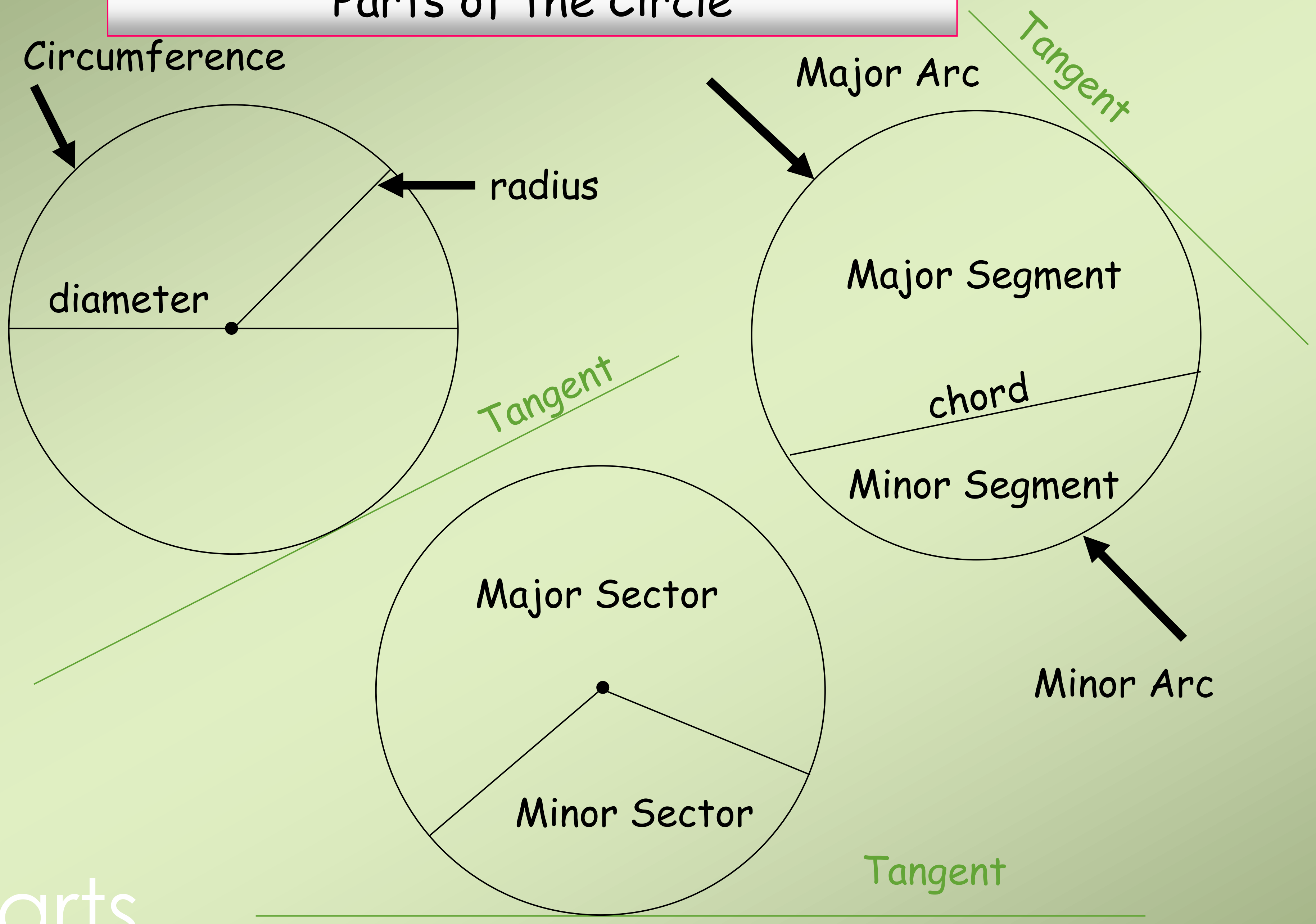
GEOMETRY GR 11

THEOREMS FOR GRADE 11 and GRADE 12 –PART 4

THEOREM 7

Parts of the Circle

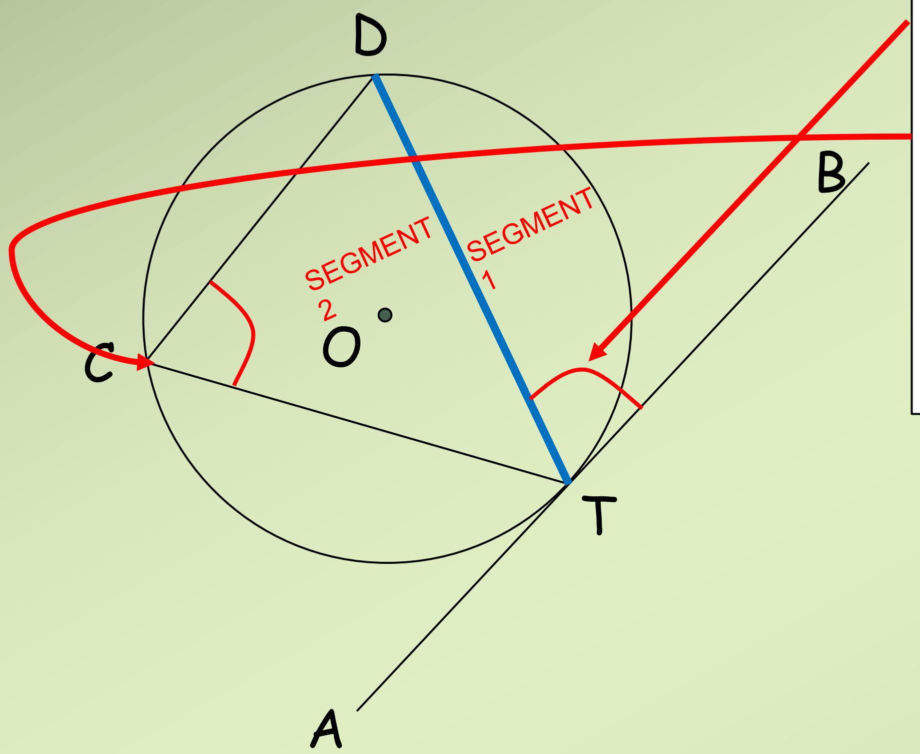


Parts

Understand what is being asked in Theorem 7:

Angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment.

