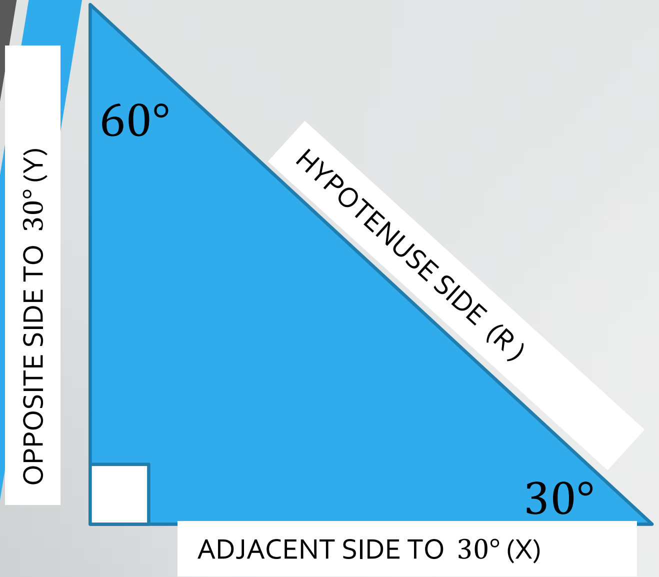
GRADE 12 TRIGONOMETRY

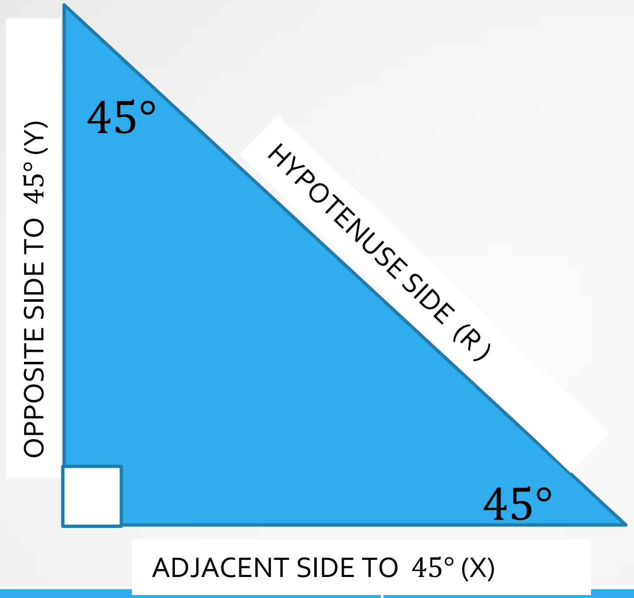
Trig Equations

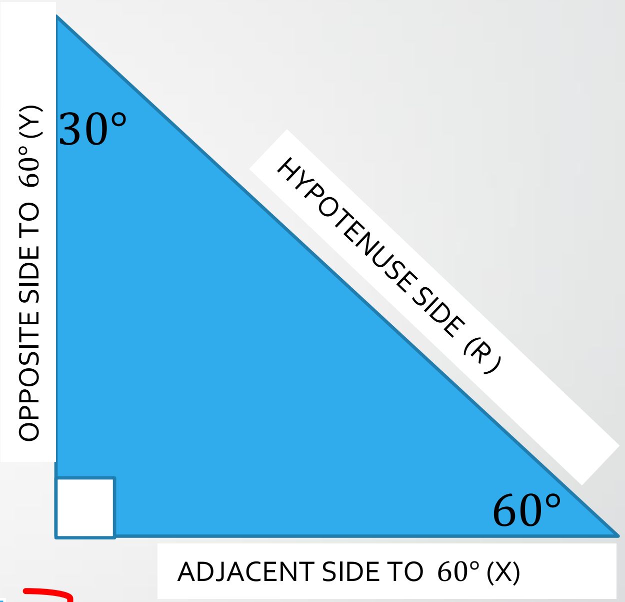
Solve problems in two and three dimensions.

