTAIN THE X COORDINATE OF POINT OF INFLICTION

$$0 = 6x - 8$$

$$8 = 6x$$

$$x = \frac{8}{6} = \frac{4}{3}$$

SUBSTITUTE X = $\frac{4}{3}$ INTO THE ORIGINAL CUBIC FUNCTION TO OBTAIN THE Y-INTERCEPT OF POINT OF INFLICTION

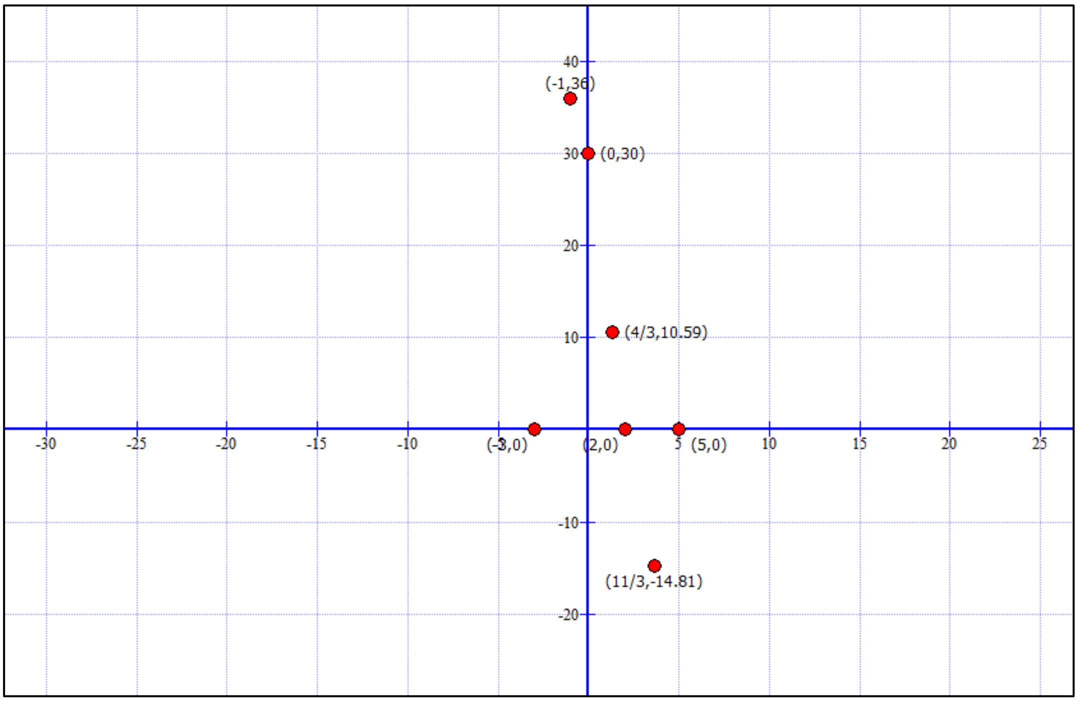
$$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 4.\left(\frac{4}{3}\right)^2 - 11.\left(\frac{4}{3}\right) + 30$$

