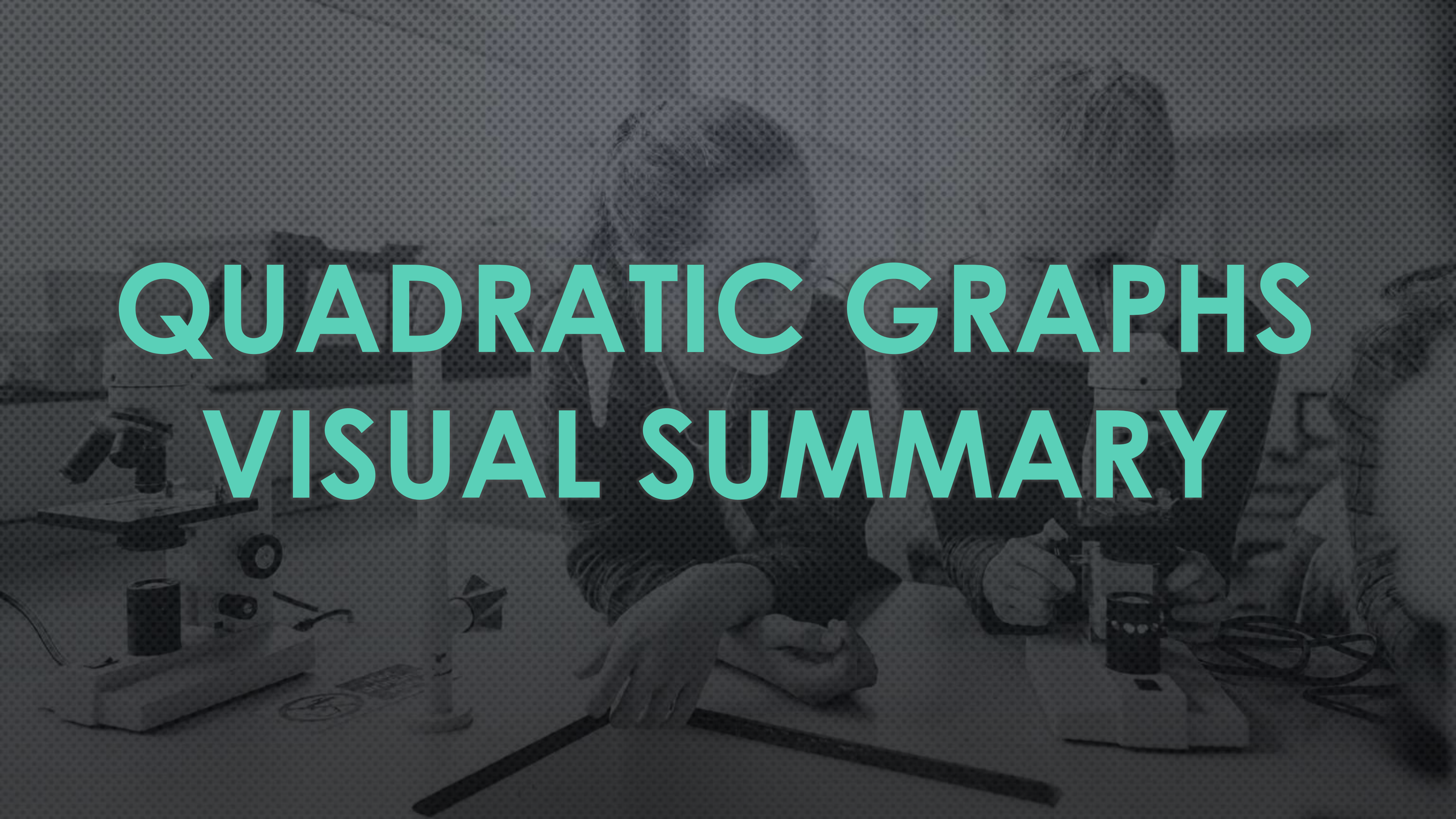