To prove that $B\hat{T}D = T\hat{C}D$



$B\hat{T}D$ is the angle between the tangent ATB and Chord TD.

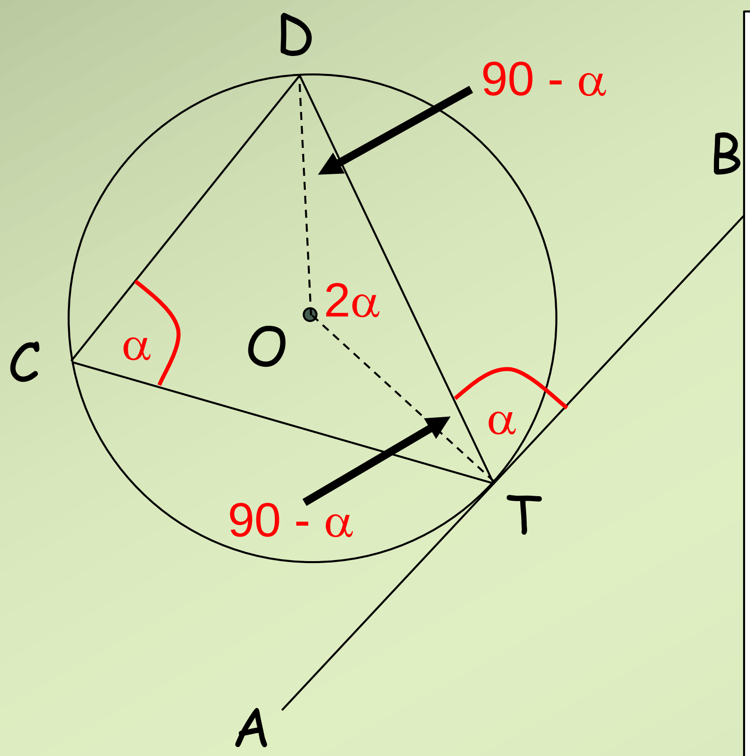
$T\hat{C}D$ is the angle that chord DT makes in the alternate segment at circumference

REMEMBER THAT A CHORD DIVIDES A CIRCLE INTO TWO SEGMENTS.
SO $T\hat{C}D$ *IS IN AN ALTERNATE SEGMENT TO* $B\hat{T}D$

Theorem 7:

Angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment.

To prove that $B\hat{T}D = T\hat{C}D$



Given: Circle with centre O

Proof

Construction: Draw OD and OT

Let angle $D\hat{T}B$ be α.

$O\hat{T}B = 90°$ (Tan ⊥ Radius)

$\therefore D\hat{T}O = 90° - \alpha$

$T\hat{D}O = 90° - \alpha$ (Angles opp equal sides)

$\therefore D\hat{O}T + D\hat{T}O + T\hat{D}O = 180°$ (angles of Δ)

$\therefore D\hat{O}T + (90° - \alpha) + (90° - \alpha) = 180°$

$\therefore D\hat{O}T = 180° - (90° - \alpha) - (90° - \alpha)$

$\therefore D\hat{O}T = 2\alpha$

$\therefore T\hat{C}D = \alpha$ (angle at centre = 2 × angle at circumference)

$\therefore B\hat{T}D = T\hat{C}D$

ATB is the tangent and OT is the radius

OT and OD are equal in length –radius therefore the opp angles in triangle are equal. $D\hat{T}O = T\hat{D}O$

$D\hat{O}T$ is an angle at the centre of circle

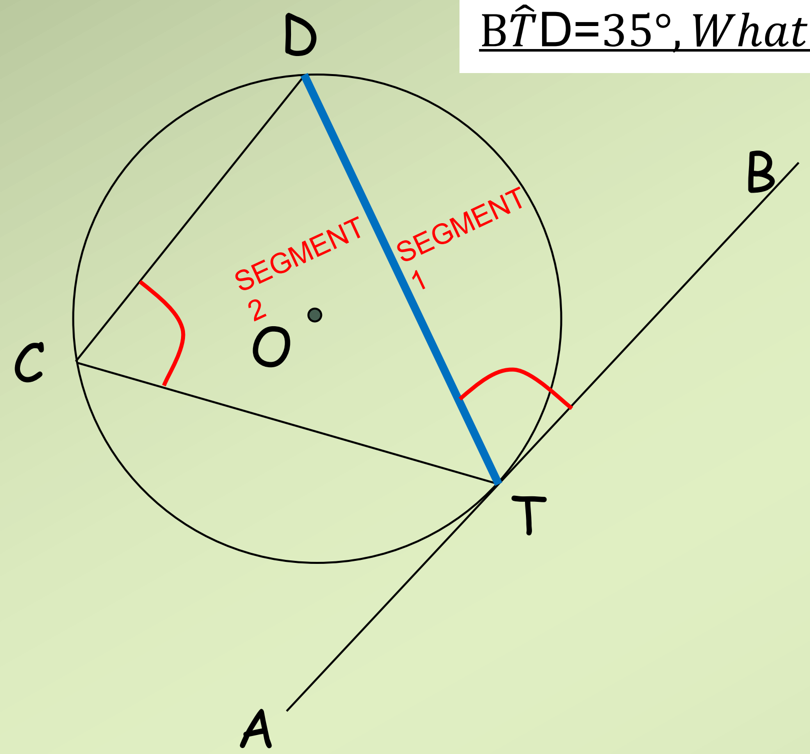
Both equal to α

Examples for Theorem 7:

Angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment.

