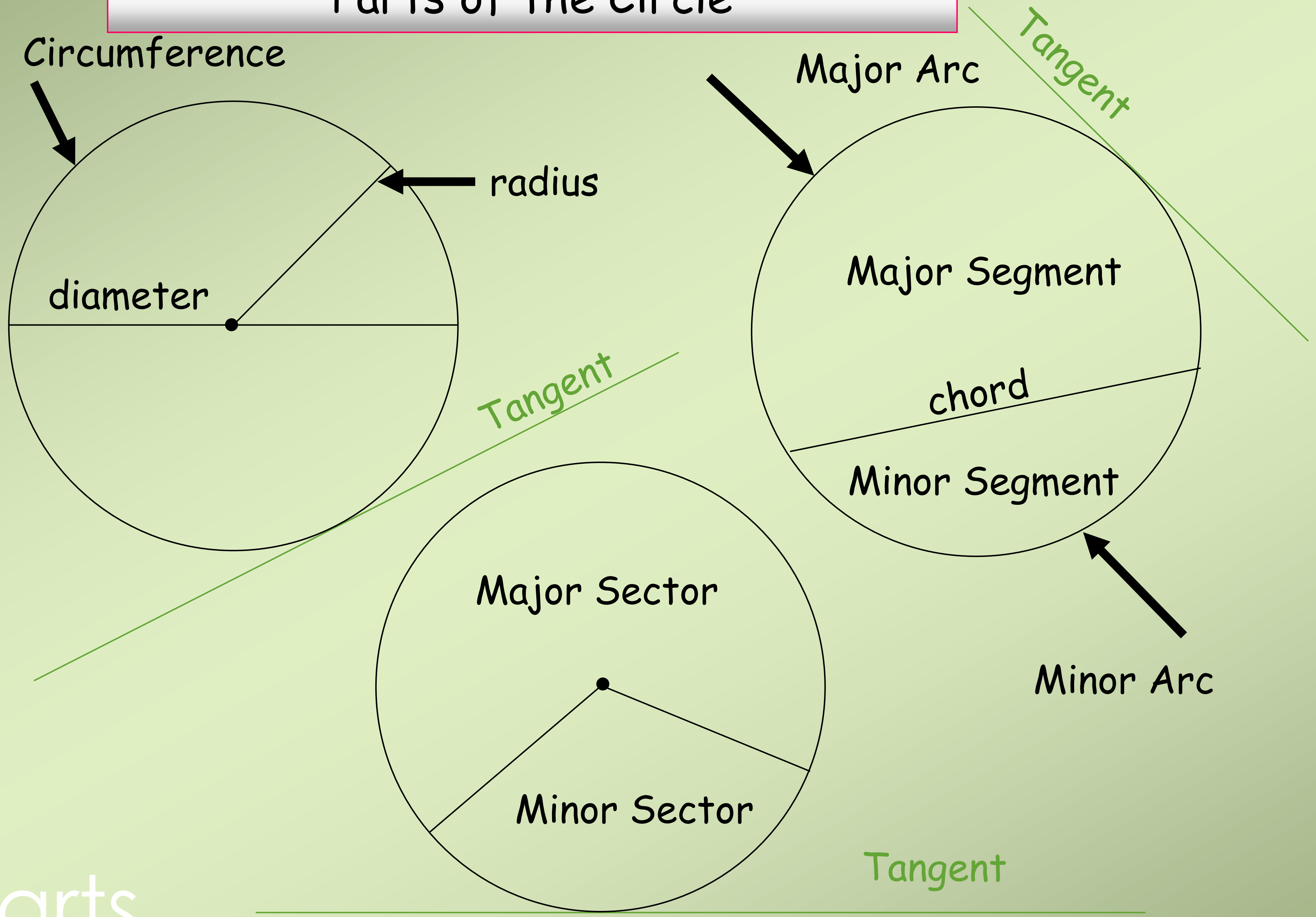
GEOMETRY GR 11

THEOREMS FOR GRADE 11 and GRADE 12 –PART 2
THEOREM 4 and 5

Parts of the Circle



Parts

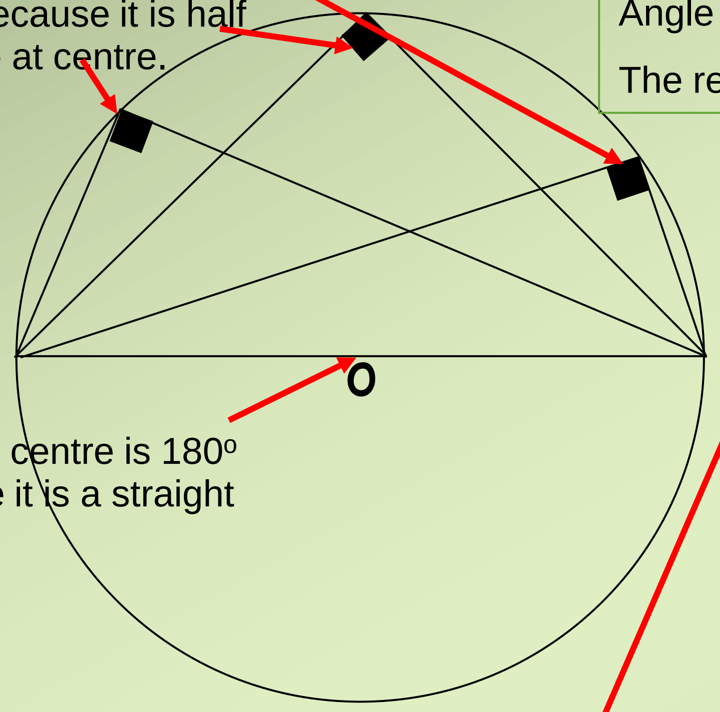
The angle in a semi-circle is a right angle.

This is just a special case of Theorem 3

Angle at centre equals 2 x angle at circle.

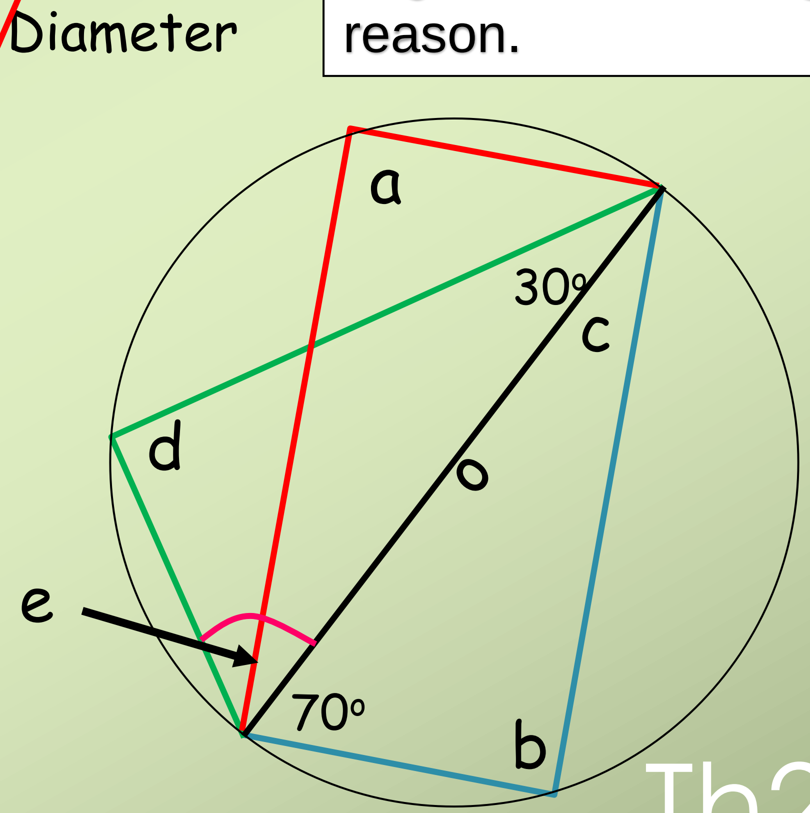
The reason angle in a semi-circle is used when doing applications

