Analytical Geometry

GRADE 10 Formulae to remember

Using the points $(x_1 ; y_1)$ and $(x_2 ; y_2)$

Mid-Point Between Two Points

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Length Between Two Points

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Definition Of Gradient

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Perpendicular Lines

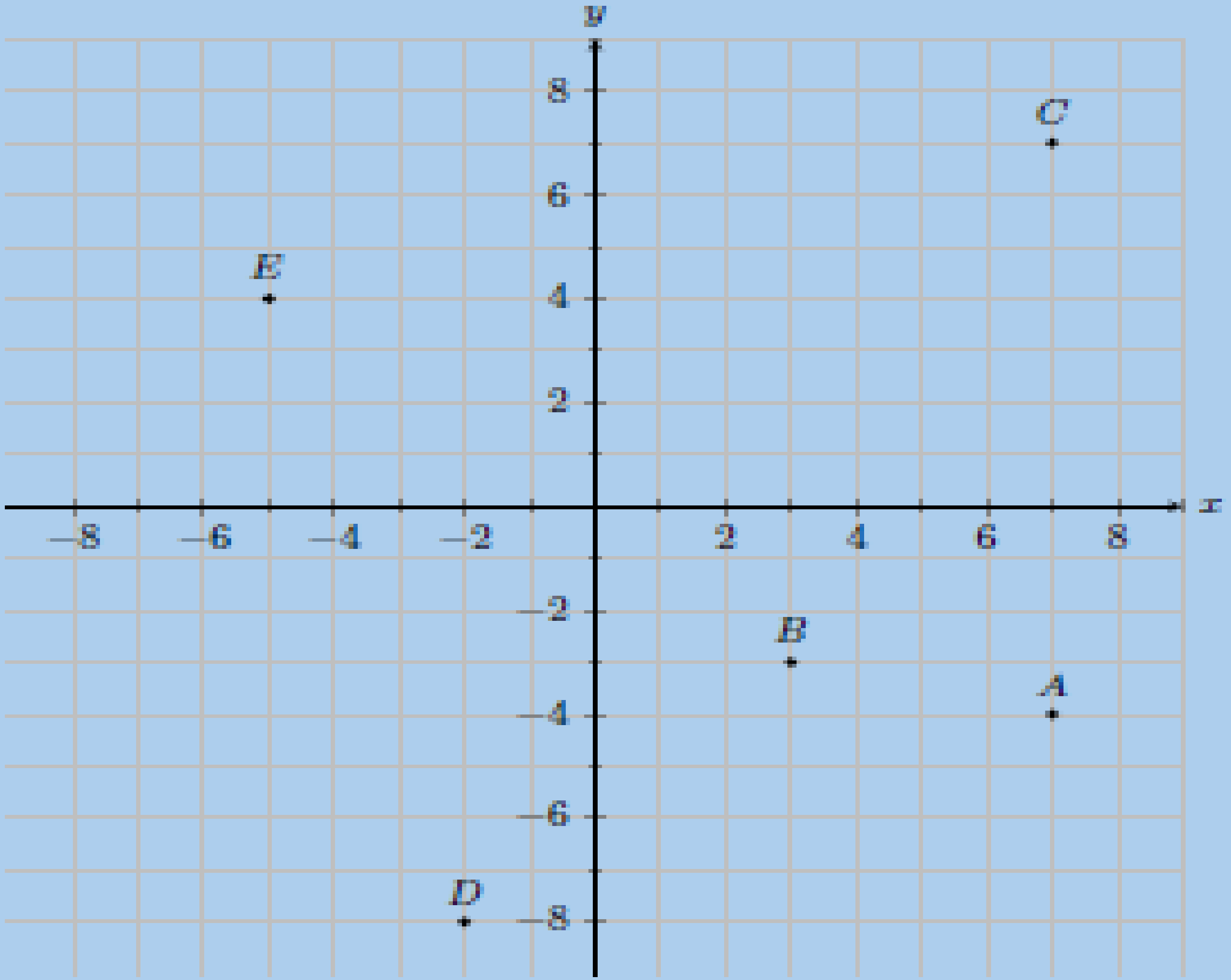
$$m_1 \times m_2 = -1$$

PARALLEL LINES HAVE EQUAL GRADIENTS

DETERMINE THE COORDINATES ON A CARTESIAN PLANE

EXAMPLE 1

1. You are given the following diagram, with various points shown:

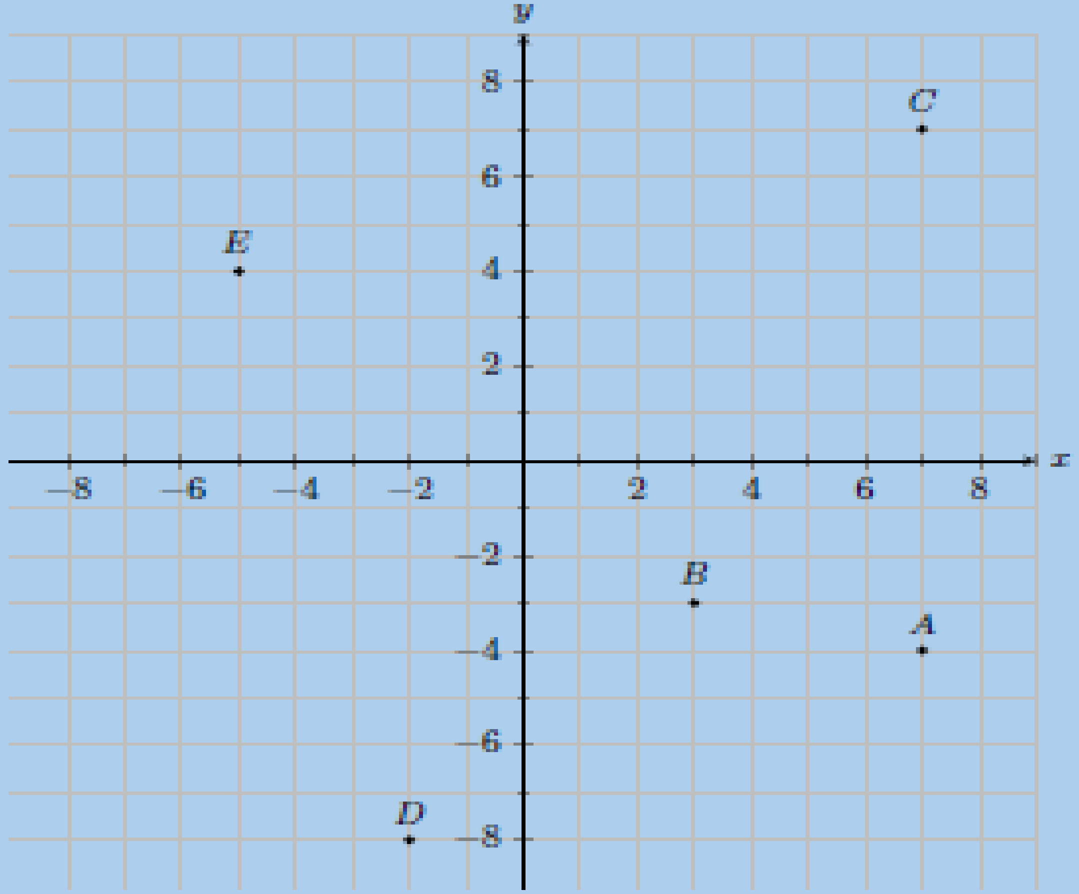


Find the coordinates of points A, B, C, D and E.

DETERMINE THE COORDINATES ON A CARTESIAN PLANE

EXAMPLE 1 ANSWERS

1. You are given the following diagram, with various points shown:



Find the coordinates of points A, B, C, D and E.

Solution:

From the graph we can read off the x and y values for each point.

$A(7;-4)$, $B(3;-3)$, $C(7;7)$, $D(-2;-8)$ and $E(-5;4)$

X-COORDINATE IS ALWAYS WRITTEN FIRST AND Y-CORDINATE IS WRITTEN SECOND WHEN WRITING A POINT IN COORDINATE FORM

DETERMINE THE LENGTH BETWEEN TWO POINTS

EXAMPLE 1

Determine the length of the line segment between the following points:

a) $P(-3;5)$ and $Q(-1;-5)$

b) $R(0{,}75;3)$ and $S(0{,}75;-4)$

c) $T(2x;y-2)$ and $U(3x+1;y-2)$

DETERMINE THE LENGTH BETWEEN TWO POINTS EXAMPLE 1 ANSWERS

a)

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1+3)^2 + (-5-5)^2} \\ &= \sqrt{(2)^2 + (-10)^2} \\ &= \sqrt{4+100} \\ &= \sqrt{104} \end{aligned}$$

b)

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0{,}75 - 0{,}75)^2 + (-4-3)^2} \\ &= \sqrt{(0)^2 + (-7)^2} \\ &= \sqrt{49} \\ &= 7 \text{ units} \end{aligned}$$