To prove that

$B\hat{T}D = 35°, What\ is\ the\ value\ of\ D\hat{C}T$

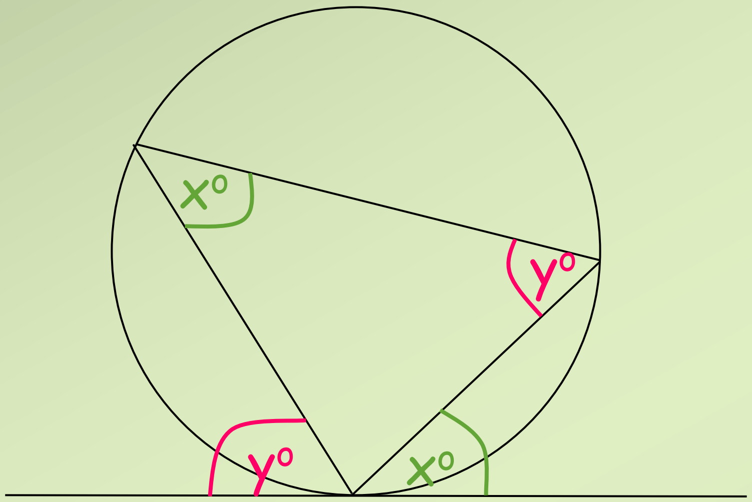


Tan-Chord Theorem is the reason used in an application when applying Theorem 7.

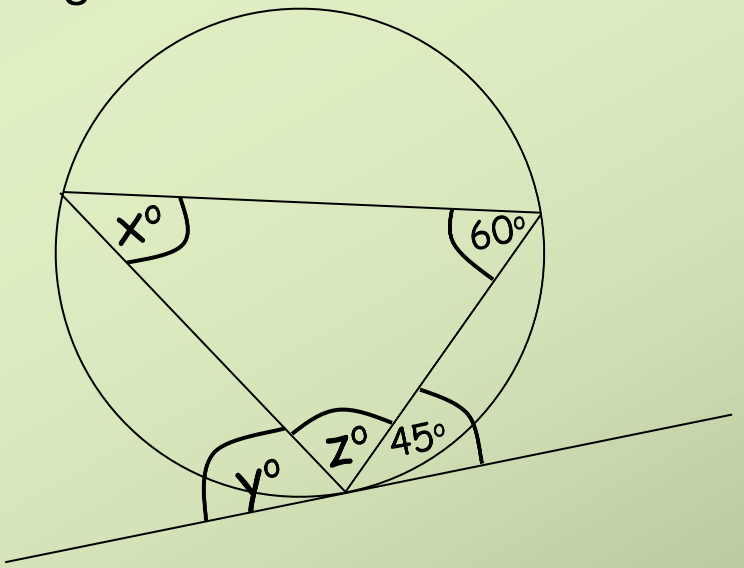
Angle between a tangent and a chord at the point of contact is equal to the angle subtended by the chord in the alternate segment.

$B\hat{T}D = 35°$	(Given)
$B\hat{T}D = T\hat{C}D$	(Tan-Chord Theorem)

Examples- Applications



Find the missing angles below giving reasons in each case.

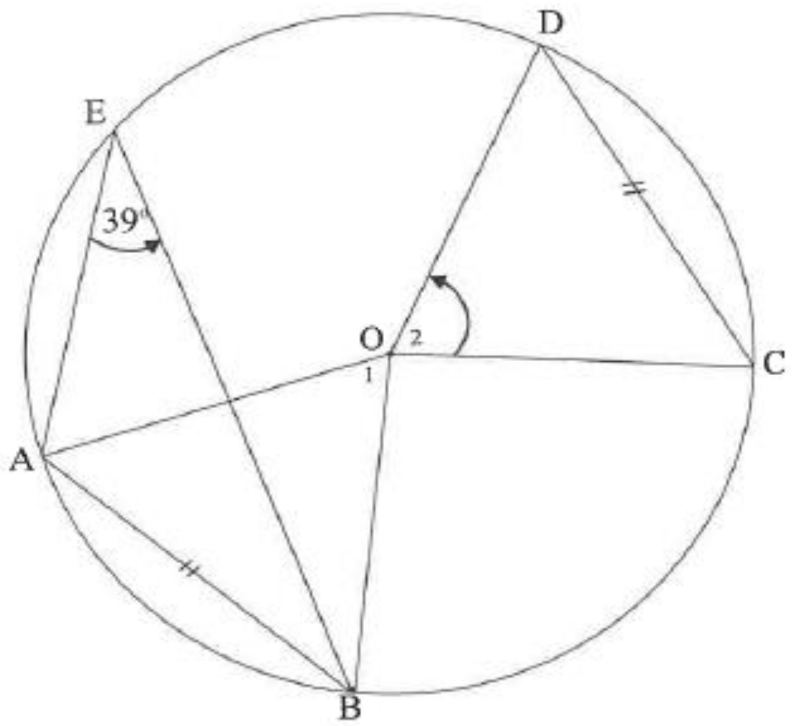


angle x =	45° (Tan Chord Theorem)
angle y =	60° (Tan Chord Theorem)
angle z =	75° (Angles of Triangle)

Example 1- Applications PAST PAPER QUESTIONS

QUESTION 9

9.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $A\hat{E}B = 39°$.



NOTE:

DC = AB which means that $\hat{O}_1 = \hat{O}_2$ BECAUSE OF **EQUAL CHORDS SUBTENDS EQUAL ANGLES**.

