GRADE 12
COUNTING PRINCIPLES AND PROBABILITY

WHAT IS NEEDED TO BE KNOWN

- dependent and independent events;
- the product rule for independent events:

  $$P\ (A\ \textit{and}\ B) = P(A) \times P(B)$$

- the sum rule for mutually exclusive events A and B:

  $$P\ (A \text{ or } B) = P(A) + P(B)$$

- the identity:

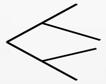
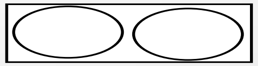
  $$P\ (A \text{ or } B) = P(A) + P(B) - P\ (A \text{ and } B)$$

- the complementary rule:

  $$P(\text{not } A) = 1 - P(A)$$

- Probability problems using Venn diagrams, trees, two – way contingency tables and other techniques

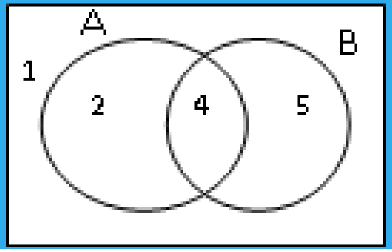
TERMINOLOGY

Probability	chance / likelihood of an event happening
Trial	the attempt made to see if an event will occur **eg:** each toss of the coin
Outcome	result of the trial – (all the elements)
Event	collection of one or more outcomes that you are interested in / subset of sample space
Set	Collection of items, labeled with a capital letter
Sample space	set of all possible outcomes. Represented as: 1. set-notation A= {1, 2, 3} 2. tree-diagram  3. Venn diagram  4. in a table – contingency tables
Random	cannot be predicted
Probability P(A) $= \frac{n(A)}{n(S)}$	Probability $= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
n(A)	number of outcomes in event A (number of ways A can take place)
n(S)	number of outcomes in the sample space (total number of possible outcomes)
P(A)	the probability of A happening (A taking place)

TERMINOLOGY

Dependent events	Independent events
• Outcome of event A **HAS** an influence on the outcome of event B • Event A happening changes the probability of event B Eg: Drawing of cards without replacement	• Outcome of event A has **NO** influence on the outcome of event B • Event A happening has no effect on the probability of event B Eg: Drawing of cards with replacement

TERMINOLOGY

<table>
<tr>
<td>INCLUSIVE EVENTS</td>
<td>These are events which do have elements in COMMON</td>
<td>
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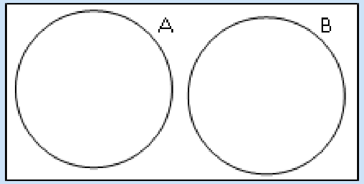
<tr>
<td>MUTUALLY EXCLUSIVE EVENTS</td>
<td>Events which have NO elements in common
n(A and B) = o</td>
<td>
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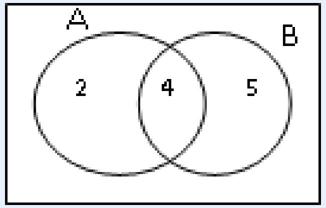
<tr>
<td>EXHAUSTIVE EVENTS</td>
<td>cover ALL the elements of the sample space</td>
<td>
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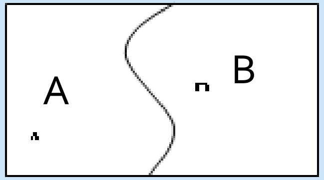
<tr>
<td>COMPLEMENTARY EVENTS</td>
<td>Events are mutually exclusive
Events are exhaustive-fill the sample space.
ALL EVENTS A AND B</td>
<td>
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TERMINOLOGY

	Rule	P(A and B)	P(A or B)
Identity (any two events A and B) **If the combined events are NOT mutually exclusive**	P(A or B) = P(A) + P(B) – P(A and B)		
Mutually exclusive events	P(A or B) = P(A) + P(B) (sum rule)	**0**	**P(A) + P(B)**
Independent events	P(A and B) = P(A).P(B) (product rule)	**P(A) . P(B)**	**P(A) + P(B) - P(A) . P(B)**
Complementary events	P(A) + P(B) = 1 P(not A) = 1 - P(A)		

EXAMPLE 1

Twelve numbers are dropped into a hat:

- Event C = $\{factors\ of\ 6\}$
- Event D = $\{factors\ of\ 9\}$

What is the probability of a chosen number is a factor of 6 or a factor of 9?

