Calculus – The Power Rule

WORKING OUT THE DERIVATIVE USING THE POWER RULE

POWER RULE

GENERAL POWER RULE

If $f(x) = x^n$ then the derivative $f'(x) = n.x^{n-1}$

EXAMPLE

If $f(x) = x^4$ then the derivative will be $f'(x) = 4x^3$

BUT FIRST LET US GO THROUGH SOME THINGS TO HELP US WITH USING THE POWER RULE EFFECTIVELY AND CORRECTLY

What is the DERIVATIVE?

The definition is:

THE RATE OF CHANGE OF A FUNCTION WITH RESPECT TO AN INDEPENDENT VARIABLE. (SLOPE OR GRADIENT)

LOOKING AT THE PREVIOUS EXAMPLE

If $f(x) = x^4$ then the derivative will be $f'(x) = 4x^3$

Which is therefore the **rate of change of function** f(x) with respect to **the independent variable x**

What is the Derivative?

Now we know what a Derivative is.

RECAP: DERIVATIVE IS THE RATE OF CHANGE OF A FUNCTION WITH RESPECT TO AN INDEPENDENT VARIABLE.

Therefore, when using the power rule to find the derivative of a function, we are finding the **rate of change** of that function with respect to **an independent variable**.

NOW NEXT THING TO KNOW IS THE **DIFFERENT NOTATIONS** **(DIFFERENT WAYS OF WRITING)**THAT ARE USED FOR THE DERIVATIVE OF A FUNCTION

Different Notations for Expressing the Derivative

Remember that $\boldsymbol{y = f(x)}$

The Different notations for the Derivative that can be used are

1. $f'(x)$
2. y'
3. $\frac{dy}{dx}$
4. $D_x[f(x)]$

When we find the Derivative of a function we DIFFERENTIATE the function

Power Rule

LET US NOW LOOK AT THE POWER RULE

If $f(x) = x^n$ then the derivative $f'(x) = n.x^{n-1}$

HOW DO WE APPLY IT?

There are two things that we do with the exponent.

1. Move the exponent to the front.
2. Subtract 1 (**Doesn't matter if the exponent is positive or negative still subtract 1**) from the exponent

Example

$f(x) = x^6$

1. Move the exponent to the front.
2. Subtract 1 from the exponent.

$$f(x) = x^6$$
$$f'(x) = 6x^{6-1}$$
$$f'(x) = 6x^5$$

Try some Examples

Example

$f(x) = x^3$ x is the independent variable and f(x) is the function

$f'(x) = 3x^3$

$f(t) = t^5$ t is the independent variable and f(t) is the function

$f'(t) = 5t^4$

$g(s) = 3s^4$ s is the independent variable and g(s) is the function

$g'(s) = 4.3s^3$

$g'(s) = 12s^3$

There is already a 3 in front of the function so when we bring the exponent to the front of the expression then we must multiply the two numbers together.

More Examples

1. If $f(x) = 3x^{-2}$, *determine f′(x)*

$f'(x) = \mathbf{-2}.3x^{-2\mathbf{-1}}$

$f'(x) = -6x^{-3}$

1. **Move the exponent to the front.**
2. **Subtract 1 from the exponent.**

REMEMBER IN THIS EXAMPLE YOU MUST MULTIPLY THE EXPONENT BROUGHT TO THE FRONT WITH THE COEFFICIENT ALREADY THERE.

2. If $y = 5x^{-3} + 4x^2$, $\quad determine \frac{dy}{dx}$

$\frac{dy}{dx} = \mathbf{-3}.5x^{-3\mathbf{-1}} + \mathbf{2}.4x^{2\mathbf{-1}}$

$\frac{dy}{dx} = -15x^{-4} + 8x^1$

With this example we have two terms. We find the derivative of each term individually.

N.B.

$$\boldsymbol{if\ f(x) = g(x) + h(x)\ \ then\ f'(x) = g'(x) + h'(x)}$$

AND

$$\boldsymbol{if\ f(x) = g(x) - h(x)\ \ then\ f'(x) = g'(x) - h'(x)}$$

Examples continued….