Involving compound angles

Recap Special Angles – Triangles not drawn to scale







	Adjacent Side (x)	Opposite Side (y)	Hypotenuse Side (r)
30°	$\sqrt{3}$	1	2
45°	$\sqrt{2}$	$\sqrt{2}$	2
60°	1	$\sqrt{3}$	2

LEARN THIS TABLE

EXAMPLE

1. $\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$
2. $\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$
3. $\tan 30° = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$
4. $\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$
5. $\cos 60° = \frac{adj}{hyp} = \frac{1}{2}$
6. $\tan 60° = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1}$
7. $\sin 45° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2}$
8. $\cos 45° = \frac{adj}{hyp} = \frac{\sqrt{2}}{2}$
9. $\tan 45° = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

RECAP COMPOUND ANGLES

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

Example 1:

Simplify: $\sin 80^\circ.\cos 50^\circ - \cos 80^\circ.\sin 50^\circ$

Answer:

$= \sin(80^\circ - 50^\circ)$

$= \sin 30^\circ$

$= \frac{1}{2}$

Example2:

Determine $\cos 75^\circ$ without using a calculator

Answer:

$\cos 75^\circ$

$= \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ.\cos 30^\circ - \sin 45^\circ.\sin 30^\circ$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

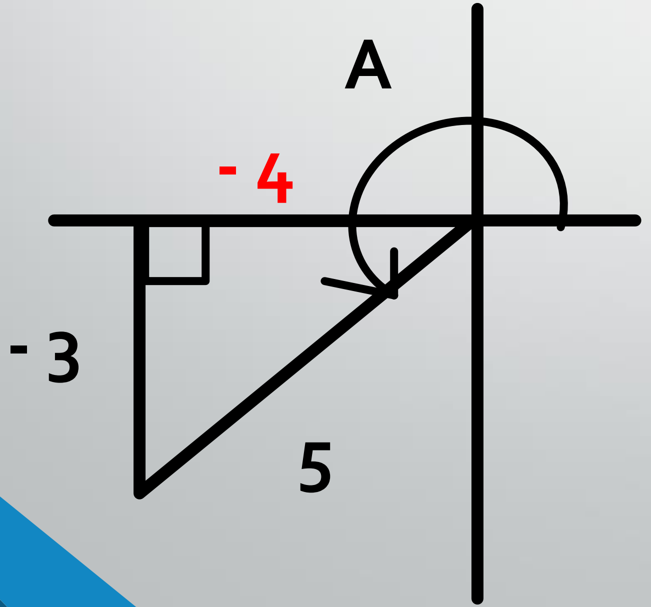
Example 3:

If $sin\,A = -\frac{3}{5}$ $and\ 90° < A < 270°$;

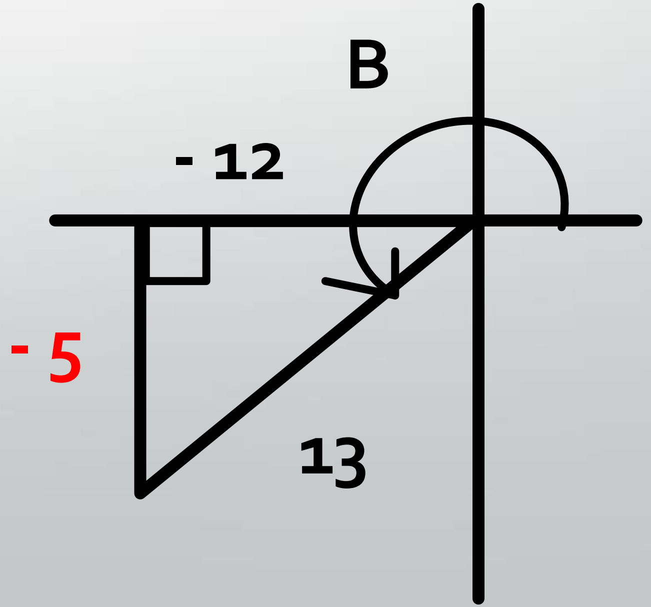
$cos\,B = -\frac{12}{13}$ $and\ 180° < B < 360°$;

determine the value of sin (A – B).

Two Sketches are needed …



Use Pythag to find missing side



Answer

sin (A – B)

Compound Angle Sin Rule

= sin A .cos B – cos A. sin B

$$= \left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \frac{16}{65}$$

Example 4:

Prove: $\cos(A + B) - \cos(A - B) = -2\sin A \cos B$

Answer

L.H.S. (Use side with more information)

$= \cos(A + B) - \cos(A - B)$

$= (\cos A \cos B - \sin A \sin B) - (\cos A \cos B + \sin A \sin B)$

Compound Angle Cos Rule

osA cosB – sinA sinB – cosA cosB – sinA sinB

$= -2\sin A \sin B$

DOUBLE ANGLES: SUMMARY

$\cos 2A =$

1. $\cos^2 A - \sin^2 A$ ***OR***

2. $2\cos^2 A - 1$ ***OR***

3. $1 - 2\sin^2 A$

$$\sin 2A = 2\sin A \cdot \cos A$$

Similarly, Half & Triple Angles ...