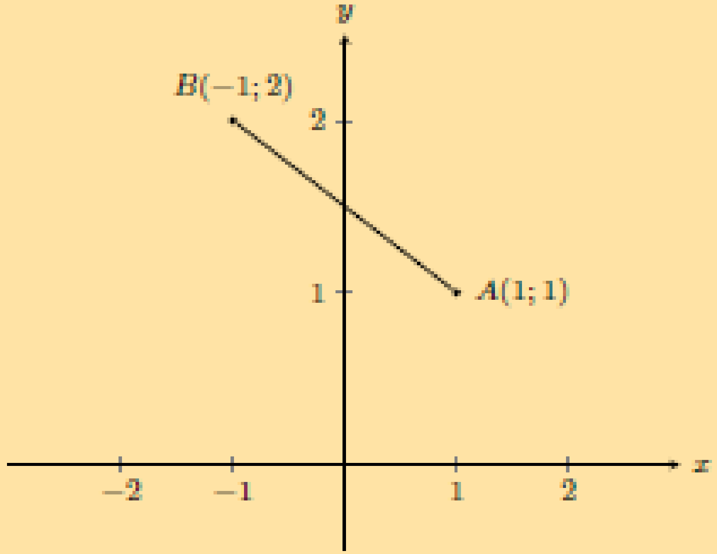
c)

$$\begin{aligned} TU &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3x + 1 - 2x)^2 + (y - 2 - y + 2)^2} \\ &= \sqrt{(x+1)^2 + (0)^2} \\ &= \sqrt{(x+1)^2} \\ &= x + 1 \text{ units} \end{aligned}$$

DETERMINE THE LENGTH BETWEEN TWO POINTS

EXAMPLE 2

You are given the following diagram:



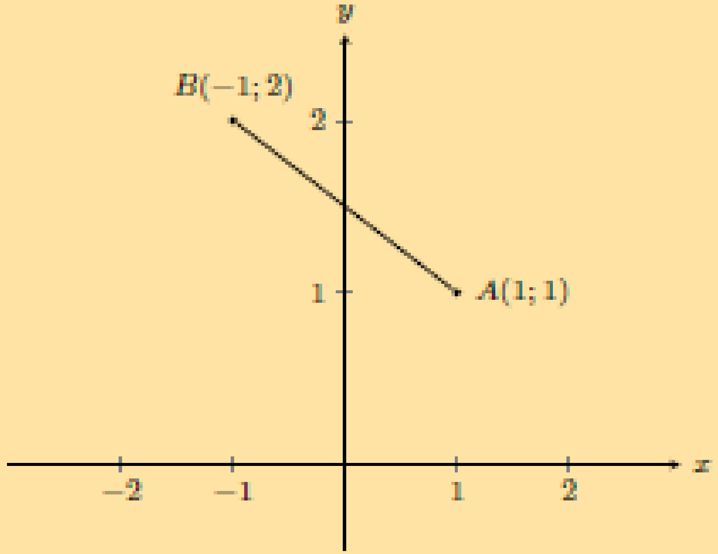
Calculate the length of line AB, correct to 2 decimal places.

$$
\begin{aligned}
d_{AB} &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\
&= \sqrt{(1 - (-1))^2 + (1 - (2))^2} \\
&= \sqrt{(1+1)^2 + (1 - 2)^2} \\
&= \sqrt{(2)^2 + (-1)^2} \\
&= \sqrt{4 + 1} \\
&= \sqrt{5} \\
&\approx 2{,}24
\end{aligned}
$$

DETERMINE THE LENGTH BETWEEN TWO POINTS

EXAMPLE 2

You are given the following diagram:



Calculate the length of line AB, correct to 2 decimal places.

$$\begin{aligned}
d_{AB} &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\
&= \sqrt{(1 - (-1))^2 + (1 - (2))^2} \\
&= \sqrt{(1+1)^2 + (1 - 2)^2} \\
&= \sqrt{(2)^2 + (-1)^2} \\
&= \sqrt{4 + 1} \\
&= \sqrt{5} \\
&\approx 2{,}24
\end{aligned}$$

DETERMINE THE LENGTH BETWEEN TWO POINTS
EXAMPLE 3

Find the length of AB for each of the following. Leave your answer in surd form.