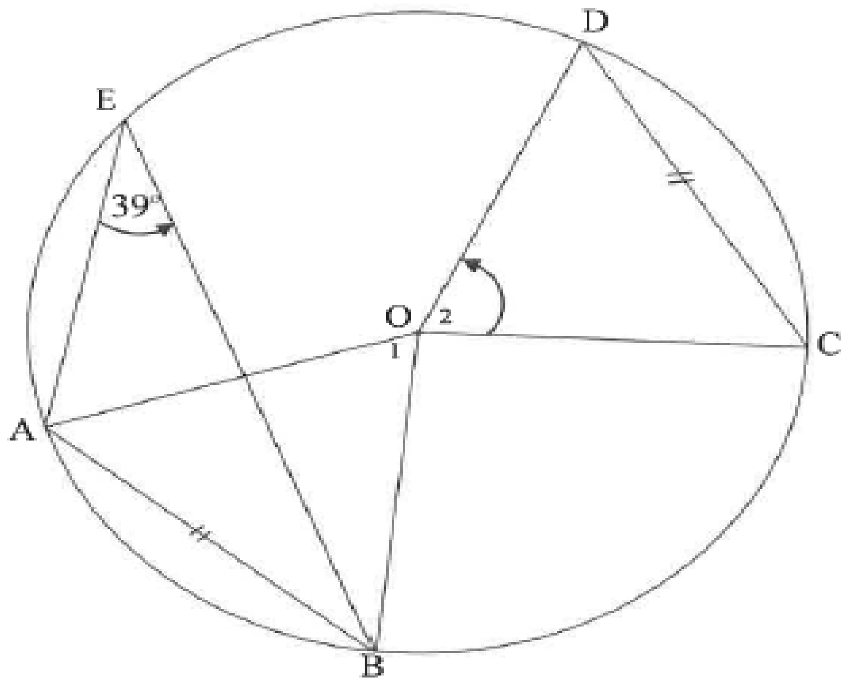
DO NOT USE VERTICALLY OPPOSITE ANGLES BECAUSE DOB and AOC ARE NOT STRAIGHT LINES

9.1.1 Determine the size of $\hat{O}_1$. (2)

9.1.2 Determine the size of $\hat{O}_2$. (2)

Answer to Example1- Applications PAST PAPER QUESTIONS

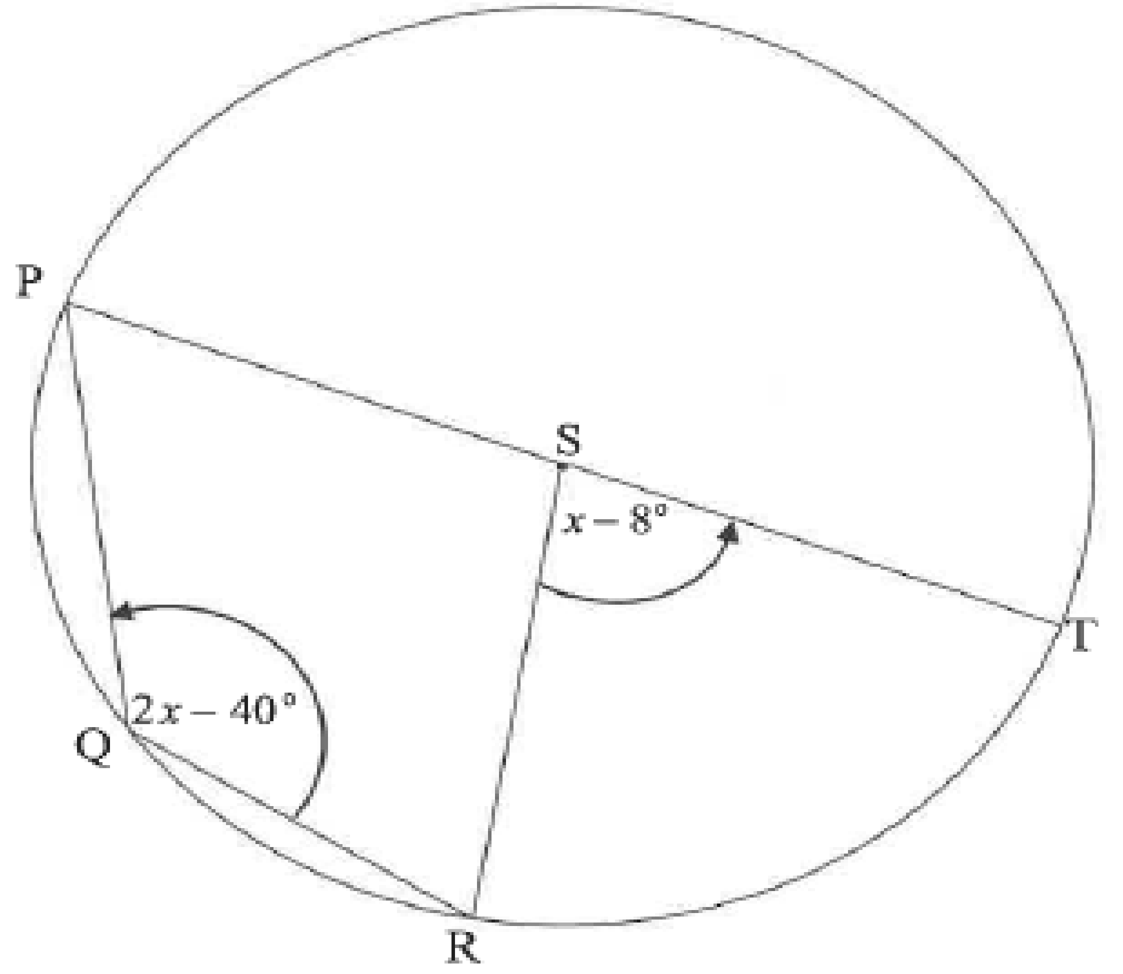
QUESTION/*VRAAG* 9



9.1.1	$\hat{O}_1 = 78°$ [angle at centre = $2 \times \angle$ at circumference] [*middelpuntshoek* = $2 \times$ *omtrekshoek*]	✓ S ✓ R (2)
9.1.2	$\hat{O}_2 = 78°$ [equal chords; equal $\angle^s$ / *gelyke koorde; gelyke hoeke*]	✓ S ✓ R (2)

