Analytical Geometry

Gr 11

Revise

Length between two points

FORMULA

$$Length = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. Determine the length of the line segment between the following points:

   a) $P(-3; 5)$ and $Q(-1; -5)$

   b) $R(0{,}75; 3)$ and $S(0{,}75; -4)$

   c) $T(2x; y-2)$ and $U(3x+1; y-2)$

   Solution:

   a)

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1+3)^2 + (-5-5)^2} \\ &= \sqrt{(2)^2 + (-10)^2} \\ &= \sqrt{4 + 100} \\ &= \sqrt{104} \\ &= 2\sqrt{26} \text{ units} \end{aligned}$$

   b)

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0{,}75 - 0{,}75)^2 + (-4-3)^2} \\ &= \sqrt{(0)^2 + (-7)^2} \\ &= \sqrt{49} \\ &= 7 \text{ units} \end{aligned}$$

   c)

$$\begin{aligned} TU &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3x + 1 - 2x)^2 + (y - 2 - y + 2)^2} \\ &= \sqrt{(x+1)^2 + (0)^2} \\ &= \sqrt{(x+1)^2} \\ &= x + 1 \text{ units} \end{aligned}$$

The Equation of line through two points.

Summary

If you know	Formulae to use
The gradient and the y-intercept	$y = mx + c$
The gradient and the coordinates of at least one point on the graph.	$y - y_1 = m(x - x_1)$ or $y = mx + c$
Two points on the line: first calculate the gradient and then substitute into $y = mx + c$.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ and $y = mx + c$

Example 1

Determine the equation of the straight line passing through the points:

1. (3; 7) and (-6; 1)
   - Answer Work out the gradient

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-7}{-6-3} = \frac{-6}{-9} = \frac{2}{3}$$

   - Substitute into y=mx+c

$$y = \frac{2}{3}x + c$$

   - *Substitute one of the points in. Here* (3; 7) *is used*

$$(7) = \frac{2}{3}.(3) + c$$

$$7 = 2 + c$$

$$7 - 2 = c$$

$$c = 5$$

   - Therefore

$y = \frac{2}{3}x + 5$

Example 2

Determine the equation of the straight line that passes through the points P(1; 2) and Q(3; 8) in the form y = ...

First calculate the gradient of PQ:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$ ✓

Then use the form $y - y_1 = m(x - x_1)$

$y - y_1 = 3(x - x_1)$ ✓

Substituting P(1; 2)

$y - 2 = 3(x - 1)$

$y - 2 = 3x - 3$

$\therefore$ The equation of linePQ is $y = 3x - 1$. ✓

Example 3

Line AB is perpendicular to CD, which has a gradient of −2. The point (3; 4) lies on AB. Determine the equation of line AB.

$m_{CD} = -2$ and CD ⊥AB.

$\therefore m_{AB} = \frac{1}{2}$

So now we have $y = \frac{1}{2}x + c$

Substitute (3; 4) to find the value of c.

$4 = \frac{1}{2}(3) + c$ ✓

$c = 4 - 1\frac{1}{2}$

$\therefore c = 2\frac{1}{2}$

equation of line AB is $y = \frac{1}{2}x + 2\frac{1}{2}$ ✓

Example 4

If the gradient of a line is −2 and the line cuts the *y*-axis at 1, then the equation of the line is $y = -2x + 1$.

Example 5

If the gradient of a line is −2 and the point (4; −1) lies on the line, find the equation of the line.

$y - y_1 = m(x - x_1)$	
$y - (-1) = -2(x - 4)$	substitute (4; −1) into the equation
$y + 1 = -2x + 8$	simplify
$y = -2x + 7$	We usually put the answer in the form $y = mx + c$.