3. If $f(x) = \sqrt[3]{x}\,, determine\ f'(x)$

Change the function to exponential form and then work out Derivative as previously done.

$$f(x) = \sqrt[3]{x}$$

$$f(x) = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

4. If $f(x) = \sqrt[3]{x^2}, determine\ f'(x)$

$$f(x) = \sqrt[3]{x^2}$$

$$f(x) = x^{\frac{2}{3}}$$

$$f(x) = \frac{2}{3}x^{\frac{2}{3}-1}$$

$$f(x) = \frac{2}{3}x^{-\frac{1}{3}}$$

Examples continued....

5. If $f(x) = 3x^3 + 6x^2 + x - 4,\ determine\ f'(x)$

$$f(x) = 3x^3 + 6x^2 + x - 4$$

$$f'(x) = 3.3x^{3-1} + 2.6x^{2-1} + 1 - 0$$

$$f'(x) = 9x^2 + 12x^1 + 1$$

Note:

1. The Derivative of a constant is 0. Therefore the derivative of -4 in the example 5 is 0.
2. The Derivative of a independent variable without a specified exponent is 1. Therefore the Derivative of x in example 5 is 1. (This only applies if x is the independent variable)

WHY is The Derivative of a independent variable without a specified power is 1.

$if\ f(x) = x$

We know that $f(x) = x^1$ if no exponent is specified.

$\therefore f'(x) = 1x^{1-1}$

$\therefore f'(x) = 1x^0$

$\therefore f'(x) = 1.1$

$\therefore f'(x) = 1$

REMEMBER: $x^0 = 1$

Any variable to the power of 0 is 1

Examples continued….

5. $yx - y = 2x^2 - 2x, determine\ \frac{dy}{dx}$

$y(x-1) = 2x(x-1)$

$y = \frac{2x(x-1)}{x-1}$

$y = 2x$

$y = 2x^1$

$\frac{dy}{dx} = \mathbf{1}.2x^{1\mathbf{-1}}$

$\frac{dy}{dx} = 2x^0$

$\frac{dy}{dx} = 2.1 = 2$

Writing it in exponential form. You do not need to do this, but I wrote it like this for explanation purposes.

Note:

1. The expression must be in the form of y = ……
2. You will need to change it first before finding the Derivative. It could involve factorisation or multiplying in. This example you will find the common factor.

TRY FACTORISE BOTH SIDES BECAUSE IT COULD HELP IN MAKING THE QUESTION EASIER TO WORK WITH.

Examples continued....

6. $Determine\ \frac{dy}{dx}\ if\ y = 4x^8 + \sqrt{x^3}$

$$y = 4x^8 + \sqrt{x^3}$$

$$y = 4x^8 + x^{\frac{3}{2}}$$

$$\frac{dy}{dx} = 8.4x^{8-1} + \frac{3}{2}x^{\frac{3}{2}-1}$$

$$\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$$

Change $\sqrt{x^3}$ into exponential form first

Examples continued….

7. If $y = ax^2 + a$

7.1 *Determine* $\frac{dy}{dx}$

From $\frac{dy}{dx}$, we know that the independent variable is x. The denominator gives us what the independent variable is. This means that a is a constant.

$\frac{dy}{dx} = \mathbf{2}.a.x^{2\mathbf{-1}} + 0$

$\frac{dy}{dx} = 2ax^1$

7.2 *Determine* $\frac{dy}{da}$

From $\frac{dy}{da}$, we know that the independent variable is now a. The denominator gives us what the independent variable is. This means that x is a constant now.

$y = a^1.x^2 + a^1$

Change the a into exponential form. REMEMBER x is now the constant.

$\frac{dy}{da} = \mathbf{1}a^{1\mathbf{-1}}.x^2 + \mathbf{1}a^{1\mathbf{-1}}$

$\frac{dy}{da} = 1a^0.x^2 + 1a^0$

$\frac{dy}{da} = x^2 + 1$

Examples continued....

8. If $y = bx^2 + b^2$

8.1 Determine $\frac{dy}{db}$

From $\frac{dy}{db}$, we know that the independent variable is b. The denominator gives us what the independent variable is. This means that x is a constant.

$$y = b^1x^2 + b^2$$

$$\frac{dy}{db} = \mathbf{1}.b^{1\mathbf{-1}}.x^2 + \mathbf{2}b^{2\mathbf{-1}}$$

$$\frac{dy}{db} = 1.b^0.x^2 + 2b^1$$

$$\frac{dy}{db} = x^2 + 2b^1$$