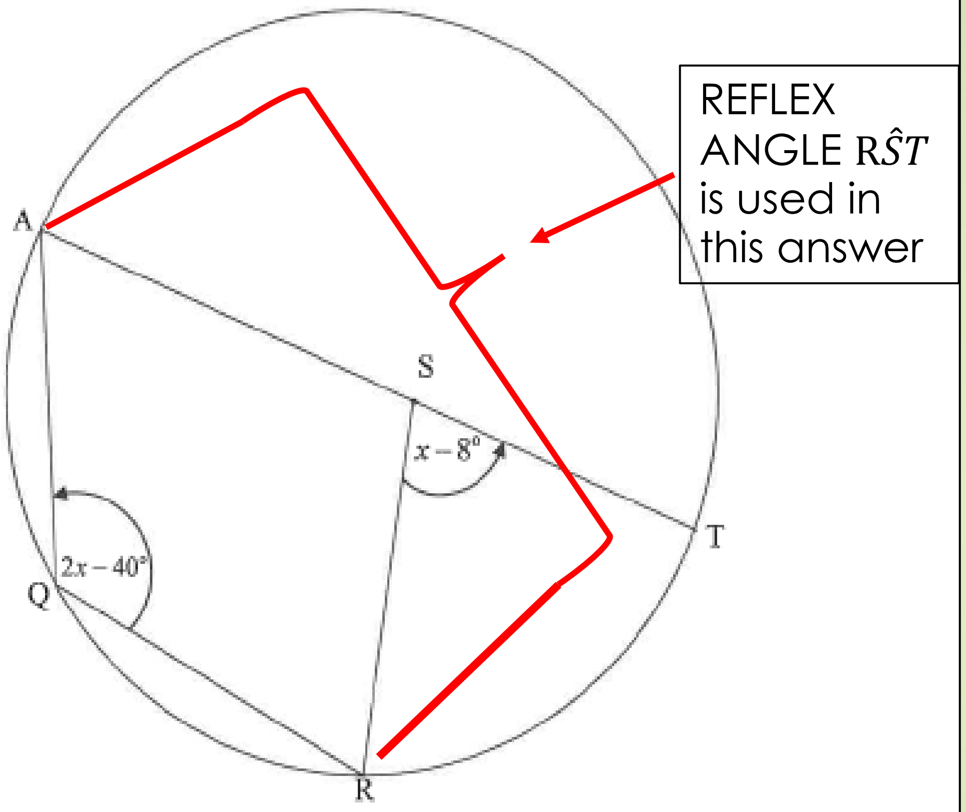
Example 2- Applications PAST PAPER QUESTIONS

9.2 In the diagram, S is the centre of circle PQRT. PT is a diameter.
$R\hat{S}T = x - 8°$ and $P\hat{Q}R = 2x - 40°$.



Determine the value of x. (4)

Answer to Example 2- Applications PAST PAPER QUESTIONS

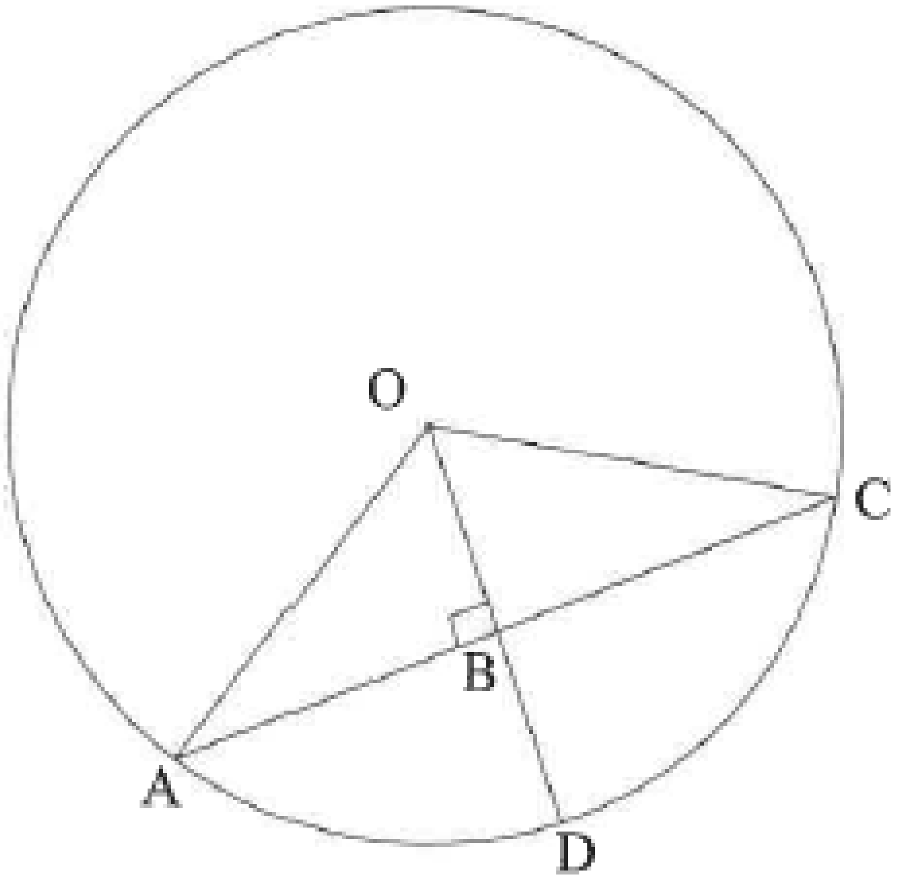


9.2	$x-8°+180° = 2(2x-40°)$ $4x-80° = 172°+x$ $3x = 252°$ $x = 84°$	[angle at centre = 2 × ∠ at circumference/ *middelpuntshoek = 2 × omtrekshoek*]	✓ S ✓ R ✓ simplification/ *vereenvoudiging* ✓ answer/