$\cos A \quad = \cos(\tfrac{1}{2}A + \tfrac{1}{2}A)$

$= \cos^2 \tfrac{1}{2}A - \sin^2 \tfrac{1}{2}A$ OR

$= 2\cos^2 \tfrac{1}{2}A - 1$ OR

$= 1 - 2\sin^2 \tfrac{1}{2}A$

$\sin A \quad = \sin(\tfrac{1}{2}A + \tfrac{1}{2}A)$

$= 2\sin \tfrac{1}{2}A \,.\, \cos \tfrac{1}{2}A$

Example 5:

Prove: $\frac{sin2A}{1+cos2A} = tanA$

Answer:

$$LHS = \frac{sin2A}{1 + cos2A}$$

$$= \frac{2sinAcosA}{1 + (2cos^2A - 1)}$$

$$= \frac{2sinAcosA}{2cos^2A}$$

$$= tanA$$

$$= RHS$$

Example 6:

Prove: $\frac{cos2x}{cosx - sinx} = cosx + sinx$

Answer:

$$LHS = \frac{cos2x}{cosx - sinx}$$

$$= \frac{cos^2x - sin^2x}{cosx - sinx}$$

$$= \frac{(cosx - sinx)(cosx + sinx)}{cosx - sinx}$$

$$= cosx + sinx$$

$$= RHS$$

Example 7:

Solve: $10cos^2x - 5sin2x + 2 = 0$

Answer:

$$10cos^2x - 5sin2x + 2 = 0$$

$$10cos^2x - 5(2sinxcosx) + 2 = 0$$

$$10cos^2x - 10sinxcosx + 2(sin^2x + cos^2x) = 0$$

$$10cos^2x - 10sinxcosx + 2sin^2x + 2cos^2x = 0$$

$$12cos^2x - 10sinxcosx + 2sin^2x = 0$$

$$6cos^2x - 5sinxcosx + sin^2x = 0$$

Factorize the Trinomial

$$(6cosx + sinx)(cosx - sinx) = 0$$

$$(6cosx + sinx)(cosx - sinx) = 0$$

$6cosx = -sinx$ OR $cosx = sinx$

$\frac{6cosx}{cosx} = \frac{-sinx}{cosx}$ $\qquad$ $\frac{cosx}{cosx} = \frac{sinx}{cosx}$

$6 = -tanx$ $\qquad$ $1 = tanx$

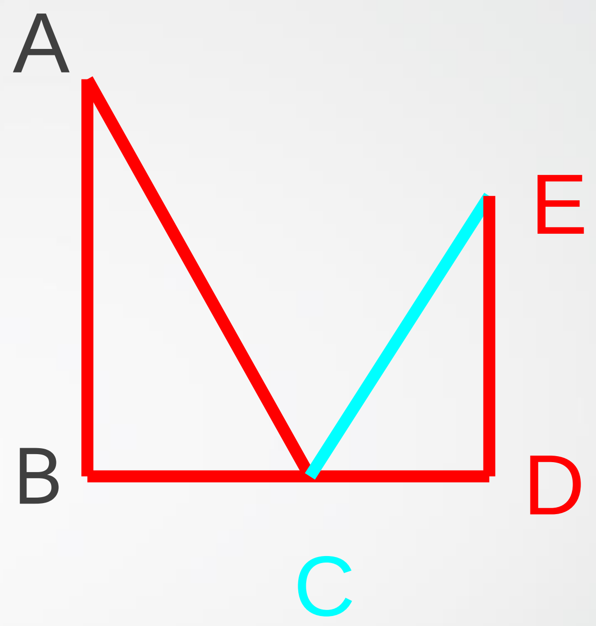
$tanx = -6$ $\qquad$ $tanx = 1$

(Q3) OR (Q1)

$x = 180° - Ref.angle$
$= 180° - 80{,}54°$
$= 99{,}46°$

$x = Ref.angle$
$= 45°$

2D & 3D TRIG



Example 1

A man at position C is rowing down the Duzi River. The angle of elevation from the top of the right bank DE to A the top of the left bank (AB) is x.