ANSWER

Events are mutually not exclusive.

P(C or D) = P(C) + P(D) – P(C and D)

Number of factors of 6 is 4 {1;2;3;6}

Number of factors of 9 is {1;3;9}

P(C or D)=$\left(\frac{4}{12}+\frac{3}{12}\right)-\left(\frac{4}{12}\times\frac{3}{12}\right)=\frac{1}{2}$

EXAMPLE 2

Independent Events Question:

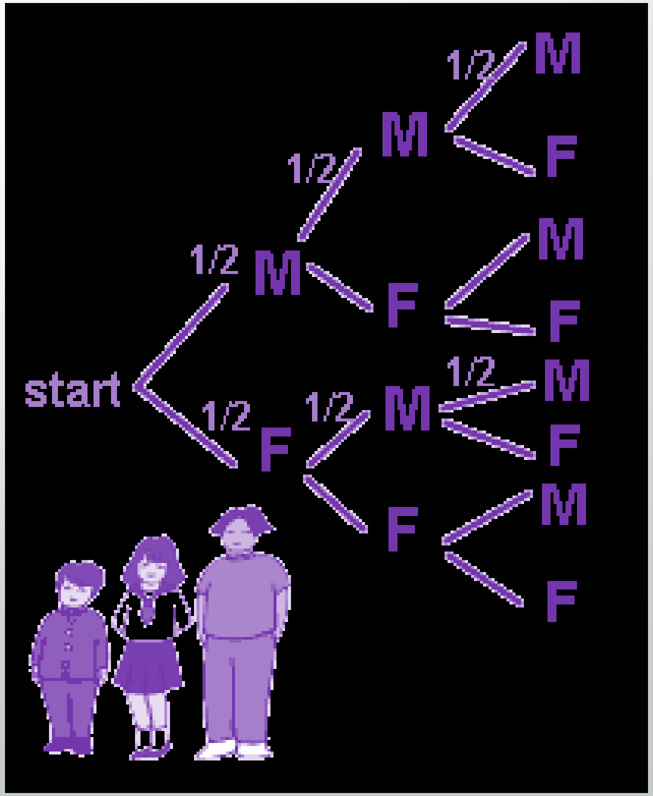
What is the probability of a couple having 3 male children?

$$P(B \text{ and } B \text{ and } B)$$

$$= P(B) \times P(B) \times P(B)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$



