INFORMAL TEST 3

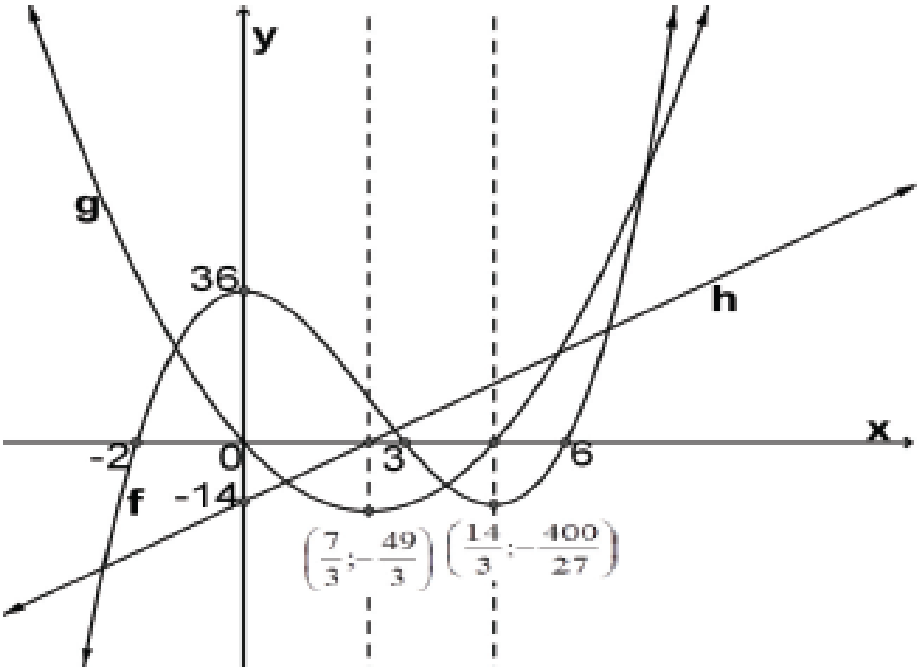
MEMO

3. INVESTIGATION 2 **TOTAL: 50**

MEMORANDUM: APPLICATIONS OF DIFFERENTIAL CALCULUS

CASE 1	
1. $f(x) = x^3 - 7x^2 + 36$	
1.1 y-intercept = 36 for x-intercepts: $(x+2)(x^2-9x+18)=0$ $(x+2)(x-3)(x-6)=0$ $x=-2 \text{ or } x=3 \text{ or } x=6$ $\therefore$ coordinates of the x-intercepts are $(-2;0)$, $(3;0)$ and $(6;0)$ For the turning points: $f'(x)=3x^2-14x$ $3x^2-14x=0$ $x(3x-14)=0$ $x=0 \text{ or } x=\frac{14}{3}$ $f(0)=36$ TP $(0;36)$ maximum $f\left(\frac{14}{3}\right)=\left(\frac{14}{3}\right)^3-7\left(\frac{14}{3}\right)^2+36$ $=-14\frac{22}{27}$ $\left(\frac{14}{3};-14\frac{22}{27}\right)$ minimum	Marks are only awarded on the graph.
1.2 $f'(x)=3x^2-14x$ $g(x)=3x^2-14x$	1 mark for the equation $g(x)=3x^2-14$✓
1.3 $g(x)=3x^2-14x$ y-intercept = 0 For the x-intercepts: $3x^2-14x=0$ $x(3x-14)=0$ $x=0 \text{ or } x=\frac{14}{3}$ $\therefore$ coordinates of the x-intercepts are $(0;0)$ and $\left(\frac{14}{3};0\right)$ For the turning point: $g'(x)=6x-14$	Marks are only awarded on the graph.

$6x - 14 = 0$ $x = \frac{14}{6}$ $x = \frac{7}{3}$ **OR** $x = \frac{-b}{2a}$ $x = \frac{-(-14)}{2(3)}$ $x = \frac{14}{6}$ $x = \frac{7}{3}$ $g\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\left(\frac{7}{3}\right)$ $= \frac{-49}{3}$ TP $\left(2\frac{1}{3}; -16\frac{1}{3}\right)$	
1.4 $g(x) = 3x^2 - 14x$ $g'(x) = 6x - 14$ $h(x) = 6x - 14$ y-intercept $= -14$ For the x-intercepts: $6x - 14 = 0$ $x = \frac{7}{3}$	1 mark for the equation: $h(x) = 6x - 14$ ✓



All values are only marked from the graphs.