Angle at circumference is 90º because it is half of angle at centre.



Angle at centre is 180º because it is a straight line

Find the unknown angles below stating a reason.



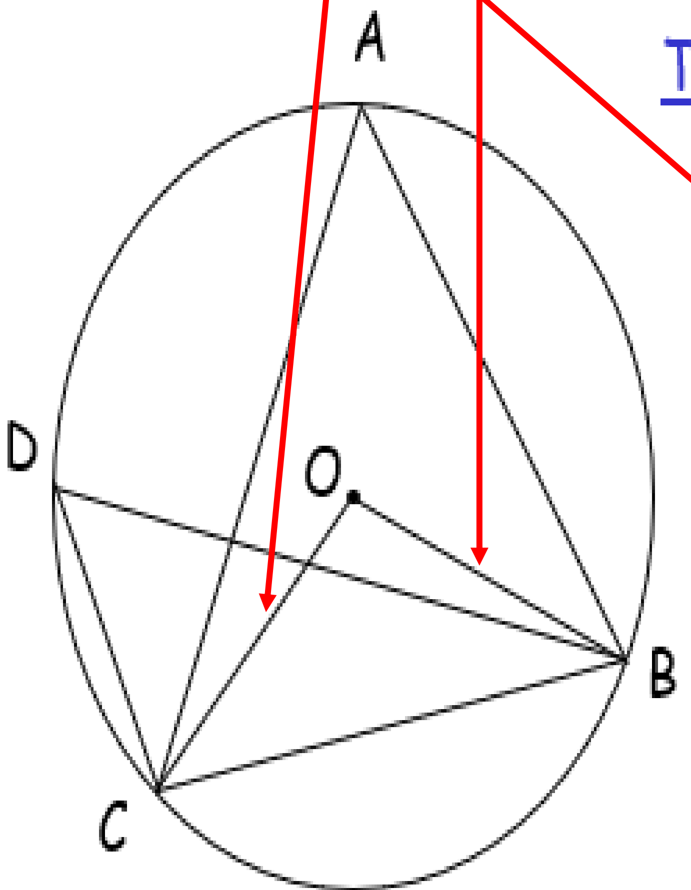
angle a =	90º angle in a semi-circle
angle b =	90º angle in a semi-circle
angle c =	20º angles of triangles
angle d =	90º angle in a semi-circle
angle e =	60º angles of triangles