The angle of depression from a girl at the top of the right bank E is also x and the man is k metres from the left bank. The man's angle of elevation to the top of the left bank is 2x. A,B,C,D and E are all on the same vertical or horizontal planes. Show that the distance between the girl and the man (CE) is given by

$$\frac{2k.\sin x}{\sin 4x}$$

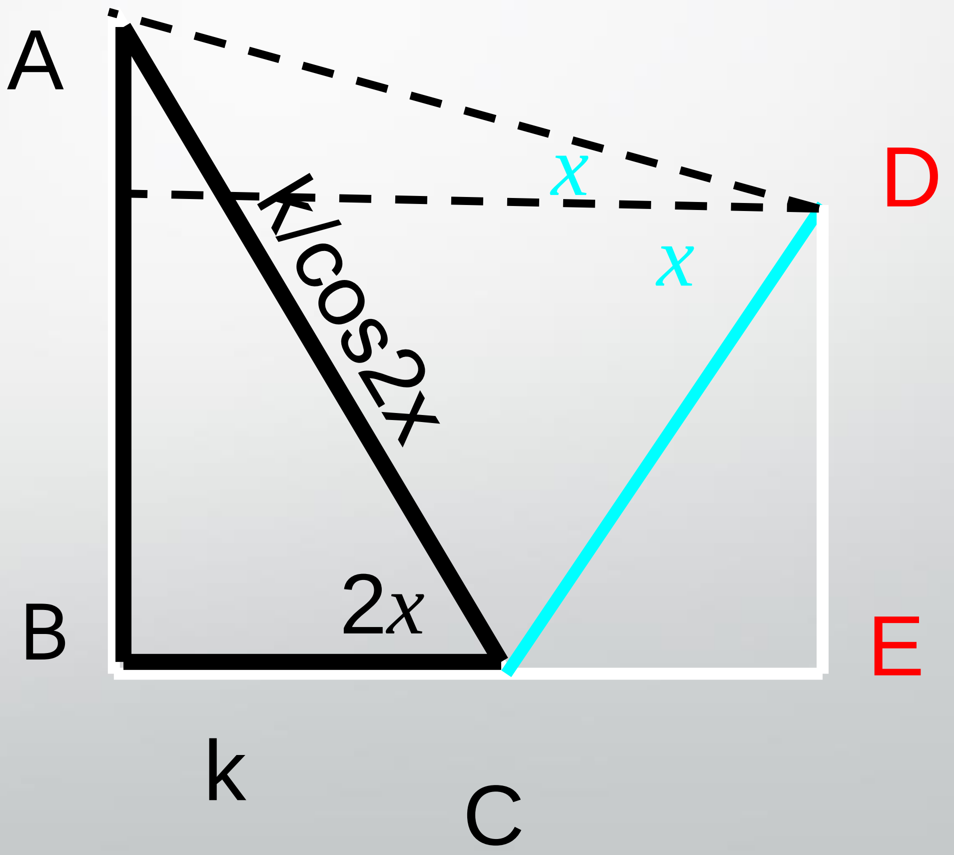
Answer:

Put the information onto the diagram

In ΔABC

cos 2x = BC / AC

AC = k / (cos 2x)