For f
x-intercepts: $x = -2$; $x = 3$ or $x = 6$ (1 mark for each x-intercept)
y-intercept: $y = 36$ (1 mark)

Turning point $(0; 36)$ (1 mark)
Turning point $\left(\frac{14}{3}, -14\frac{22}{27}\right)$ (1 mark for each coordinate)
Shape (1 mark)

(8)

For g
x-intercepts: $x = 0$ *or* $x = \frac{14}{3}$ (1 mark for each intercept)
Turning point $\left(\frac{7}{3}, -16\frac{1}{3}\right)$ (1 mark for both coordinates)

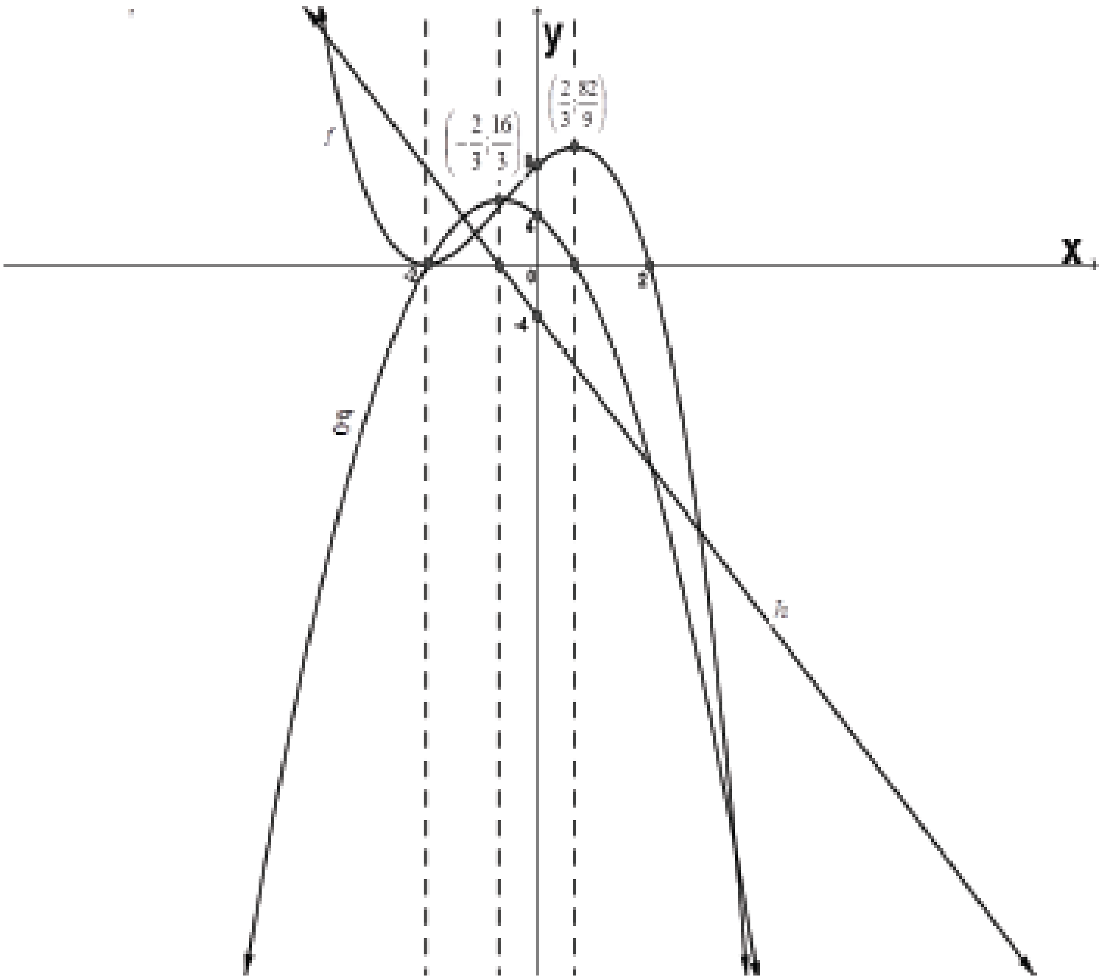
(3)

For h
x-intercept: $x = \frac{7}{3}$
y-intercept: $y = -14$ (1 mark for each) (2)

1.5 The x-intercepts of the quadratic function and the x-coordinate of the turning point of the cubic are equal, i.e. $x = 0$ and $x = \frac{14}{3}$	1 mark for the statement✓ (1)
1.6 $f''(x) = 6x - 14$ $6x - 14 = 0$ $x = \frac{7}{3}$	$f''(x) = 6x - 14$✓ $6x - 14 = 0$✓ Answer ✓ (3)
OR $\frac{x_1 + x_2}{2} = \frac{0 + \frac{14}{3}}{2}$ $= \frac{7}{3}$	✓ formula ✓ substitution ✓ answer
1.7 The axis of symmetry of g, the x-intercept of h and the point of inflection of f is $x = \frac{7}{3}$	✓ answer (1)
CASE 2	
2. $f(x) = -x^3 - 2x^2 + 4x + 8$	
2.1 y-intercept = 8 for x-intercepts: $(x + 2)(x^2 - 4) = 0$ $(x + 2)(x - 2)(x + 2) = 0$ $x = -2$ *or* $x = 2$ $\therefore$ coordinates of the x-intercepts are $(-2; 0)$ and $(2; 0)$ For the turning points: $f'(x) = -3x^2 - 4x + 4$ $-3x^2 - 4x + 4 = 0$ $3x^2 + 4x - 4 = 0$ $(3x - 2)(x + 2) = 0$ $x = \frac{2}{3}$ or $x = -2$ $f(-2) = 0$ TP $(-2; 0)$ minimum $f\left(\frac{2}{3}\right) = -\left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) + 8$ $= 9\frac{1}{9}$ TP $\left(\frac{2}{3}; 9\frac{1}{9}\right)$ maximum	Marks are only awarded on the graph.

