

Theorem 1

The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

line from centre \perp to chord

GIVEN: $OB \perp AC$

To Prove: $AB = BC$

Construction: Draw OA and OC

Hint
Do congruency

Given:
 $OB \perp AC$

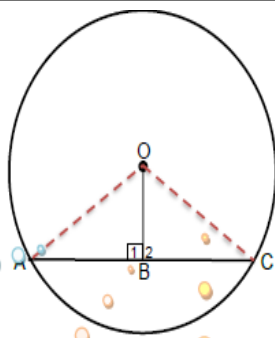
Construction:
Draw OC and OA

Proof:

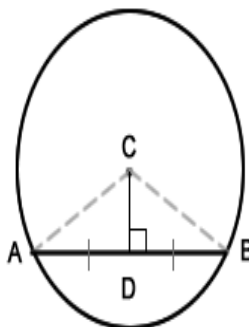
In $\triangle OAB$ and $\triangle OCB$

- (i) $OA = OC$ [radii]
- (ii) $OB = OB$ [common]
- (iii) $\hat{B}_1 = \hat{B}_2 = 90^\circ$ [given; $OB \perp AC$]

$\therefore \triangle OAB \cong \triangle OCB$ [hyp; 90° \angle ; side]
 $AB = BC$ [from congruency]



Theorem 2



The perpendicular bisector of a chord contains the centre of the circle

Statement

Reason

In $\triangle CAD$ and $\triangle CBD$,

$AD = DB$

Given

$\angle CDA = \angle CDB$

$CD \perp$ to AB

$CD = CD$

Reflexive property

$\triangle CAD$ and $\triangle CBD$

SAS

$CA = CB$

Congruent triangles

Q is center of circle

The center of the circle is the only point within the circle that has points on the circumference equal distance from it.

Theorem 3

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).

\angle at centre = $2 \times \angle$ at circumference

Given: Circle with centre O and arc AB subtended \hat{AOB} at the centre and \hat{ACB} on the circumference.

To Prove: $\hat{BOC} = 2\hat{BAC}$

Construction: Draw AO extended.

Proof:

In diagram A, B and C:

Let $\hat{A}_1 = x$ and $\hat{A}_2 = y$

$OA = OB$ [radii]

$\hat{A}_1 = \hat{B} = x$ [\angle s opp equal sides]

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext \angle of \triangle]

$\therefore \hat{O}_1 = 2x$

$\therefore \hat{O}_1 = 2\hat{A}_1$

Similarly $\hat{O}_2 = 2y$

$\therefore \hat{O}_2 = 2\hat{A}_2$

In diagrammes A and C:

$\therefore \hat{O}_1 + \hat{O}_2 = 2x + 2y$

$= 2(x + y)$

$\hat{AOB} = 2\hat{ACB}$

In diagram B :

$\hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

$\hat{AOB} = 2\hat{ACB}$

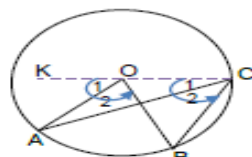
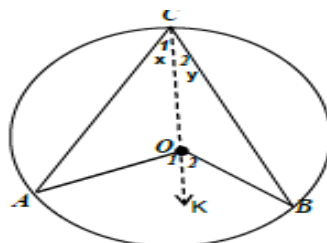


Diagram B

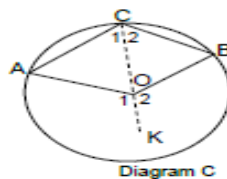
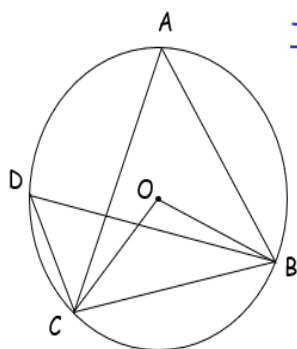


Diagram C

Hint:
ext \angle of \triangle isosceles \triangle

Theorem 4

To Prove that angles subtended by an arc or chord in the same segment are equal.

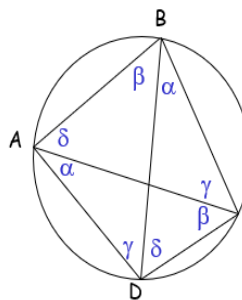


To prove that angle CAB = angle BDC

- With centre of circle O draw lines OB and OC.
- Angle COB = 2 x angle CAB (Theorem 1).
- Angle COB = 2 x angle BDC (Theorem 1).
- 2 x angle CAB = 2 x angle BDC
- Angle CAB = angle BDC

Theorem 5

To prove that the opposite angles of a cyclic quadrilateral are supplementary (Sum to 180°).



To prove that angles A + C and B + D = 180°

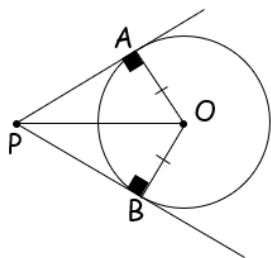
- Draw straight lines AC and BD
- Chord DC subtends equal angles α (same segment)
- Chord AD subtends equal angles β (same segment)
- Chord AB subtends equal angles γ (same segment)
- Chord BC subtends equal angles δ (same segment)
- $2(\alpha + \beta + \gamma + \delta) = 360^\circ$ (Angle sum quadrilateral)
- $\alpha + \beta + \gamma + \delta = 180^\circ$
- Angles A + C and B + D = 180° QED

α	β	γ	δ
alpha	beta	gamma	delta

Theorem 5

Theorem 6

To prove that the two tangents drawn from a point outside a circle are of equal length.

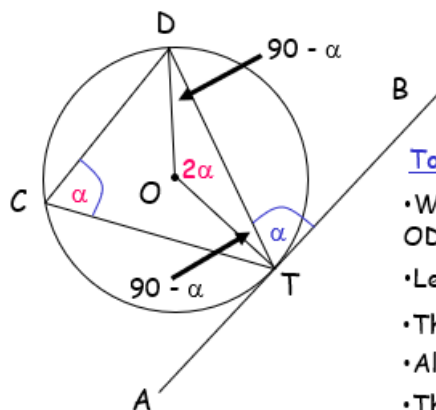


To prove that AP = BP.

- With centre of circle at O, draw straight lines OA and OB.
- OA = OB (radii of the same circle)
- Angle PAO = PBO = 90° (tangent radius).
- Draw straight line OP.
- In triangles OBP and OAP, OA = OB and OP is common to both.
- Triangles OBP and OAP are **congruent** (RHS)
- Therefore AP = BP. QED

Theorem 7

To prove that the angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.



To prove that angle BT D = angle TCD

- With centre of circle O, draw straight lines OD and OT.
- Let angle DTB be denoted by α .
- Then angle DTO = 90 - α (Theorem 4 tan/rad)
- Also angle TDO = 90 - α (Isos triangle)
- Therefore angle TOD = 180 - (90 - α + 90 - α) = 2 α (angle sum triangle)

• Angle TCD = α (Theorem 1 angle at the centre)

• Angle BT D = angle TCD QED