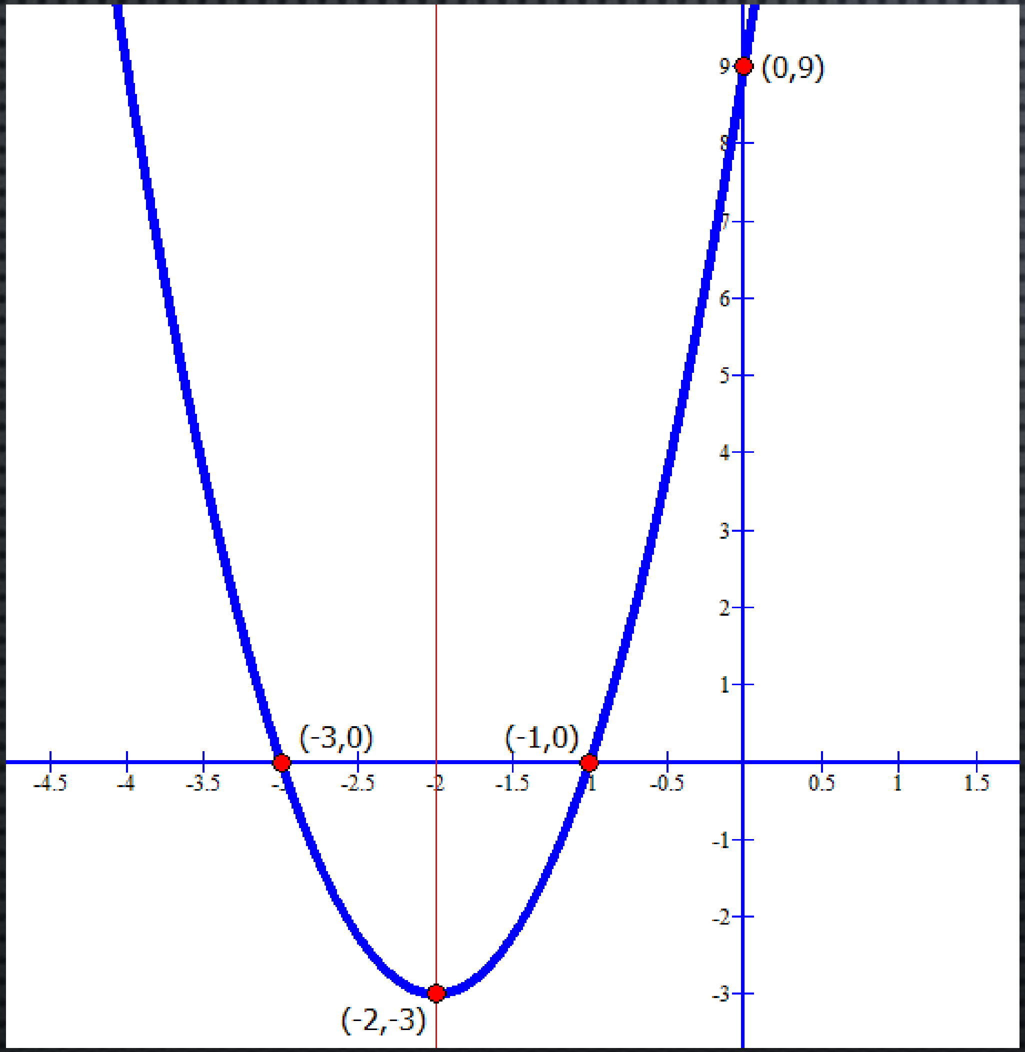
QUADRATIC GRAPHS VISUAL SUMMARY