2.2 $f'(x) = -3x^2 - 4x + 4$ $g(x) = -3x^2 - 4x + 4$	1 mark for equation of $g(x) = -3x^2 - 4x + 4$ ✓
2.3 y-intercept = 4 x-intercept $-3x^2 - 4x + 4 = 0$ $(3x - 2)(x + 2) = 0$ $x = -2$ or $x = \frac{2}{3}$ TP $g'(x) = -6x - 4$ $-6x - 4 = 0$ $x = \frac{-4}{6}$ $x = \frac{-2}{3}$ $g\left(\frac{-2}{3}\right) = -3\left(\frac{-2}{3}\right)^2 - 4\left(\frac{-2}{3}\right) + 4$ $g\left(\frac{-2}{3}\right) = 5\frac{1}{3}$ TP $\left(\frac{-2}{3}; \frac{16}{3}\right)$ **OR** $x = \frac{-b}{2a}$ $x = \frac{-(-4)}{2(-3)}$ $x = \frac{-2}{3}$ $g\left(\frac{-2}{3}\right) = -3\left(\frac{-2}{3}\right)^2 - 4\left(\frac{-2}{3}\right) + 4$ TP $\left(\frac{-2}{3}; \frac{16}{3}\right)$	Marks are only awarded on the graph.
2.4 $f''(x) = -6x - 4$ $h(x) = -6x - 4$ y-intercept = -4 x-intercept $-6x - 4 = 0$ $x = \frac{-2}{3}$	1 mark only for equation $h(x) = -6x - 4$✓ (1) Other marks are awarded on the graph.
2.5 The x-intercepts of the quadratic function and the x-coordinate of the turning points of the cubic are equal, i.e. $x = \frac{2}{3}$ and $x = -2$	1 mark for the statement✓ (1)

2.6 $$f''(x) = -6x - 4$$ $$-6x - 4 = 0$$ $$x = \frac{-2}{3}$$	$f''(x) = -6x - 4$ ✓ $-6x - 4 = 0$ ✓ $x = \frac{-2}{3}$ ✓ (3)
OR $$\frac{x_1 + x_2}{2} = \frac{-2 + \frac{2}{3}}{2}$$ $$= \frac{-2}{3}$$	✓ formula ✓ substitution ✓ answer
2.7 The axis of symmetry of g, the x-intercept of h and the x-coordinate of the point of inflection of f are $$x = \frac{-2}{3}$$	✓ answer (1)
3. The point of inflection of the cubic function is the same as the axis of symmetry of the graph of the first derivative and also the x-intercept of the graph of the second derivative	✓ ✓ conclusion (2)
4.	
4.1.1 for increasing: $x < 2$ *or* $x > 4$	$x < 2$ ✓ $x > 4$ ✓ (2)
4.1.2 for decreasing: $2 < x < 4$	For both values of x ✓ For correct inequality ✓ (2)
4.2 The x-values of the turning points $x = 2$ $x = 4$	$x = 2$ ✓ $x = 4$ ✓ (2)
4.3 $x = 2$ is the relative maximum since for f increasing $x < 2$ $x = 4$ is the relative minimum since for f increasing $x > 4$	$x = 2$ maximum ✓ $x = 4$ minimum ✓ (2)



All values are only marked from the graphs.

2.1 For f

Each x-intercept 1 mark $x = 2$ and $x = -2$ ✓✓ (2 marks)

y-intercept 1 mark $y = 8$ ✓

For the turning point (-2; 0) 1 mark ✓

For the turning point (0, 67; 9,11) or $\left(\frac{2}{3}; 9\frac{1}{9}\right)$ 1 mark for x-coordinate and 1 mark for y-coordinate ✓✓(2 marks)

Shape of the graph 1 mark ✓ (7)

2.3 For g

x-intercepts: $x = -2$ ✓ and $x = \frac{2}{3}$ ✓ (1 mark for each)

y-intercept: $y = 4$ ✓ (1 mark)

Turning point $\left(\frac{-2}{3}, 5\frac{1}{3}\right)$ ✓ (1 mark both coordinates) (4)

2.4 For h

x-intercept: $x = \frac{-2}{3}$ ✓ (1 mark)

y-intercept: $y = -4$ ✓ (1 mark) (2)