a) $A(2;7)$ and $B(-3;5)$

Solution:

$$\begin{aligned} d_{AB} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(2 - (-3))^2 + (7 - 5)^2} \\ &= \sqrt{(5)^2 + (2)^2} \\ &= \sqrt{29} \end{aligned}$$

b) $A(-3;5)$ and $B(-9;1)$

Solution:

$$\begin{aligned} d_{AB} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(-3 - (-9))^2 + (5 - 1)^2} \\ &= \sqrt{(6)^2 + (4)^2} \\ &= \sqrt{52} \end{aligned}$$

c) $A(x;y)$ and $B(x+4;y-1)$

Solution:

$$\begin{aligned} d_{AB} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(x - (x + 4))^2 + (y - (y - 1))^2} \\ &= \sqrt{(x - x - 4)^2 + (y - y + 1)^2} \\ &= \sqrt{(-4)^2 + (1)^2} \\ &= \sqrt{17} \end{aligned}$$

DETERMINE THE GRADIENT BETWEEN TWO POINTS

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 1

$A(-5; -9)$ and $B(3; 2)$

Solution:

Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$

$$x_1 = -5 \quad y_1 = -9 \quad x_2 = 3 \quad y_2 = 2$$

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-9)}{3 - (-5)} \\ &= \frac{11}{8} \end{aligned}$$

DETERMINE THE GRADIENT BETWEEN TWO POINTS

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 2

$A(x-3;y)$ and $B(x;y+4)$

Solution:

Let the coordinates of A be $(x_1;y_1)$ and the coordinates of B be $(x_2;y_2)$

$$x_1 = x - 3 \quad y_1 = y \quad x_2 = x \quad y_2 = y + 4$$

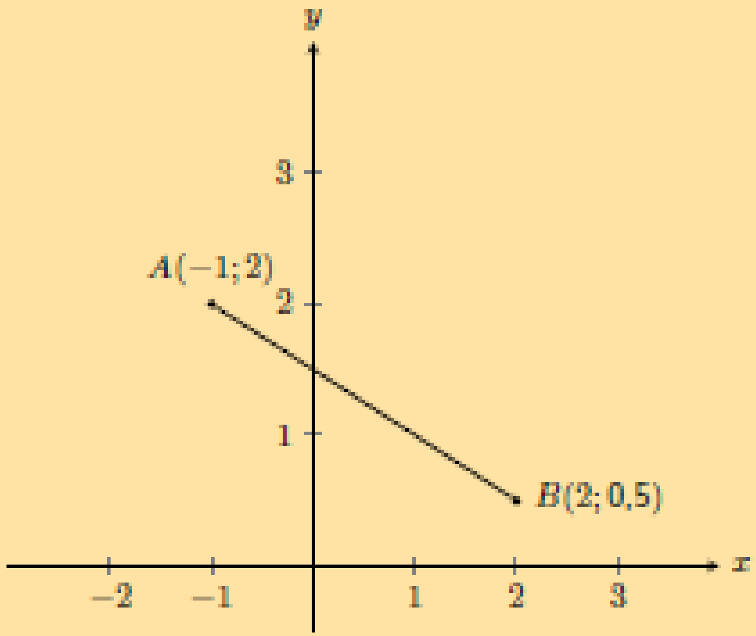
$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{y + 4 - y}{x - (x - 3)} \\ &= \frac{4}{3} \end{aligned}$$

DETERMINE THE GRADIENT BETWEEN TWO POINTS

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 4

You are given the following diagram:



Calculate the gradient (m) of line AB.

Solution:

Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$

$$x_1 = -1 \quad y_1 = 2 \quad x_2 = 2 \quad y_2 = 0{,}5$$

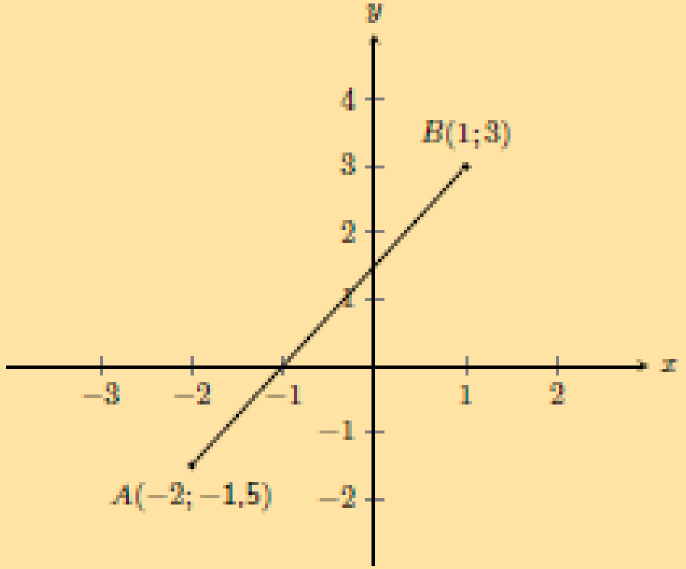
$$\begin{aligned} m &= \frac{y_B - y_A}{x_B - x_A} \\ &= \frac{(0{,}5) - (2)}{(2) - (-1)} \\ &= \frac{-1{,}5}{3} \\ &= -0{,}5 \end{aligned}$$

DETERMINE THE GRADIENT BETWEEN TWO POINTS

$$\text{Gradient} = \frac{\text{Difference in } y}{\text{Difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 5

You are given the following diagram:



Calculate the gradient (m) of line AB.

