Grade 12

Calculus-First Principles

Exercise 1

Find the $f'(x)$ of the following functions using **FIRST PRINCIPLES:**

a. $f(x) = x^2 - 5$

b. $f(x) = x^2 + 2$

c. $f(x) = 4 - 7x$

ANSWERS

a.

$f(x+h) = (x+h)^2 - 5$ $= x^2 + 2xh + h^2 - 5$	✓ $x^2 + 2xh + h^2 - 5$
$f(x+h) - f(x) = x^2 + 2xh + h^2 - 5 - (x^2 - 5)$ $= 2xh + h^2$	✓simplification
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$	
$= \lim_{h\to 0} \frac{2xh + h^2}{h}$	✓factorisation
$= \lim_{h\to 0} \frac{h(2x+h)}{h}$	
$= \lim_{h\to 0} (2x+h)$	✓ $\lim_{h\to 0}(2x+h)$
$= 2x$	✓ $2x$ (5)

b.

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$	
$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2 - (x^2 + 2)}{h}$	✓ $x^2 + 2xh + h^2 + 2$
$= \lim_{h\to 0} \frac{2xh + h^2}{h}$	✓ $\lim_{h\to 0} \frac{2xh + h^2}{h}$
$= \lim_{h\to 0} \frac{h(2x+h)}{h}$	✓ $\lim_{h\to 0} \frac{h(2x+h)}{h}$
$= \lim_{h\to 0} (2x+h)$	
$= 2x$	✓ answer (4)

c.

$f(x) = 4 - 7x$	
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$	✓ $4 - 7(x+h)$
$= \lim_{h\to 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$	✓substitution
$= \lim_{h\to 0} \frac{h(-7)}{h}$	✓simplification
$= -7$	✓answer (4)