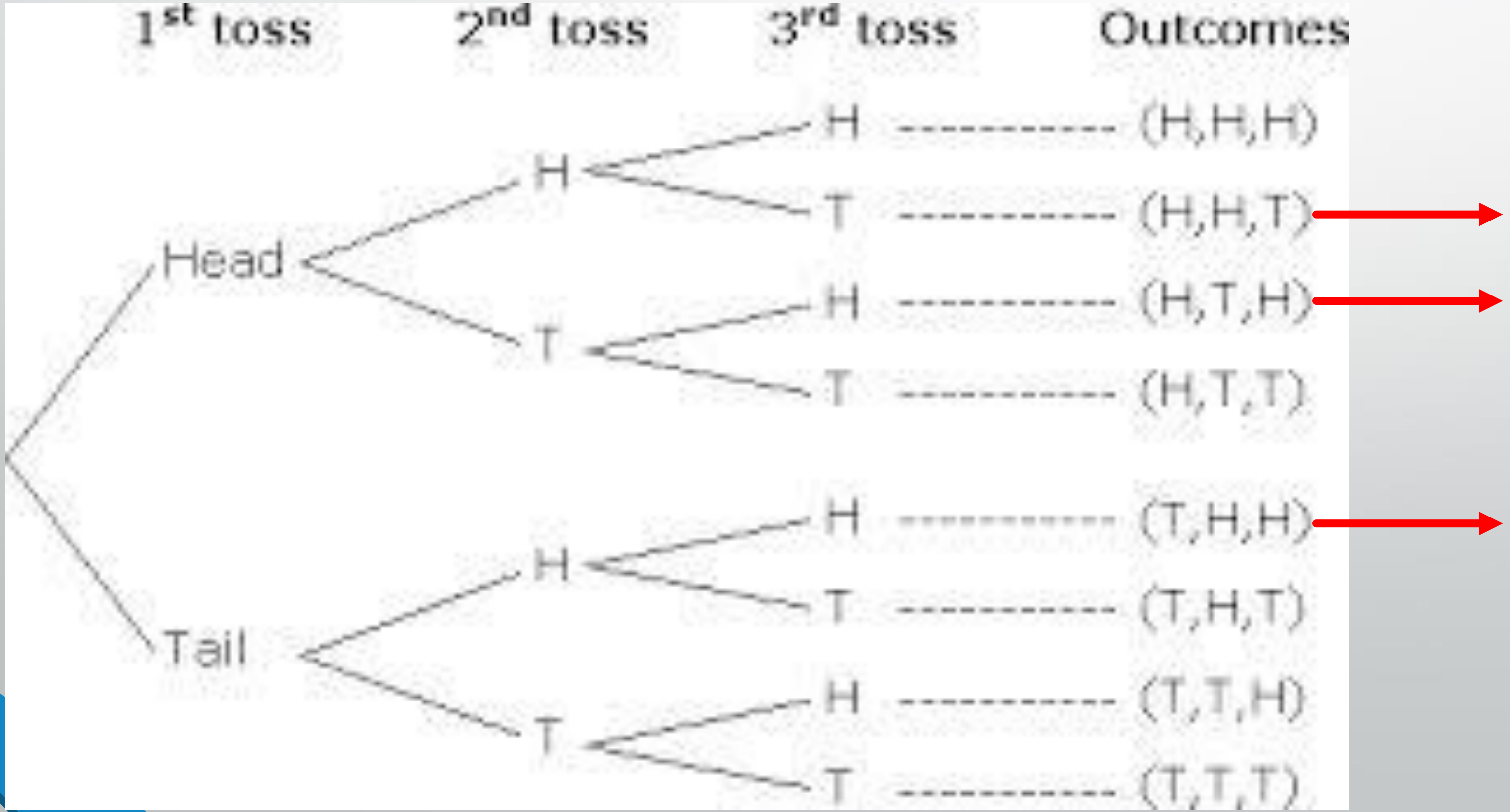
Example 3:
Independent Events Tree Diagram

Use a tree diagram to determine the probability of getting two Heads and one Tail when tossing a fair coin three times?

Answer

There are 8 outcomes.

Of those 8 outcomes only 3 have two heads and one tail.



Answer:

$$\frac{3}{8}$$

Example 4:
Independent Events Question

If the probability of having an accident is 0,25 and the probability of using a motorbike is 0,64, then determine the probability of having an accident while using a motorbike.

Answer

P(A and B) = P(A).P(B)

$P(accident\ and\ using\ motorbike)$

$= \ P(accident) \times \ P(using\ a\ motorbike)$

$= 0{,}25 \ \times 0{,}64$

$= 0{,}16$

DEPENDENT EVENTS

- The outcome of the 1st event effects the outcome of the 2nd event
- Can use the Product Rule:

$$P(A\ and\ B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B)$$

<u>Example 5:</u>

What is the probability of drawing two Aces from a standard deck of cards, if the first card drawn is an Ace and is not replaced?

$$P(Ace) \times P(Ace) = \frac{4}{52} \times \frac{3}{\mathbf{51}} = \frac{1}{221}$$