Th2

Theorem 4 - Angles subtended by a chord of the circle, on the same side of the chord, are equal.

To Prove that angles subtended by an arc or chord in the same segment are equal.

Constructions (if not given)



To prove that angle CAB = angle BDC

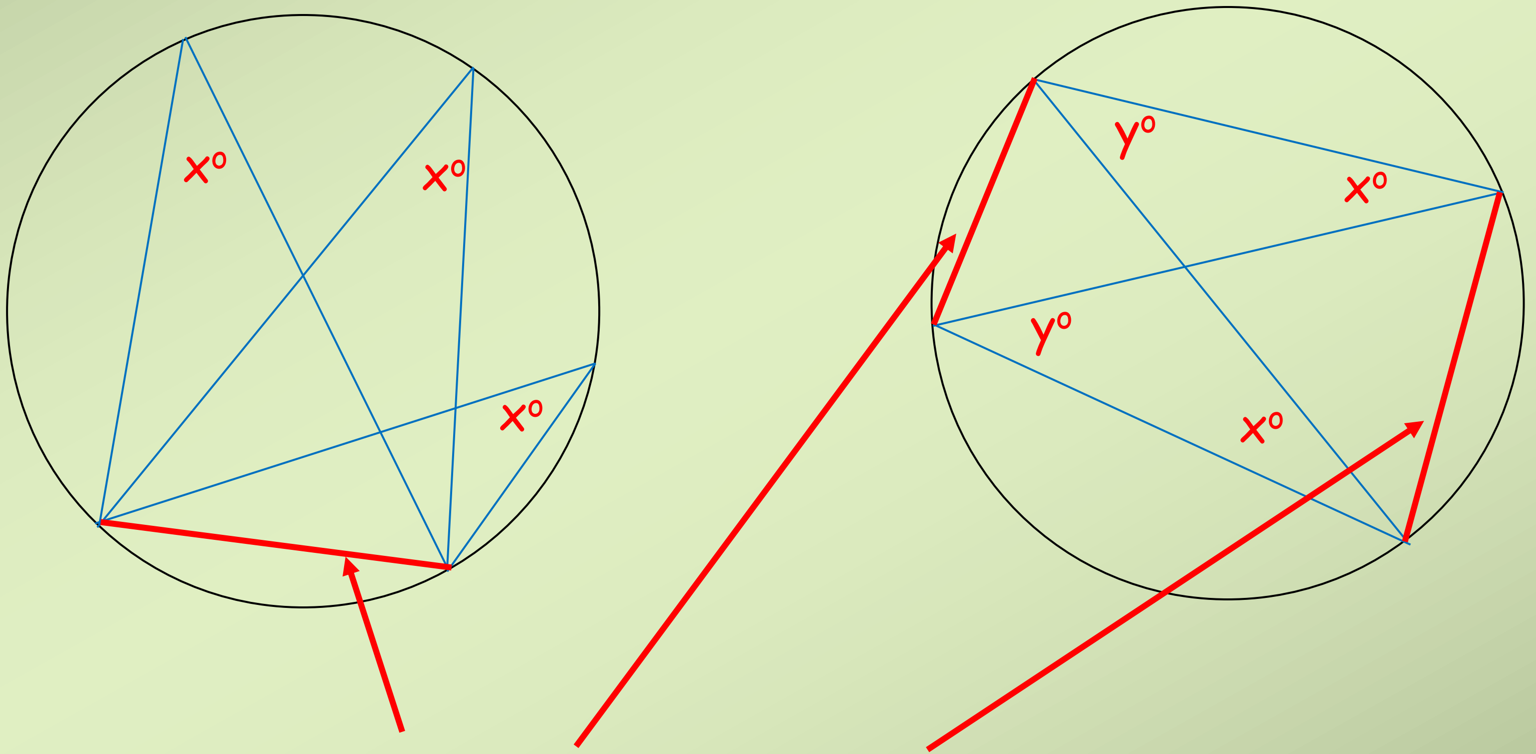
- With centre of circle O draw lines OB and OC.
- Angle COB = 2 x angle CAB   angle at centre = 2 x angle at circumference
- Angle COB = 2 x angle BDC   angle at centre = 2 x angle at circumference
- 2 x angle CAB = 2 x angle BDC
- Angle CAB = angle BDC

Use of angle at centre = 2 x angle at circumference to prove this theorem

Learn this theorem like the other theorems.

