GEOMETRY GR 11

SIMULTANEOUS EQUATIONS

Simultaneous Equations

To solve two unknown variables you need two equations.

Two equations will be given to you to solve two unknown variables.

Let us look at a easy example

Example 1- with two linear equations

Solve for the unknown variables

$x + 2y = 5$

$y - 3x + 5 = 0$

Answer

$x + 2y = 5$ ……………………………………………….Equation 1

$y - 3x + 5 = 0$ ……………………………………………..Equation 2

Step 1- Make one of the variables the subject of one of the equations. Work with the easier looking equation. In this example we will work with Equation 1 first and make x the subject of equation 1. Once you have made x the subject of the equation, name this equation – Equation 3.

$x + 2y = 5$

$x = 5 - 2y$………………………………………………….Equation 3 ← **Equation 1 is now named Equation 3**

Simultaneous Equations

Example 1 - with two linear equations

Solve for the unknown variables

$x + 2y = 5$

$y - 3x + 8 = 0$

Answer

$x + 2y = 5$ ……………………………………………….Equation 1

$y - 3x + 8 = 0$ ……………………………………………Equation 2

Step 2 – Substitute Equation 3 into the Equation 2

$y - 3x + 8 = 0$ ……………………………………………Equation 2

$y - 3(5 - 2y) + 8 = 0$

$y - 15 + 6y + 8 = 0$ …………………………………….Equation 4 ← **Equation 2 is now named Equation 4**

Step 3 – Solve the unknown variable – y in this case.

$y - 15 + 6y + 8 = 0$

$7y - 7 = 0$

$7y = 7$

$y = 1$

Simultaneous Equations

Example 1- with two linear equations

Solve for the unknown variables

$x + 2y = 5$

$y - 3x + 8 = 0$

Answer

$x + 2y = 5$ ……………………………………………………Equation 1

$y - 3x + 8 = 0$ ……………………………………………...Equation 2

Step 4 – Substitute the solution of Equation 4 (The solved unknown) into the Equation 3.

$y = 1$

$x = 5 - 2y$……………………………………………………Equation 3

$x = 5 - 2(1)$

$x = 3$

Therefore x = 3 and y = 1

Simultaneous Equations

If you are asked to solve variables and the one equation is linear and the other is quadratic, the same steps are followed with slight changes.

- Step 1: Make one of the unknowns of the linear equation the subject of the equation (- get x or *y* alone on one side of the equation).
- Step 2: Substitute the x or *y value* (whichever is the subject of the equation from the previous step) into the quadratic equation.
- Step 3: Solve the one unknown. (You may need to use the Quadratic Formula). You will get two answers for the unknown.
- Step 4: Substitute the solved unknown values into the linear equation to solve for the other unknown.

Example 2- with 1 linear equation and 1 quadratic equation

Solve for the unknown variables

$y + 2x - 2 = 0$ ………………………………(1)

$2x^2 + y^2 = 3xy$ ………………………………..(2)

Simultaneous Equations

<u>If you are asked to solve variables and the one equation is linear and the other is quadratic, the same steps are followed with slight changes.</u>

<u>Example 2- with 1 linear equation and 1 quadratic equation</u>

Solve for the unknown variables

$y + 2x - 2 = 0$ ……………………………….(1)

$2x^2 + y^2 = 3xy$ ………………………………….(2)

<u>Answer:</u>

Step 1: Make one of the unknowns of the linear equation the subject of the equation (- get x or y alone on one side of the equation).

$y + 2x - 2 = 0$ ……………………………(1)

$y = 2 - 2x$… ……………………………(3) ← **Equation 1 is now named Equation 3**

<u>If you are asked to solve variables and the one equation is linear and the other is quadratic, the same steps are followed with slight changes.</u>

<u>Example 2- with 1 linear equation and 1 quadratic equation</u>

<u>Answer:</u>

- **<u>Step 2</u>**: Substitute the x or *y value* (whichever is the subject of the equation from the previous step) into the quadratic equation.

Substitute Equation 3 into Equation 2

$y = 2 - 2x$ ………………………………(3)

$2x^2 + y^2 = 3xy$ ……………………………..(2)

$2x^2 + (2 - 2x)^2 = 3x.(2 - 2x)$……………………………..(2)

Simultaneous Equations

<u>If you are asked to solve variables and the one equation is linear and the other is quadratic, the same steps are followed with slight changes.</u>

<u>Example 2- with 1 linear equation and 1 quadratic equation</u>

<u>Answer:</u>

<u>Step 3</u>: Solve the one unknown. (You may need to use the Quadratic Formula). You will get two answers for the unknown.