Example 6:
Dependent Events Tree Diagram

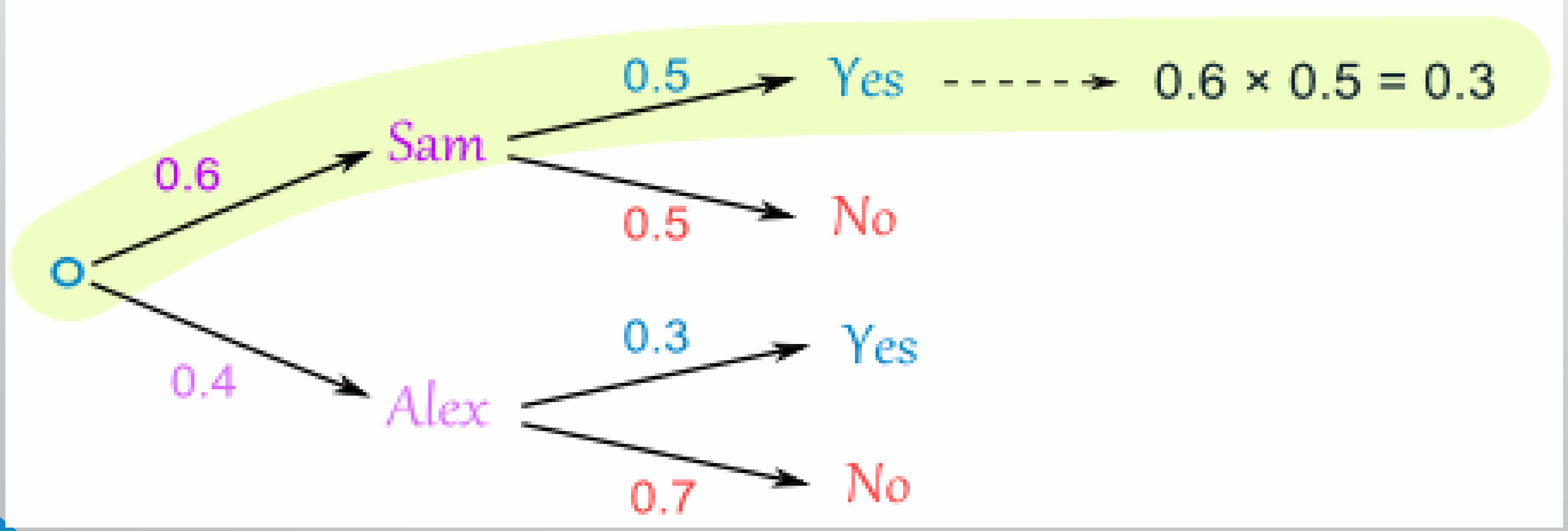
Sam and Alex work together.

Sam has 0,6 chance of being picked to work somewhere else.

If Alex gets picked to work somewhere else, the chance that he says yes is 0,3.

Use a tree diagram to determine the probability of the boss picking Sam and Sam saying "yes" to the working somewhere else.

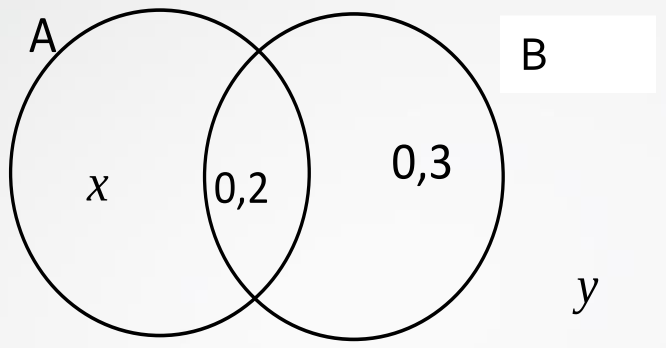
$$P(A \text{ and } B) = P(A) \times P(B)$$



Example 7:
Dependent Events Venn Diagram

If A and B are independent events, find the values of x and y.

All working must be shown.



$$P(A\ and\ B) = P(A) \times P(B)$$

P(A) = x+0,2

P(B) = 0,2+0,3 =0,5

A and B are independent events, therefore,

$$P(A) \times P(B) = P(A\text{ and }B)$$

That is,

$$(x + 0{,}2) \times 0{,}5 = 0{,}2$$

$$x + 0{,}2 = 0{,}4$$

$$x = 0{,}2$$

y + x+0,2+0,3 =1 BECAUSE THEN IT IS ALL THE POSSIBILITIES. 1 MEANS ALL.

$$0{,}2 + 0{,}2 + 0{,}3 + y = 1$$

$$\therefore\ y = 0{,}3$$

Example 8:
Dependent Events Question:

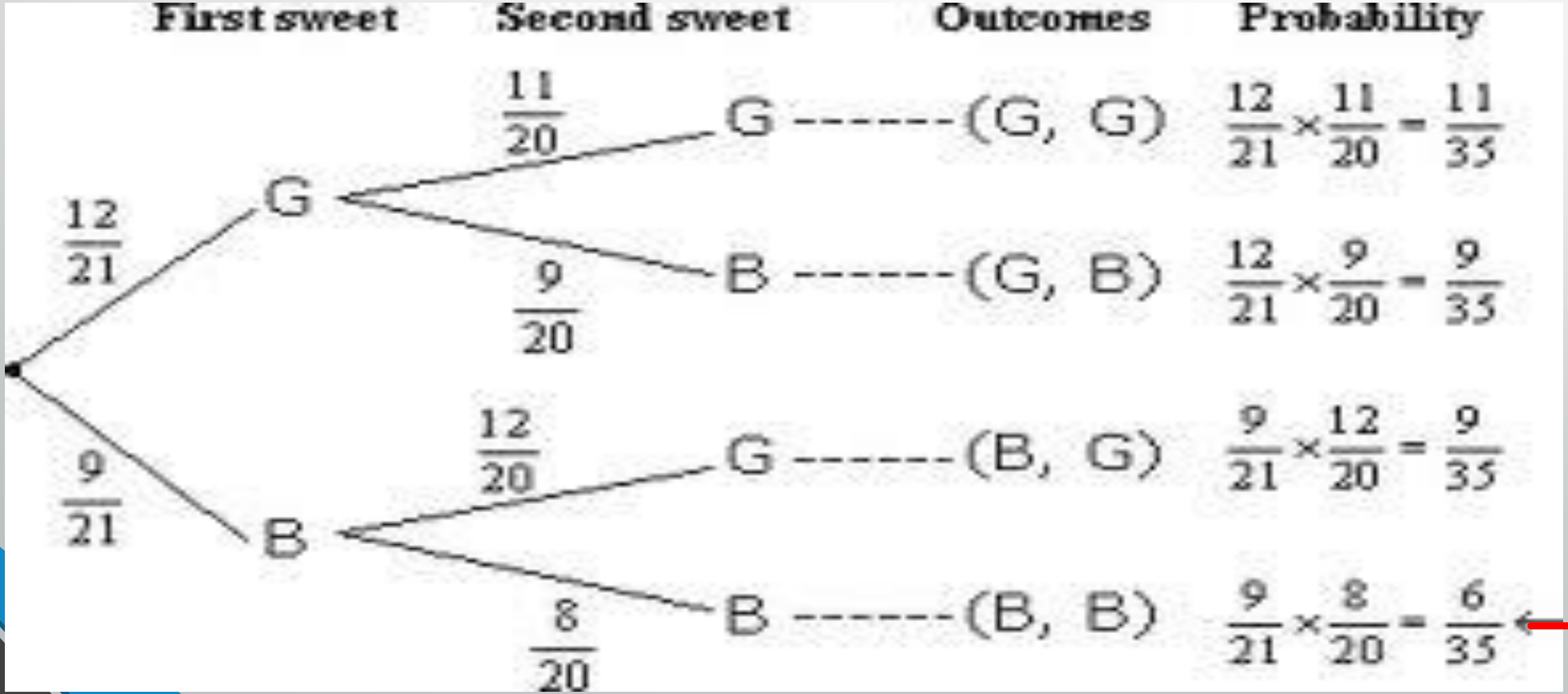
Use a tree diagram to determine the probability of choosing two blue sweets (without replacement), if there are 12 Green sweets and 9 Blue sweets.

TOTAL NUMBER OF SWEETS IS 21.

P(GREEN) $=\frac{12}{21}$

P(BLUE)$=\frac{9}{21}$

$$P(A \text{ and } B) = P(A) \times P(B)$$



Example 9:
Dependent Events Question:

If $P(A) = \frac{3}{8}$ and $P(B) = \frac{1}{4}$, find $P(A \text{ or } B)$ if:

(a) *A* and *B* are mutually exclusive events.

(b) *A* and *B* are independent events.

(a) *P(A or B) = P(A) + P(B) – P(A and B)*

$$= \frac{3}{8} + \frac{1}{4} - 0$$
$$= \frac{5}{8}$$
$$= 0{,}625$$

(b) *P(A or B) = P(A) + P(B) – P(A) × P(B)*

$$= \frac{3}{8} + \frac{1}{4} - \left(\frac{3}{8}\right)\left(\frac{1}{4}\right)$$
$$= \frac{5}{8} - \frac{3}{32}$$
$$= \frac{17}{32}$$
$$= 0{,}53125$$

Example 10:

The table summarises the results of all the driving tests taken at a Test Centre in Cape Town during the first week of January.