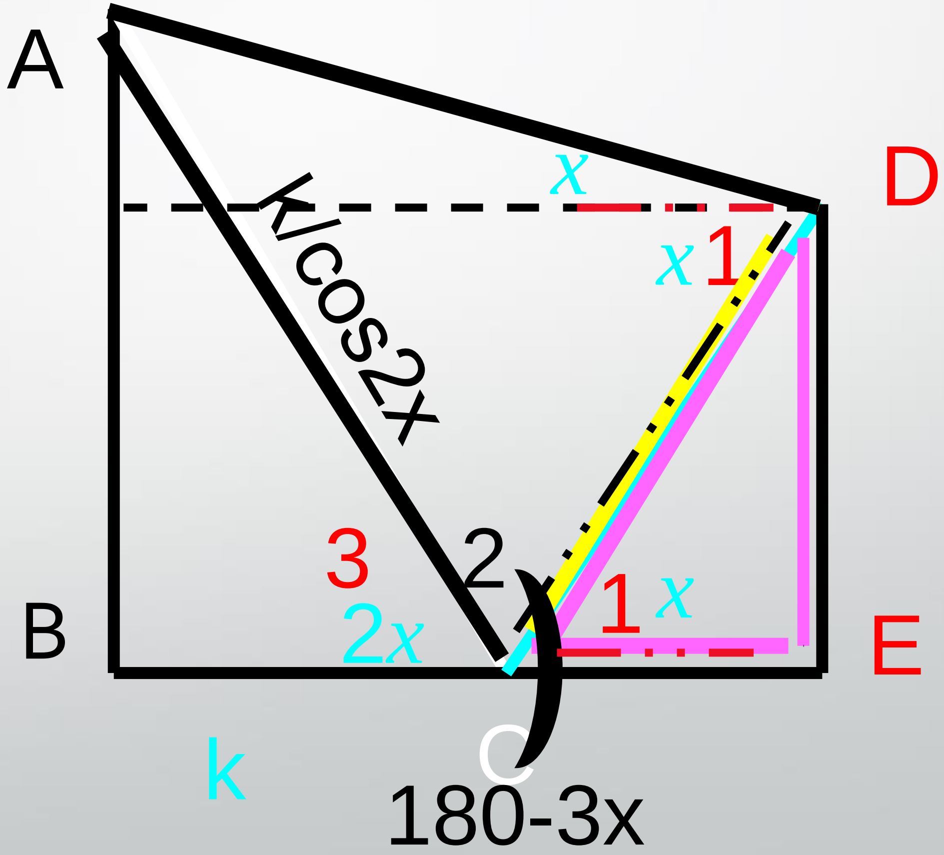
In ΔDEC

$C_1 = D_1 = x$ ………… Alt ∠'s

In ΔACD

$C_2 = 180 – C_3 – C_1$

$= 180 - 3x$



$A_1 = 180 - C_2 - D_1 - D_2$ sum of ∠'s of ΔACD

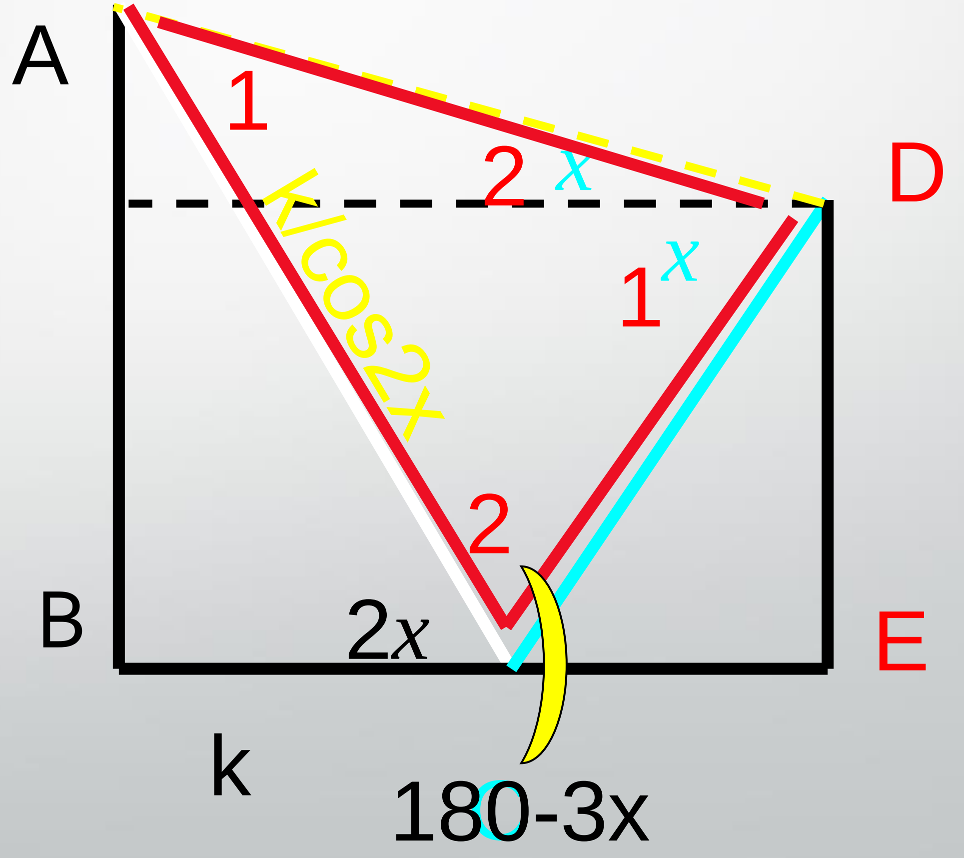
$= 180 - (180 - 3x) - 2x$

$= x$

$$\frac{CD}{\sin A_1} = \frac{AC}{\sin D_{1+2}}$$

$$\frac{CD}{\sin x} = \frac{AC}{\sin 2x}$$

$$CD = \sin x \cdot \frac{k}{\cos 2x} \cdot \frac{1}{\sin 2x}$$



$$CD = \sin x.\frac{k}{\cos 2x}.\frac{1}{\sin 2x}$$

We need sin4x in the solution so use the double angles formula

sin2x = 2 cos x. sin x so