GRAPHS – QUADRATIC GRAPHS (PARABOLA)



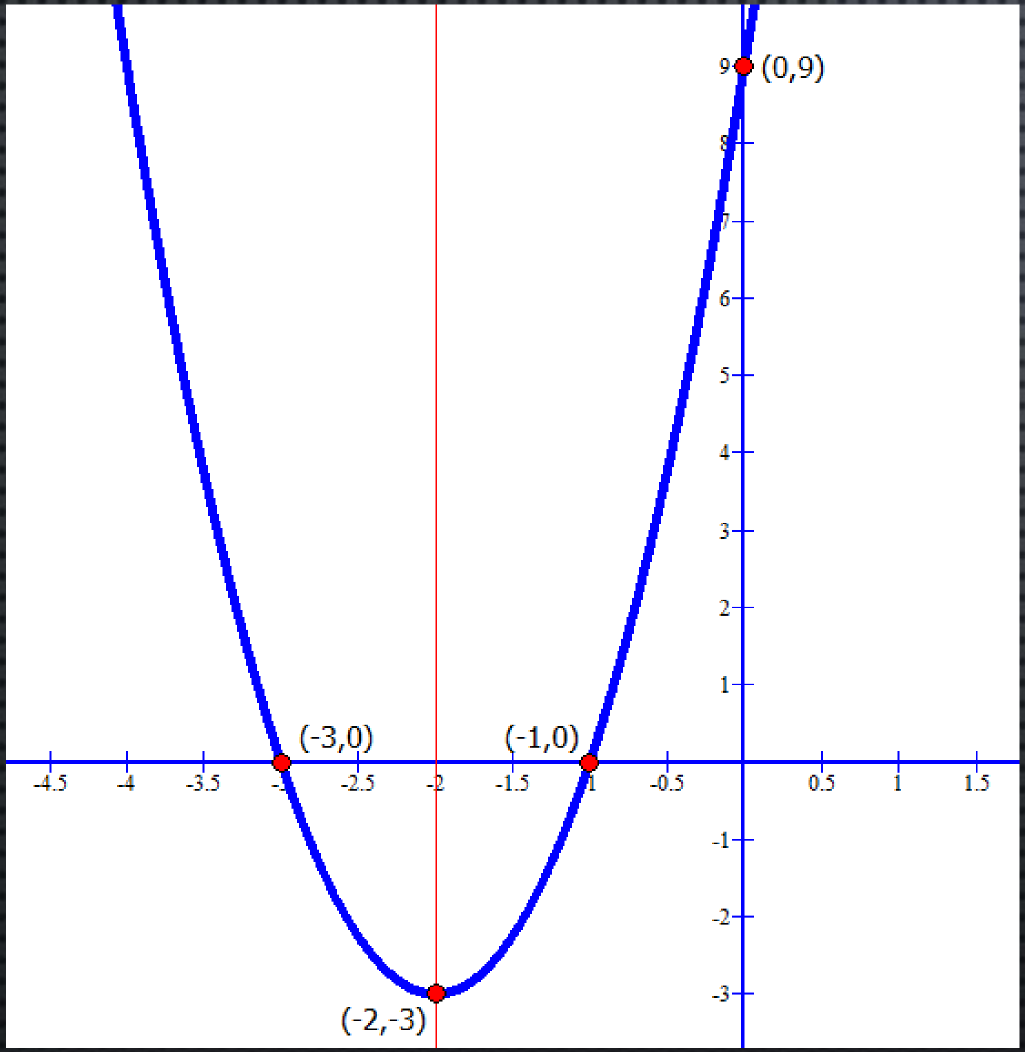
$f(x) = ax^2 + bx + c$ **STANDARD FORM**

$f(x) = 3x^2 + 12x + 9$

1. **Shape:** "Happy Face" because a>0 
2. **Y-Intercept (when x=0):**
   (0 ; 9)
3. **X-Intercepts (when y=0)**: Solve using factorisation.
   $0 = 3x^2 + 12x + 9$
   (-2;0) and (-1;0)
4. **Axis of symmetry:**
   $x = \frac{-b}{2a}$
   $x = \frac{-(12)}{2(3)} = \frac{-12}{6} = -2$
   $x = -2$
5. **Turning Point:**
   The point is a coordinate. The x-coordinate is the axis of symmetry.

   Substitute x=-2 into $f(x) = 3x^2 + 12x + 9$ to get the y-value
   $y = 3.(-2)^2 + 12.(-2) + 9$
   $y = 12 - 24 + 9$
   $y = -3$
   $\therefore (-2; -3)$ **is the turning point.**
   PLOT THE POINTS IN YELLOW ABOVE AND JOIN THE POINTS
   Domain: $x \in \mathbb{R}$
   Range: $y > -3$

GRAPHS – QUADRATIC GRAPHS (PARABOLA)