$2x^2 + (2 - 2x)^2 = 3x.(2 - 2x)$……………………………………..(2)

$2x^2 + (2 - 2x).(2 - 2x) = 3x.(2 - 2x)$

$2x^2 + (4 - 8x + 4x^2) = 6x - 6x^2$

$12x^2 - 14x + 4 = 0$

$6x^2 - 7x + 2 = 0$

$(3x - 2)(2x - 1) = 0$

$(3x - 2) = 0 \quad OR \quad (2x - 1) = 0$

$x = \frac{2}{3} \quad OR \quad x = \frac{1}{2}$

Simultaneous Equations

<u>If you are asked to solve variables and the one equation is linear and the other is quadratic, the same steps are followed with slight changes.</u>

<u>Example 2- with 1 linear equation and 1 quadratic equation</u>

<u>Step 4</u>: Substitute the solved unknown values into the linear equation to solve for the other unknown.

<u>Substitute the values into Equation 3</u>

$y = 2 - 2x$... ....(3)

$y = 2 - 2.\left(\frac{2}{3}\right) \quad OR \quad y = 2 - 2.\left(\frac{1}{2}\right)$

$y = \frac{2}{3} \quad OR \quad y = 1$

$Therefore\, x = \frac{2}{3}\ and\ y = \frac{2}{3} \qquad OR \qquad x = \frac{1}{2}\ \ and\ y = 1$

Simultaneous Equations

Example 3 – Try on your own first before looking at answer – Exam Type Question

1.2 Solve for x and y simultaneously:

$y + x = 2$ and $x^2 + 3xy + 8 = 0$ (6)

Example 4 – Try on your own first before looking at answer – Exam Type Question

1.2 Solve simultaneously for x and y:

$3x - y = 2$ and $2y + 9x^2 = -1$ (6)

Answers to Example 3 and 4

- Example 3

1.2	$y+x=2$ and/en $x^2+3xy+8=0$ $\therefore\ y=2-x$ $x^2+3x(2-x)+8=0$ $x^2+6x-3x^2+8=0$ $-2x^2+6x+8=0$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or $x=-1$ $y=2-4$ or / *of* $y=2-(-1)$ $y=-2$ $\quad y=3$	✓ $y=2-x$ ✓substitution/*verv.* ✓std form/*stand. vorm* ✓factors or using formula/ *faktore of gebruik formule* ✓both x-values/*wrdes* ✓both y-values/*wrdes* (6)
	OR/*OF* $y+x=2$ and $x^2+3xy+8=0$ $\therefore\ x=2-y$ $(2-y)^2+3(2-y)y+8=0$ $4-4y+y^2+6y-3y^2+8=0$ $-2y^2+2y+12=0$ $y^2-y-6=0$ $(y-3)(y+2)=0$ $y=3$ or $y=-2$ $x=2-3$ or $x=2-(-2)$ $x=-1$ or $x=4$	✓ $x=2-y$ ✓substitution/*verv.* ✓std form/*stand. vorm* ✓factors or using formula/ *faktore of gebruik formule* ✓both y-values/*wrdes* ✓both x-values/*wrdes* (6)

Answers to Example 3 and 4

- Example 4

1.2	$2y+9x^2=-1$......(1) $3x-y=2$ ...... (2) $y=3x-2$ ....(3) $2(3x-2)+9x^2=-1$ $6x-4+9x^2=-1$ $9x^2+6x-3=0$ $3x^2+2x-1=0$ $(3x-1)(x+1)=0$ $x=\frac{1}{3}$ or $x=-1$ $y=-1$ or $y=-5$	✓ $y=3x-2$ ✓substitution ✓ standard form ✓factors ✓both x values ✓both y values (6)
	OR/OF $2y+9x^2=-1$......(1) $3x-y=2$ ...... (2) $x=\frac{y+2}{3}$ $2y+9\left(\frac{y+2}{3}\right)^2=-1$ $2y+9\left(\frac{y^2+4y+4}{9}\right)=-1$ $2y+y^2+4y+4+1=0$ $y^2+6y+5=0$ $(y+5)(y+1)=0$ $y=-1$ or $y=-5$ $x=\frac{1}{3}$ or $x=-1$	**OR/OF** ✓ $x=\frac{y+2}{3}$ ✓substitution ✓ standard form ✓factors ✓both y values ✓both x values (6)