sin 4x = 2 cos 2x . sin 2x

That means cos 2x = sin4x / (2. sin 2x)

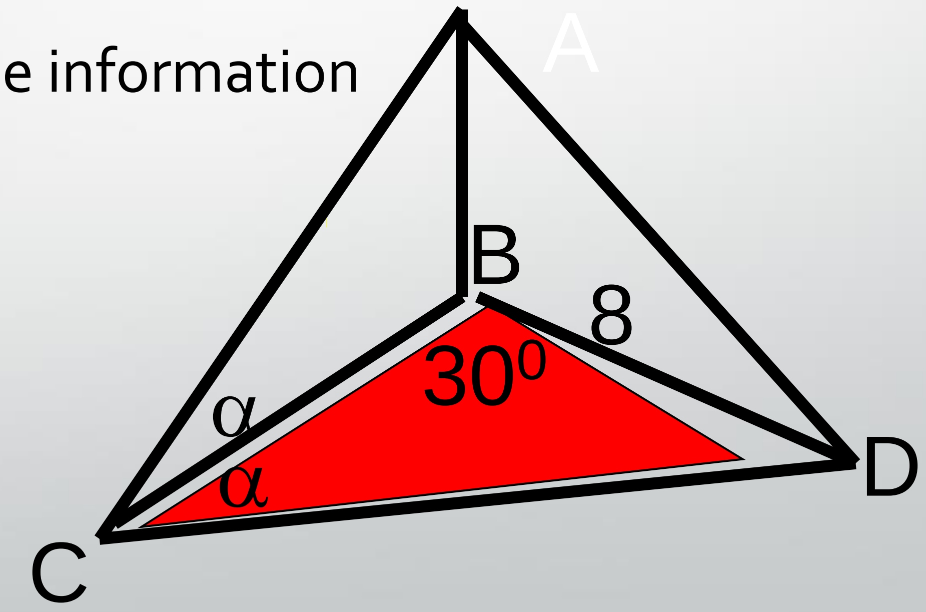
$$CD = \frac{k.\sin x}{\frac{\sin 4x}{2.\sin 2x}}.\frac{1}{\sin 2x}$$

$$\therefore CD = \frac{2k.\sin x}{\sin 4x}.$$

Example 2

B, C and D are 3 points on the same horizontal plane. A flagpole AB is falling over so two wire stays AC and AD are used to shore it up. Without using a calculator, use the information in the diagram to show that the pole, AB is

$4(1+\sqrt{3}\tan\alpha)$ high.



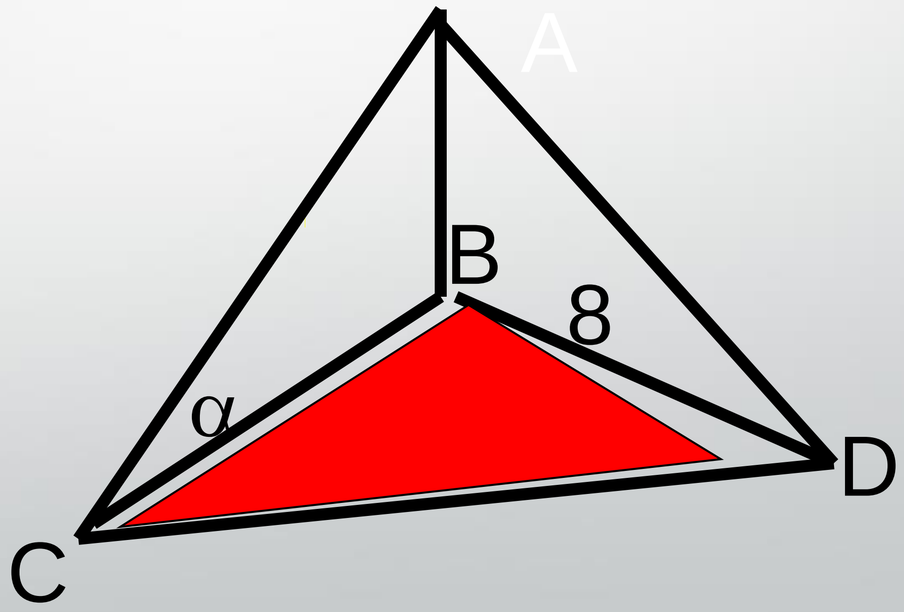
Start in Δ BCD. We need to find CB.

