INFORMAL TEST 3

INSTRUCTIONS

1. Answer all the questions.
2. Clearly show all calculations you have used in determining your answers.
3. Round answers off to TWO decimal places, unless stated otherwise.
4. Number your answers correctly according to the numbering system used in this question paper.
5. Write neatly and legibly.

OBJECTIVE: Investigating the point of inflection of a cubic graph and its relationship with the graphs of the first and the second derivatives.

CASE 1

Given: $f(x) = x^3 - 7x^2 + 36$

1.1 Draw the graph of f neatly on graph paper. Clearly indicate all intercepts and coordinates of turning points. (8)

1.2 Determine the first derivative of f, and name it g. (1)

1.3 Draw the graph of g on the same set of axes as f. Clearly show all intercepts and the turning point. (3)

1.4 Determine the second derivative of f and name it h. Then sketch the graph of h on the same set of axes as f and g. Clearly show all intercepts of the graph with the axes. (3)

1.5 What do you notice regarding the x-intercepts of the quadratic function and the x-coordinates of the turning points of the cubic function? (1)

1.6 The point of inflection can be determined by solving $f''(x) = 0$. It can also be determined by calculating the midpoint of the turning points of the cubic graph. Hence, determine the point of inflection of f. (3)

1.7 What do you notice regarding the axis of symmetry of g, the x-intercept of h and the x-coordinate of the point of inflection of f? (1)

[20]

CASE 2

Given: $f(x) = -x^3 - 2x^2 + 4x + 8$

2.1 Draw the graph of f neatly on graph paper. Clearly indicate all intercepts and coordinates of turning points. (7)

2.2 Determine the first derivative of f, and name it g. (1)

2.3 Draw the graph of g on the same set of axes as f, and clearly show all intercepts and the turning point. (4)

2.4 Determine the second derivative of f and name it h, then sketch the graph of h on the same set of axes as f and g. Clearly show all intercepts of the graph with the axes. (3)

2.5 What do you notice regarding the x-intercepts of the quadratic function and the (1)

x-coordinates of the turning points of the cubic function?

2.6 The point of inflection can be determined by solving $f''(x) = 0$. It can also be determined by calculating the midpoint of the turning points of the cubic graph. Hence, determine the point of inflection of f. (3)

2.7 What do you notice regarding the axis of symmetry of g, the x-intercept of h and the x-coordinate of the point of inflection of f? (1)

[20]

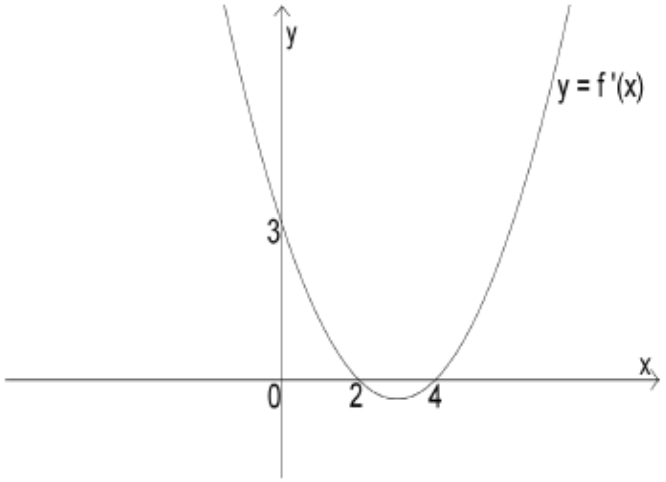
3. CONCLUSION

Based on the two cases, what conclusion can you draw about the point of inflection of a cubic function in relation to the graphs of the first and second derivatives?

[2]

4. APPLICATION

The parabola shown below is the graph of the derivative of a function f.



4.1 For what value(s) of x is f:

4.1.1 Increasing (2)

4.1.2 Decreasing (2)

4.2 Give the abscissae of the turning point(s) of $y = f(x)$. (2)

4.3 Classify the stationary point(s). (2)

[8]

TOTAL: 50