Solution:

Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$

$$x_1 = -2 \quad y_1 = -1,5 \quad x_2 = 1 \quad y_2 = 3$$

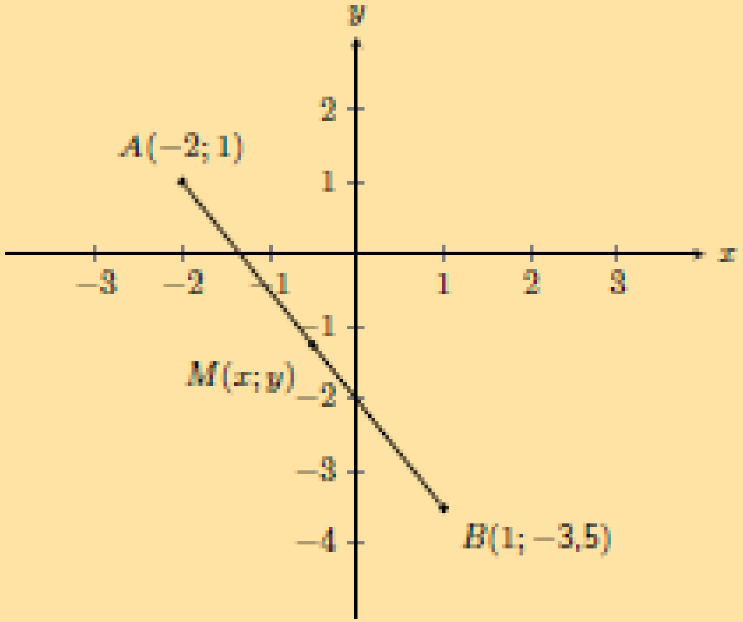
$$\begin{aligned} m &= \frac{y_B - y_A}{x_B - x_A} \\ &= \frac{(3) - (-1,5)}{(1) - (-2)} \\ &= \frac{4,5}{3} \\ &= 1,5 \end{aligned}$$

DETERMINE THE MIDPOINT BETWEEN TWO POINTS

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

EXAMPLE 1

You are given the following diagram:



Calculate the coordinates of the mid-point (M) between point $A(-2;1)$ and point $B(1;-3{,}5)$.

Solution:

Let the coordinates of A be $(x_1; y_1)$ and the coordinates of B be $(x_2; y_2)$.

$$x_1 = -2 \quad y_1 = 1 \quad x_2 = 1 \quad y_2 = -3{,}5$$

Substitute values into the mid-point formula:

$$\begin{aligned} M(x;y) &= \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right) \\ x &= \frac{x_1+x_2}{2} \\ &= \frac{-2+1}{2} \\ &= -0{,}5 \\ y &= \frac{y_1+y_2}{2} \\ &= \frac{1+(-3{,}5)}{2} \\ &= -1{,}25 \end{aligned}$$

The mid-point is at $M(-0{,}5; -1{,}25)$.

DETERMINE THE MIDPOINT BETWEEN TWO POINTS

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

EXAMPLE 2

$C(5;9)$, $D(23;55)$

Solution:

$$\begin{aligned} M_{CD} &= \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right) \\ &= \left(\frac{5+23}{2};\frac{9+55}{2}\right) \\ &= \left(\frac{28}{2};\frac{64}{2}\right) \\ &= (14;32) \end{aligned}$$

EXAMPLE 3

$E(x+2;y-1)$, $F(x-5;y-4)$

Solution:

$$\begin{aligned} M_{EF} &= \left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right) \\ &= \left(\frac{x+2+x-5}{2};\frac{y-1+y-4}{2}\right) \\ &= \left(\frac{2x-3}{2};\frac{2y-5}{2}\right) \end{aligned}$$

Equation of a Straight Line

Summary

If you know	Formulae to use
The gradient and the y-intercept	$y = mx + c$
The gradient and the coordinates of at least one point on the graph.	$y - y_1 = m(x - x_1)$ or $y = mx + c$
Two points on the line: first calculate the gradient and then substitute into $y = mx + c$.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ and $y = mx + c$