$f(x) = ax^2 + bx + c$ **STANDARD FORM**

$f(x) = 3x^2 + 12x + 9$

NOW TAKE THE SAME FUNCTION AND APPLY COMPLETING THE SQUARE ON THE FUNCTION

$y = 3x^2 + 12x + 9$

FACTORISE

$y = 3(x^2 + 4x + 3)$

Take the value of b (which is 4 in this case) and do the following:
$(\frac{4}{2})^2 = (2)^2 = 4$

ADD 4 and SUBTRACT 4 on RIGHT SIDE HAND OF FUNCTION

$y = 3[(x^2 + 4x + 4) + 3 - 4]$

$y = 3[(x + 2)^2 - 1]$

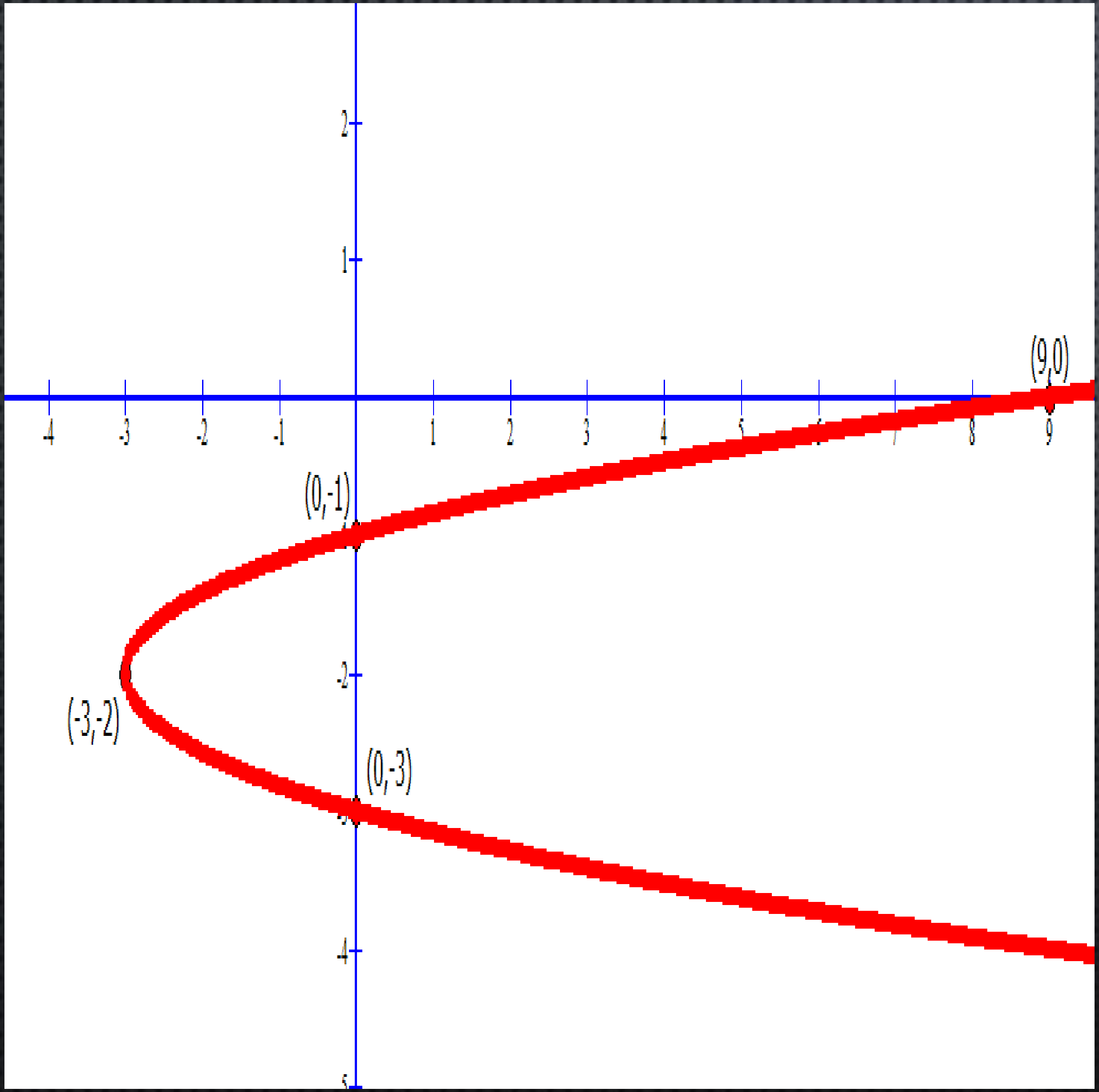
$y = 3(x + 2)^2 - 3$

NOW IT IS IN THE FORM OF $y = a(x - p)^2 + q$ $\quad y = 3(x - (-2)^2) - 3$

THEREFORE (p;q) is the Turning Point of the Function (-2; -3)