OR/OF Join T and R/ *verbind T en R* $\hat{T} = 180° - (2x - 40°)$ [opp ∠'s of cyclic quad/ *teenoorst. ∠e van koordevierhoek*] $\hat{R} = \hat{T} = 220° - 2x$ [∠s opp. = sides / ∠e teeoor gelyke sye] $x - 8° + 220° - 2x + 220° - 2x = 180°$ [*sum of* int ∠s *of* Δ / *som binne* ∠e *van* Δ] $-3x = -252°$ $x = 84°$	(4) ✓ S ✓ R ✓ S ✓ answer/ *antwoord*

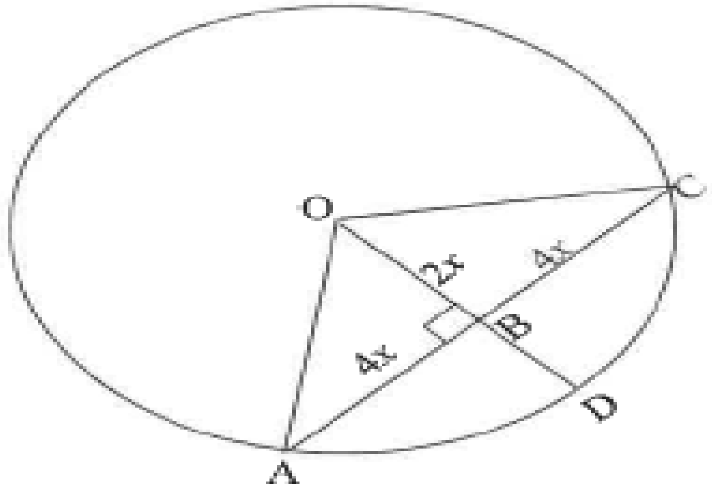
Example 3- Applications PAST PAPER QUESTIONS

9.3 In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B. OB = $2x$ units and AC = $8x$ units.



Show that the length of BD is $2x\left(\sqrt{5}-1\right)$ units. (5)

Answer to Example 3- Applications PAST PAPER QUESTIONS

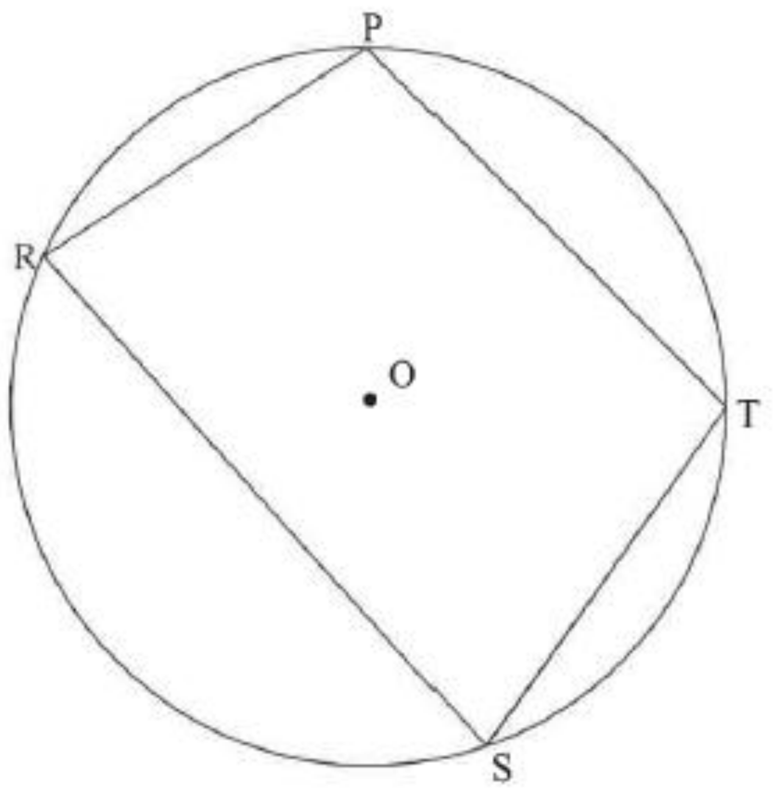


9.3	$AB = BC = 4x$ [line from centre ⊥ to chord/ *lyn van middelpunt ⊥ aan koord*] $OA^2 = (4x)^2 + (2x)^2$ [Pythagoras] $OA = \sqrt{16x^2 + 4x^2}$ $= \sqrt{20x^2}$ $= 2\sqrt{5}\,x$ $OD = OA = 2\sqrt{5}\,x$ [radii] $BD = 2\sqrt{5}\,x - 2x$ $= 2x(\sqrt{5} - 1)$	✓ S ✓ R ✓ Substitution/ *vervanging* ✓ length of OA / *lente van* OA ✓ $BD = 2\sqrt{5}\,x - 2x$ (5) **[13]**