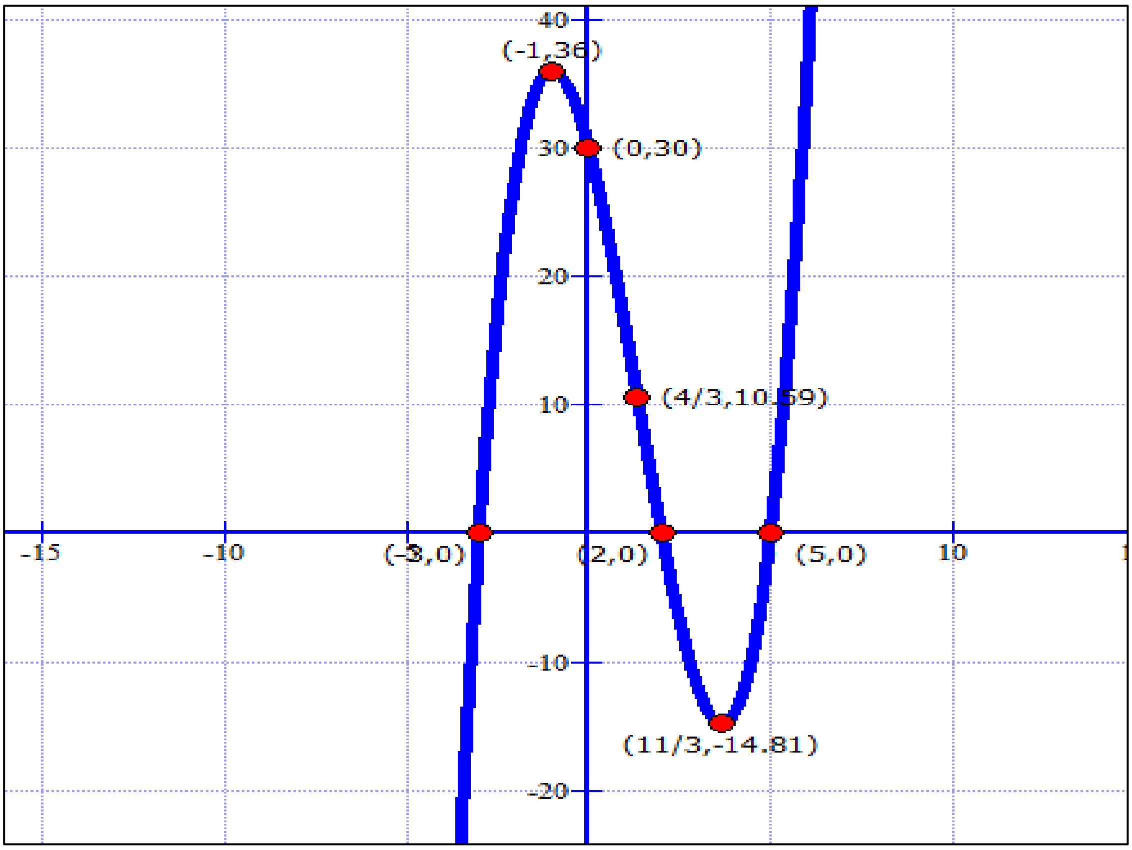
$$f\left(\frac{4}{3}\right) = 10.59$$

THEREFORE, THE POINT OF INFLICTION IS $(\frac{\mathbf{4}}{\mathbf{3}}; \mathbf{10.59})$

NOW ONCE ALL THE ABOVE IS DONE YOU CAN DRAW THE CUBIC GRAPH. EACH POINT WILL BE PLOTTED AND LABELLED ON THE GRAPH.

PLOT POINTS AND THEN JOIN THE POINTS BEARING IN MIND THE SHAPE.





Example 2

Try yourself

1. $f(x) = -x^3 - x^2 + x + 10$

a) Write down the coordinates of the y-intercept of f

b) Show that (2; 0) is the only x-intercept.

c) Calculate the coordinates of the turning points of f

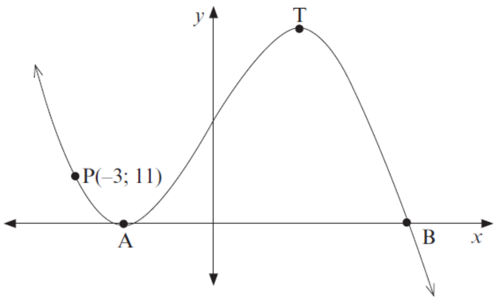
d) Sketch the graph of f. Show all intercepts with axes and all turning points.

e) Determine the point of inflection.

Example 3

Try yourself

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$. A and T are turning points of g. A and B are the x-intercepts of g. P(–3; 11) is a point on the graph.



a) Determine the length of AB.

b) Determine the x-coordinate of T.

c) Determine the equation of the tangent to g at P(–3; 11) in the form $y = \ldots$

d) Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots.

e) Determine the x-coordinate of the point of inflection. (14)

HINT FOR C) ABOVE TO WORK OUT THE EQUATION OF A LINE – THE TANGENT IS A LINE:

If you know	Formulae to use
The gradient and the y-intercept	$y = mx + c$
The gradient and the coordinates of at least one point on the graph.	$y - y_1 = m(x - x_1)$ or $y = mx + c$
Two points on the line: first calculate the gradient and then substitute into $y = mx + c$.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ and $y = mx + c$

WORK OUT THE GRADIENT AT THE POINT P (-3;11)

USE P(-3;11) TO DETERMINE THE EQUATION