THEREFORE x = p is the axis of symmetry x=-2

GRAPHS – QUADRATIC GRAPHS (PARABOLA) - INVERSE



$f(x) = ax^2 + bx + c$ **STANDARD FORM**

$f(x) = 3x^2 + 12x + 9$

$y = 3x^2 + 12x + 9$

x= $3y^2 + 12y + 9$

X BECOMES Y and Y BECOMES X

- **x-Intercept :**
  (9 ; 0)
- **y-Intercepts**
  (0;-2) and (0;-1)
- **Axis of symmetry:**
  $y = -2$
- **Turning Point:**
  $(-3; -2)$ is the turning point.
  PLOT THE POINTS IN YELLOW ABOVE AND JOIN THE POINTS
  Domain: $x > \ -3$
  Range: $y \in \mathbb{R}$

Graphs – Quadratic Graphs (Parabola) - Example

Example 1

Sketch the graph of $f(x) = x^2 - 5x - 6$

1. **Shape**

2. **Y-Intercept (when x=0)**

3. **X-Intercepts (when y=0)**

4. **Axis of symmetry**

5. **Turning Point**

GRAPHS – QUADRATIC GRAPHS (PARABOLA) - EXAMPLE

Answer to Example 1

Sketch the graph of $f(x) = x^2 - 5x - 6$

a = 1
b = -5
c = -6

1. **Shape**

   $a > 0$

1. **Y-Intercept (when x=0)**

   $y = x^2 - 5x - 6$

   $y = (0)^2 - 5.(0) - 6$

   $y = -6$ **(0;-6)**

1. **X-Intercepts (when y=0)**

   $y = x^2 - 5x - 6$

   $(0) = x^2 - 5x - 6$

   $0 = x^2 - 5x - 6$

   $0 = (x-6)(x+1)$

   $x = 6 \text{ or } x = -1$

   (6;0) and (-1;0) are x-intercepts