Example 4- Applications PAST PAPER QUESTIONS

QUESTION 10

10.1 In the diagram below, O is the centre of the circle and PTSR is a cyclic quadrilateral.

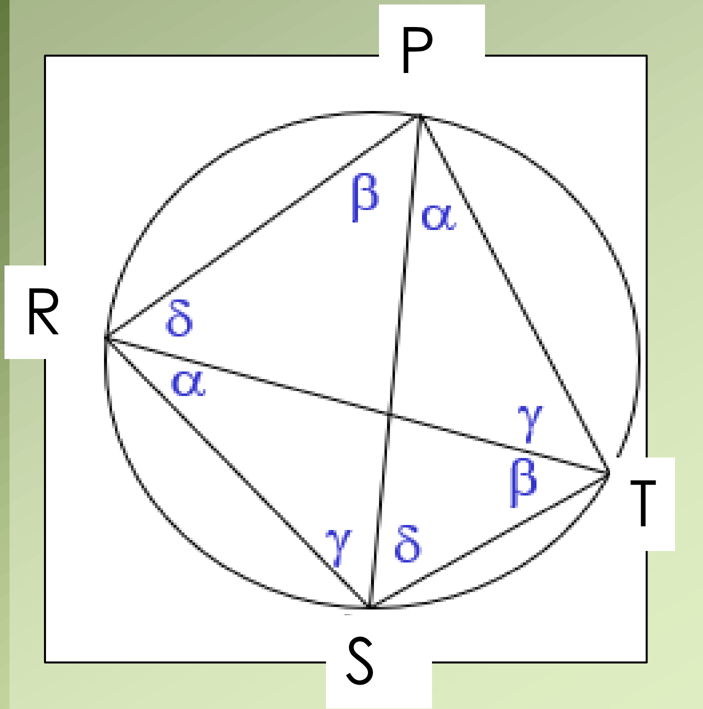


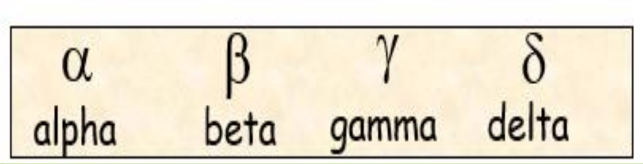
Prove the theorem that states that $\hat{P} + \hat{S} = 180°$. (5)

This is a Theorem that you need to know.

Opposite angles of a Cyclic Quad are supplementary

First possible Answer to Example 4- Applications PAST PAPER QUESTIONS





Opposite angles of cyclic quad are supplementary (adds up to 180^0)

<u>Given: Cyclic Quad PRST</u>

<u>Proof</u>

<u>*Construction:*</u> Draw PS and RT

<u>*USING CHORD ST*</u>

$S\hat{P}T = S\hat{R}T = \alpha$ (angle subtended by same chord or arc)

<u>*USING CHORD RS*</u>

$R\hat{T}S = R\hat{P}S = \beta$ (angle subtended by same chord or arc)

<u>*USING CHORD PR*</u>

$R\hat{T}P = P\hat{S}R = \gamma$ (angle subtended by same chord or arc)

<u>*USING CHORD PT*</u>

$P\hat{S}T = P\hat{R}T = \delta$ (angle subtended by same chord or arc)

$S\hat{P}T + S\hat{R}T + R\hat{T}S + R\hat{P}S + R\hat{T}P + P\hat{S}R + P\hat{S}T + P\hat{R}T = 360^0$ (Angles of Quad $= 360^0$)

$\alpha + \alpha + \beta + \beta + \gamma + \gamma + \delta + \delta = 360^0$

$2\alpha + 2\beta + 2\gamma + 2\delta = 360^0$

$2(\alpha + \beta + \gamma + \delta) = 360^0$

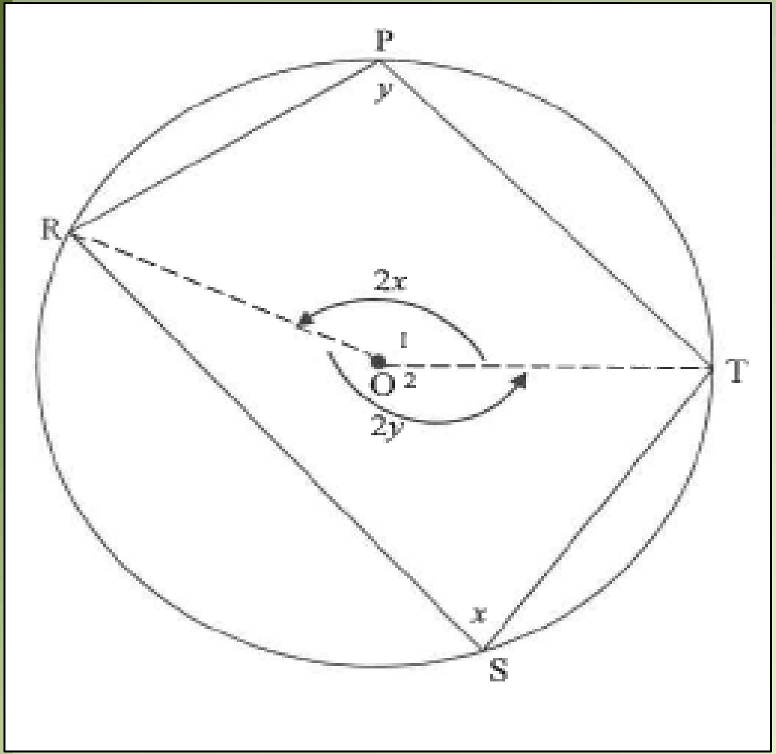
$\alpha + \beta + \gamma + \delta = 180^0$

$\therefore P\hat{R}S + P\hat{T}S = 180^0$ and $R\hat{P}T + R\hat{S}T = 180^0$

$\therefore \boldsymbol{\hat{P} + \hat{S} = 180°}$

Second possible Answer to Example 4- Applications PAST PAPER QUESTIONS

This is an alternate answer to the one that you learnt.

