CALCULUS-OPTIMISATION

- Calculus can be applied to many real-life situations, in which a set of equations need to be maximized (e.g. to make a profit) or minimized (e.g. to reduce costs)
- You will often need to set up equations – so learn the formulae for perimeter, area, total surface area and volume
- To maximize or minimize an equation, we will **differentiate** and put the equation **equal to 0**
- **… i.e. the maximum or minimum point of an equation / graph is the turning point and the gradient equals 0!**

CALCULUS-OPTIMISATION

<u>Example 1:</u>

The length of a rectangle is x and its breadth is $(25 - x)$. Determine the dimensions of the rectangle in order for it to have a maximum area.

$$Area = length \times breadth$$
$$= x \times (25 - x)$$
$$= 25x - x^2$$

What do we have to do to maximize or minimize?

CALCULUS-OPTIMISATION

$$Area = 25x - x^2$$

$$\frac{dArea}{dx} = 25 - 2x$$

$$0 = 25 - 2x$$

$$2x = 25$$

$$x = 12{,}5$$

Differentiate and put the equation equal to zero!

$$Length = x = 12{,}5\ units$$

$$Breadth = 25 - x$$

$$= 25 - 12{,}5$$

$$= 12{,}5\ units$$

But the question asked for the dimensions of the rectangle …

CALCULUS-OPTIMISATION

<u>Example 2:</u>

A cylinder has a radius of *x* and a height of (300 – *x*). Find the value of *x* in order for the volume of the cylinder to be a maximum.

$$Volume = \pi r^2 h$$
$$= \pi\,(x)^2(300 - x)$$
$$= 300\pi x^2 - \pi x^3$$

What do we have to do to maximize or minimize?

CALCULUS-OPTIMISATION

$$Volume = 300\pi x^2 - \pi x^3$$

$$\frac{dVolume}{dx} = 300\pi x - 3\pi x^2$$

$$0 = 300\pi x - 3\pi x^2$$

$$0 = 100x - x^2$$

$$0 = x(100 - x)$$

$$x = 0 \ \ or \ x = 100$$

Differentiate and put the equation equal to zero!

Since the question asked for a maximum, x = 100 …

$$Volume = 300\pi x^2 - \pi x^3$$

$$= 300\pi(100)^2 - \pi(100)^3$$

$$= 6\ 283\ 185{,}31$$

CALCULUS-OPTIMISATION
SPEED, VELOCITY AND ACCELERATION

- Average speed is the distance travelled divided by the time taken.

$$\boldsymbol{Average\ speed = \frac{change\ in\ distance}{change\ in\ time}}$$

Example 1

I took 4 hours to travel to Johannesburg , a distance of 456km.

$$\boldsymbol{Average\ speed\ = \frac{456}{4}\ =\ 114\ kph.}$$

CALCULUS-OPTIMISATION
Speed, velocity and acceleration

<u>Example 2</u>

A stone falls at a distance of $s(t) = 2x^2 - 3$ meters, where t is time in seconds. Determine the average speed of the stone as it falls.

Note! s represents distance and not speed!

Distance travelled in …

2 seconds: $s(2) = 2(2)^2 - 3 = 5$ m

3 seconds: $s(3) = 2(3)^2 - 3 = 15$ m

$$Average\ speed = \frac{15-5}{3-2} = 10\ m\ per\ sec$$

CALCULUS-OPTIMISATION
SPEED, VELOCITY AND ACCELERATION

- Instantaneous speed – or VELOCITY - is the speed taken at one particular point in time

$$Velocity = derivative\ of\ distance$$
$$v = d'(Distance)$$

<u>Example 1</u>

Determine the velocity at 3 seconds, if the distance of a particle can be given by: s(t) = 20 – 5t (where t = seconds).

$s'(t) = -5.$

Note! s'(t) is the derivative of distance = velocity!

CALCULUS-OPTIMISATION
Speed, velocity and acceleration

Example 2

Determine the speed I was travelling at the exact time of 4 seconds, when the speed trap camera flashed after I had covered a distance of 120x meters.

$$Average\ speed = \frac{120x}{4} = 30x\ meters\ per\ sec$$

$$Acceleration = d'(Speed)$$

$$= 30\ meters\ per\ second$$

CALCULUS-OPTIMISATION
SPEED, VELOCITY AND ACCELERATION

- **Instantaneous velocity – or ACCELERATION - is the velocity taken at one particular point in time**

$$Acceleration = derivative\ of\ velocity$$
$$a = d'(Velocity)$$

<u>Example 1</u>

Determine the acceleration of a particle, if the velocity can be given by v(t) = 6t (where t = seconds).

$v'(t) = 6.$

Note! v'(t) is the derivative of velocity = acceleration!

CALCULUS-OPTIMISATION
Speed, velocity and acceleration

<u>Example 2</u>

If the distance of an object can be represented by the equation: $s(t) = 4t^2 - 6t + 1$, determine the acceleration of the object.

$s(t) = 4t^2 - 6t + 1$ … Distance

$s'(t) = 8t - 6$ … Velocity

$s''(t) = 8$ … Acceleration