1. **Axis of symmetry**

   $x = \frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2} = 2\frac{1}{2}$

5. **Turning Point**

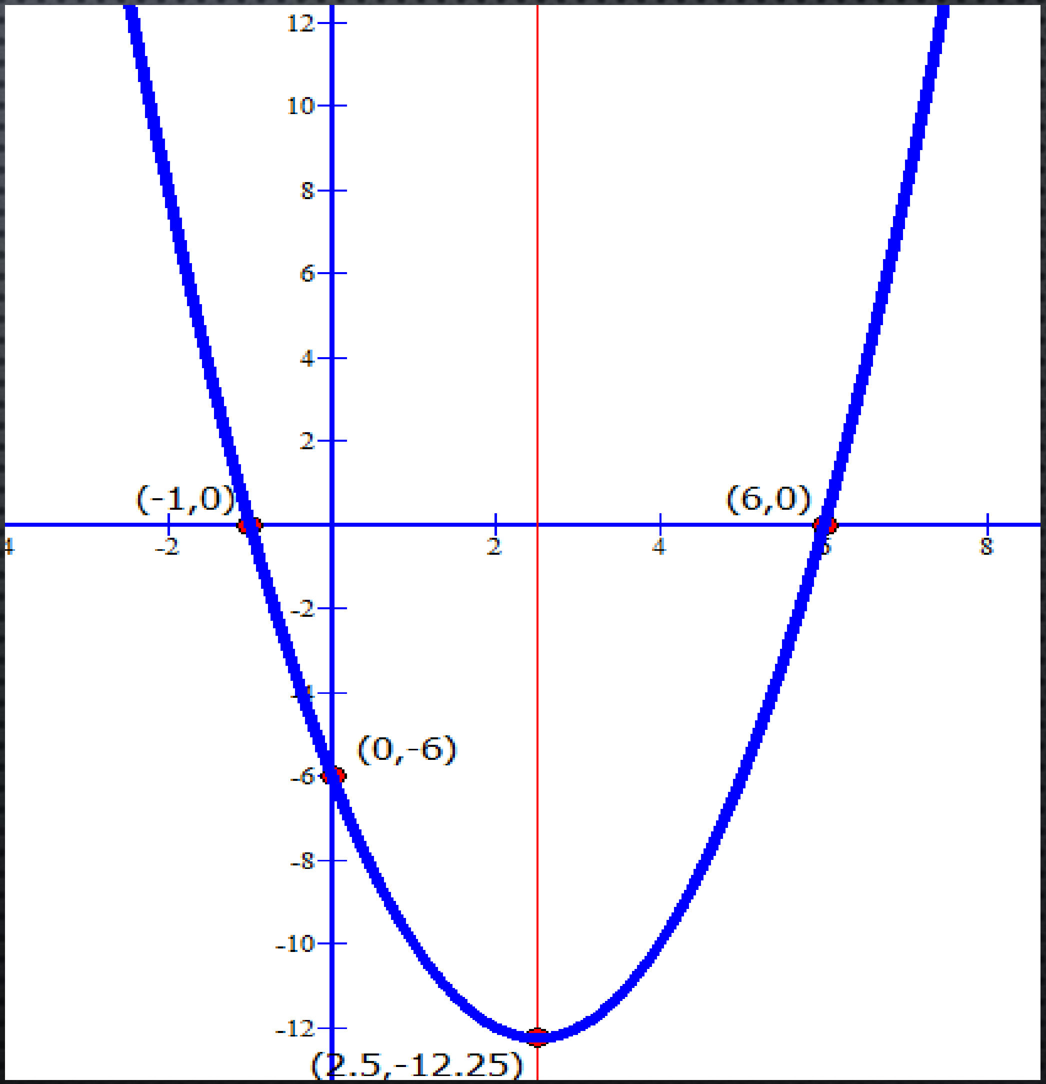
   $y = (\frac{5}{2})^2 - 5.(\frac{5}{2}) - 6$

   $y = \frac{25}{4} - \frac{25}{2} - 6$

   $y = -\frac{49}{4} = -12\frac{1}{4}$

   THEREFORE TURNING POINT IS $(2\frac{1}{2}; -12\frac{1}{4})$

PLOT THE POINTS TO DRAW THE GRAPH



GRAPHS – QUADRATIC GRAPHS (PARABOLA) – DETERMINE EQUATIONS

METHOD 1

Given the x-intercepts and one other point $(x; y)$

- Use the formula: $y = a(x - x_1)(x - x_2)$

x_1 and x_2 are the two x-intercepts

- Substitute the values of the x-intercepts.
- Substitute the given point $(x; y)$ which is not the x-intercept.
- Solve for *a*.
- Write the equation in the form $f(x) = ax^2 + bx + c$

METHOD 2

Given the turning point $(p; q)$ and one other point $(x; y)$

- Use the formula: $y = a(x + p)^2 + q$.
- Substitute the co-ordinates of the turning point (*p*; *q*).
- Substitute the given point (x ; y).
- Solve for *a*.
- Write the equation in the form $y = a(x + p)^2 + q$ or by multiplying out in the form $f(x) = ax^2 + bx + c$ depending on what is asked in the question