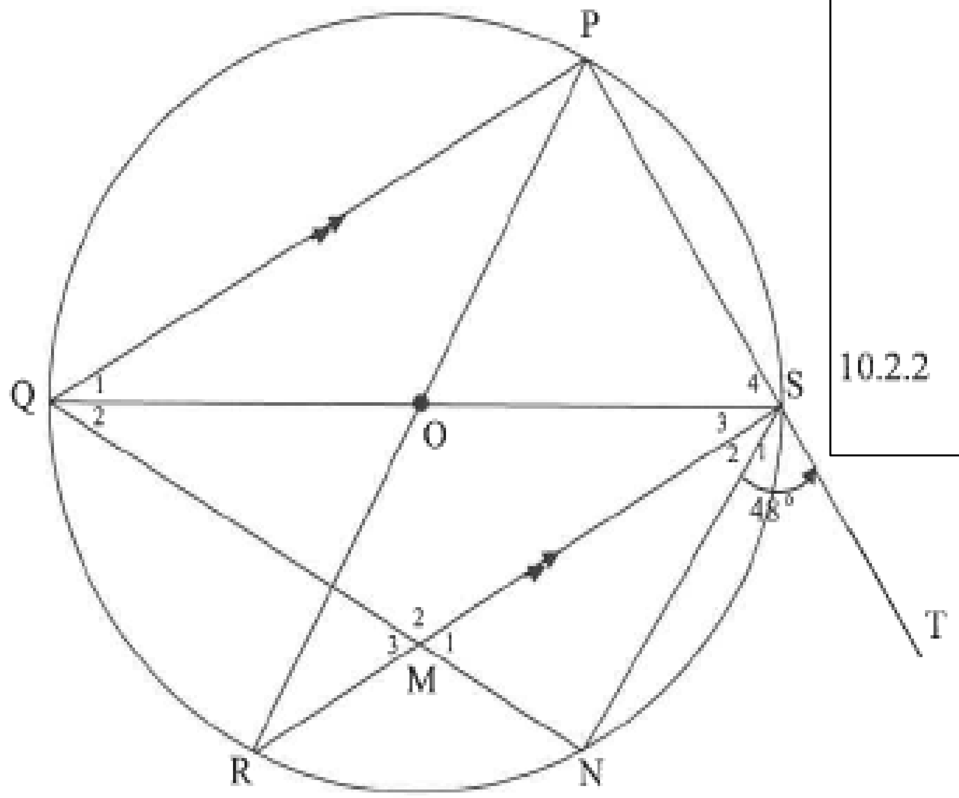


10.1	Construction: Draw radii OR and OT	✓ construction/
	Let $\hat{S} = x$ and $\hat{P} = y$	
	$\hat{O}_1 = 2\hat{S}$ [angle at centre = 2 times angle at circumference/]	✓ S ✓ R
	$\hat{O}_1 = 2x$	
	Similarly/ $\hat{O}_2 = 2y$	✓ S
	$2x + 2y = 360°$ [anlges around point $x + y = 180°$	✓ S/R
	$\therefore \hat{S} + \hat{P} = 180°$	

Example 5- Applications PAST PAPER QUESTIONS

10.2 In the figure, QS and PR are diameters of the circle with centre O such that $PQ \parallel SR$. PS is produced to T. N is a point on the circle such that $\hat{Q}_1 = \hat{Q}_2$. SN is drawn.

RS intersects QN at M. $\hat{S}_1 = 48^\circ$



10.2.1 Determine, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

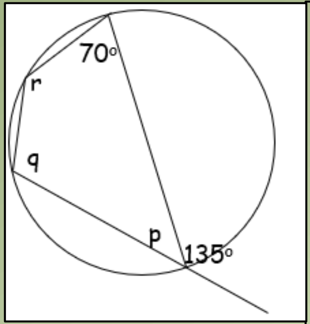
(b) $\hat{R}$ (2)

(c) $\hat{M}_1$ (2)

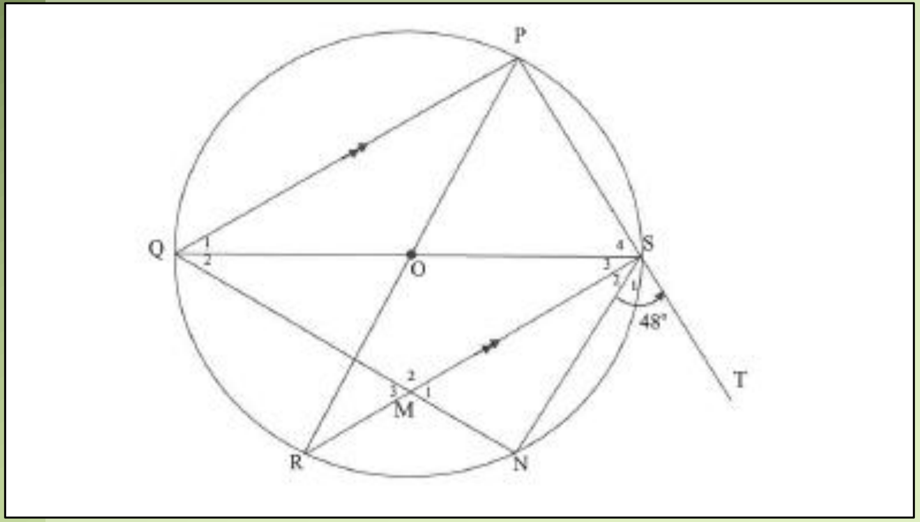
10.2.2 Prove that ST is a tangent to the circle passing through M, N and S. (2)

[14]

Answer to Example 5- Applications PAST PAPER QUESTIONS



SOMETHING TO NOTE:
Exterior Angle of a Cyclic is Equal to the interior opposite angle.
Example on left r = 135°.
This is used in 10.2.1 (a)



10.2.1(a)	$\hat{Q} = \hat{S}_1 = 48°$ [ext ∠ of cyclic quad/ *buite ∠ van 'n koodervierhoek*] $\hat{Q}_1 = \hat{Q}_2 = 24°$ [QS bisects/ *halveer* $P\hat{Q}N$]	✓S ✓R ✓S
10.2.1(b)	$\hat{R} = \hat{Q}_1 = 24°$ [∠s in the same segment/ *in dieselfde segment*]	✓S ✓R
10.2.1(c)	$\hat{M}_1 = \hat{Q} = 48°$ [corresp/ *ooreenkomst* ∠s , PQ || SR]	✓S ✓R

10.2.2	$\hat{M}_1 = \hat{S}_1 = 48°$ ∴ ST is a tangent to circle MNS.[converse tan − chord theorem] ∴ *ST is 'n raaklyn aan MNS* [*omgekrd raaklyn-koord st.*]	✓S ✓R