$\angle BDC = 180^{o} – (30^{o} + \alpha)$

$$\frac{BC}{\sin D} = \frac{DB}{\sin C}$$

$$\frac{BC}{\sin[\,(180 - (30 + \alpha)]} = \frac{8}{\sin \alpha}$$

$$BC = \frac{8.\sin(\,30 + \alpha)}{\sin \alpha}$$



Moving into Δ ABC – which is right angled at B

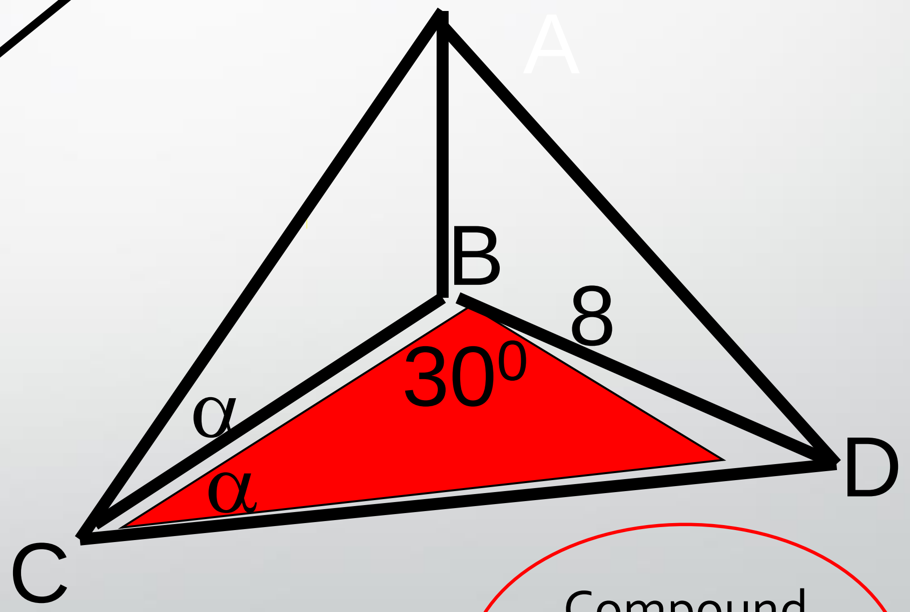
– we need to find AB.

$$\tan\alpha = \frac{AB}{CB}$$

$$BC = \frac{8.\sin(30+\alpha)}{\sin\alpha}$$

AB = BC. tan α

$$= \frac{8.\sin(30+\alpha)}{\sin\alpha} \;.\tan\alpha$$



$$= \frac{8(\sin 30.\cos\alpha + \cos 30.\sin\alpha)}{\sin\alpha} .\tan\alpha$$

Compound Angle Sin Rule

$$= \frac{8(\sin 30.\cos\alpha + \cos 30.\sin\alpha)}{\sin\alpha} \;.\; \tan\alpha$$

Special Triangles

$$= \frac{(8.\frac{1}{2}.\cos\alpha + 8.\frac{\sqrt{3}}{2}.\sin\alpha)}{\sin\alpha} \;.\; \tan\alpha$$

$$= (\frac{4\cos\alpha}{\sin\alpha} + \frac{4.\sqrt{3}.\sin\alpha}{\sin\alpha}) \;.\; \tan\alpha$$

$$= (4\frac{\cos\alpha}{\sin\alpha} \;.\tan\alpha \quad + 4.\sqrt{3}.\frac{\sin\alpha}{\sin\alpha}$$

$$= \frac{4}{\tan\alpha}.\tan\alpha + 4.\sqrt{3}.\tan\alpha$$

$$\therefore AB = 4\,(1 + \sqrt{3}.\tan\alpha\,)$$