METHOD 3

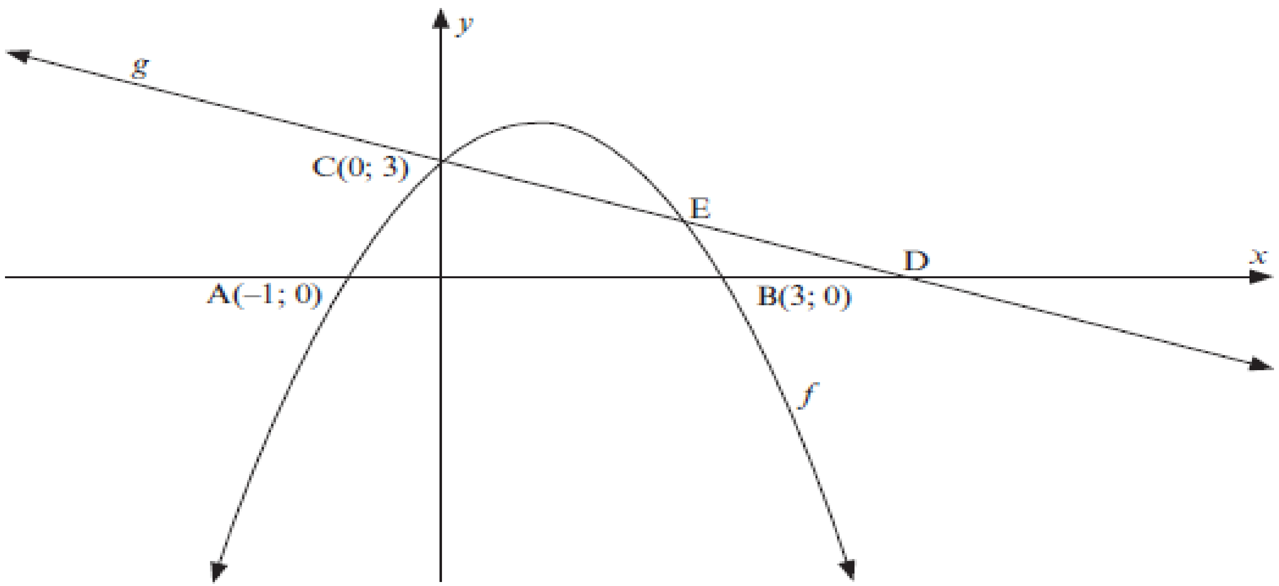
Given the co-ordinates of three points on the parabola

- Use the formula: $y = ax^2 + bx + c$.
- Substitute the coordinates of the points into $y = ax^2 + bx + c$.
- Solve the equations simultaneously for *a; b and c*.

GRAPHS – QUADRATIC GRAPHS (PARABOLA) – EXAMPLE 2

The sketch represents the graph of the parabola given by $f(x) = ax^2 + bx + c$ and the straight line defined by $g(x) = mx + c$

Points A, B ,C and D are the intercepts on the axes. E is the point of intersection of the two graphs.



2.1 Write down the co-ordinates of point D if D is the image of B after B has been translated two units to the right. (1)

2.2 Determine the equation of g. (3)

2.3 Determine the equation of the function f in the form $f(x) = ax^2 + bx + c$. (4)

2.4 Determine the coordinates of E. (4)

2.5 Write down the values of x for which $f(x) \geq g(x)$. (2)

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GRAPHS – QUADRATIC GRAPHS (PARABOLA) – ANSWER TO EXAMPLE 2

2.1 $D(5; 0)$ ✓ (1)

2.2 $g(x) = mx + 3$

$0 = m(5) + 3$ or $m_g = \frac{3-0}{0-5} = -\frac{3}{5}$ ✓

$m = -\frac{3}{5}$ ✓

$g(x) = -\frac{3}{5}x + 3$ ✓ (3)

2.3 $f(x) = a(x + 1)(x - 3)$ ✓

$3 = a(0 + 1)(0 - 3)$ ✓

$a = 1$ ✓

$f(x) = -(x + 1)(x - 3)$

$f(x) = -x^2 + 2x + 3$ ✓ (4)

2.4 $-\frac{3}{5}x + 3 = -x^2 + 2x + 3$ ✓

$x^2 - \frac{13}{5}x = 0$

$x\left(x - \frac{13}{5}\right) = 0$ ✓

$x = 0$ *or* $x = \frac{13}{5} = 2{,}60$ ✓

$g\left(\frac{13}{5}\right) = -\frac{3}{5}\left(\frac{13}{5}\right) + 3$

$= \frac{36}{25}$

$= 1{,}44$ ✓

$\therefore E\left(\frac{13}{5}, \frac{36}{25}\right)$ *or* $E\left(2\frac{3}{5}; 1\frac{11}{25}\right)$ *or* $E(2{,}60; 1{,}44)$ (4)

2.5 $0 \leq x \leq \frac{13}{5}$ ✓✓ (2)

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