	Male	Female	Totals
Pass	32	43	75
Fail	8	15	23
Totals	40	58	98

A person is chosen at random from those who took their test during the first week of January.

a) Find the probability that the person who failed was a female.
b) The person chosen is a male. Find the probability that he passed the test.

(a) $P(FF) = \frac{15}{98}$
$= 0{,}153$

(b) $P(MP) = \frac{32}{40}$
$= 0{,}8$

Example 11:
Dependent Events Question

What is the probability of picking two blue marbles from a hat containing 4 blue; 8 red and 6 green marbles, if the first marble isn't replaced?

Total number of marbles: 18

$$P(B \text{ and } B) = P(B) + P(B)$$

$$= \frac{4}{18} + \frac{3}{17}$$

$$= \frac{61}{153}$$

MUTUALLY EXCLUSIVE EVENTS

- Events that cannot happen at the same time are mutually exclusive events
- Sum Rule for Mutually Exclusive Events:

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

Example 12:

What is the probability of getting a 1 **or** a 5 when rolling a dice?

$$P(1 \text{ or } 5) = P(1) + P(5)$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6} = \frac{1}{2}$$

MUTUALLY EXCLUSIVE EVENTS

- Note!

  $$P(A\ and\ B) = 0$$

  $$P(A \cap B) = 0$$

Example 13:

What is the probability of getting a 1 and a 5 when rolling a dice?

$$P(1\ and\ 5) = 0$$

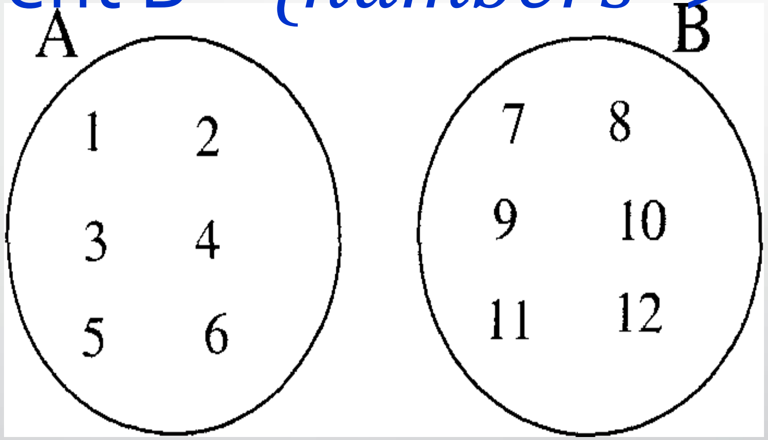
Mutually Exclusive Venn Diagram

Example 14:

Twelve numbers are dropped into a hat:

Event A = $\{numbers \ \leq 6\}$

Event B = $\{numbers \ > 6\}$



$a)\ P(A\ \cup\ B)$
$= P(A) + P(B)$

$b)\ P(A\ \cap\ B)$
$= 0$

Mutually Exclusive Events Question:

Example 15:

What is the probability of picking a red or blue ball from a bag containing 3 red; 7 blue; and 5 green balls?

$$P(R\ or\ B) = P(R) + P(B)$$

$$= \frac{3}{15} + \frac{7}{15}$$

$$= \frac{2}{3}$$

COMBINED EVENTS

- If the combined events are NOT mutually exclusive, then

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 16:

What is the probability of getting an even number or the number 4 when rolling a fair die?

$$P(E\ or\ 4) = P(E) + P(4) - P(E\ and\ 4)$$

$$= \frac{3}{6} + \frac{1}{6} - \left(\frac{3}{6} \times \frac{1}{4}\right)$$

$$= \frac{13}{24}$$

COMPLEMENTARY EVENTS

- Given an event “G”, a complementary event is an event that is not “G”
- Complementary Rule:

$$P(not\ A) = 1 - P(A)$$

$$P(A') = 1 - P(A)$$

Example 17:

What is the probability of not getting a 6 when rolling a fair die?

$$P(6') = 1 - P(6)$$

$$= 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$