Theorem 4
Examples

Angles subtended by an arc or chord in the same segment are equal.

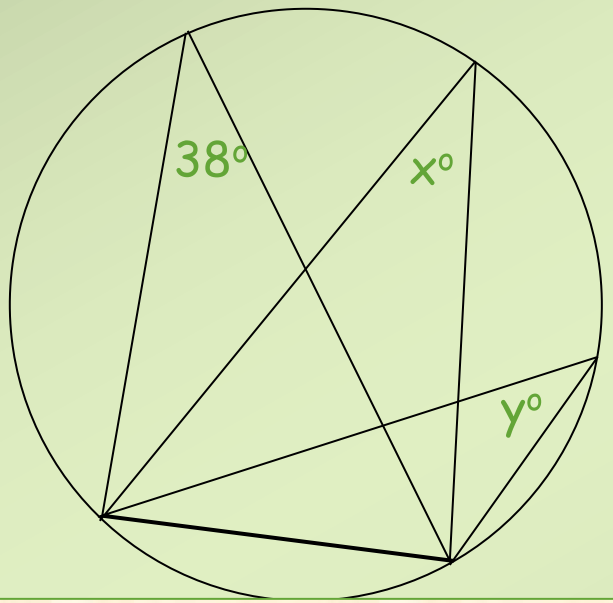


Red lines are the chords. The chords come together at points on circumference. These angles will be equal.

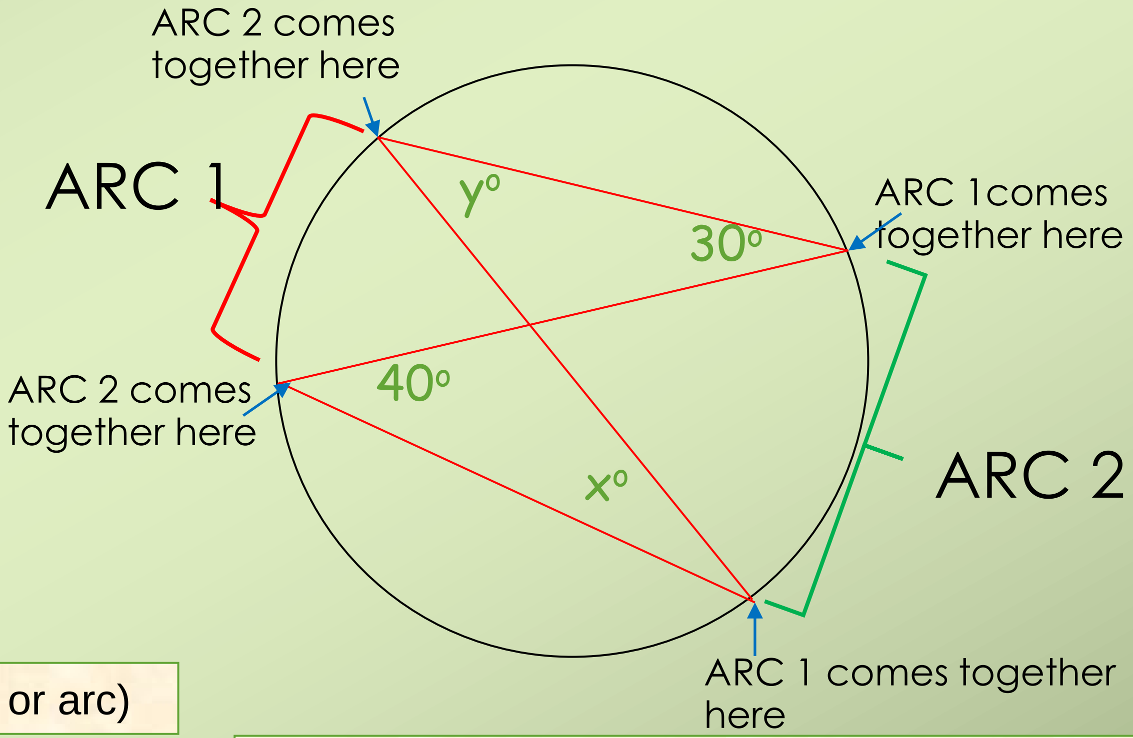
Theorem 4
Examples

Angles subtended by an arc or chord in the same segment are equal.

Find the unknown angles in each case



Angle x = angle y = 38° (angle subtended by same chord or arc)

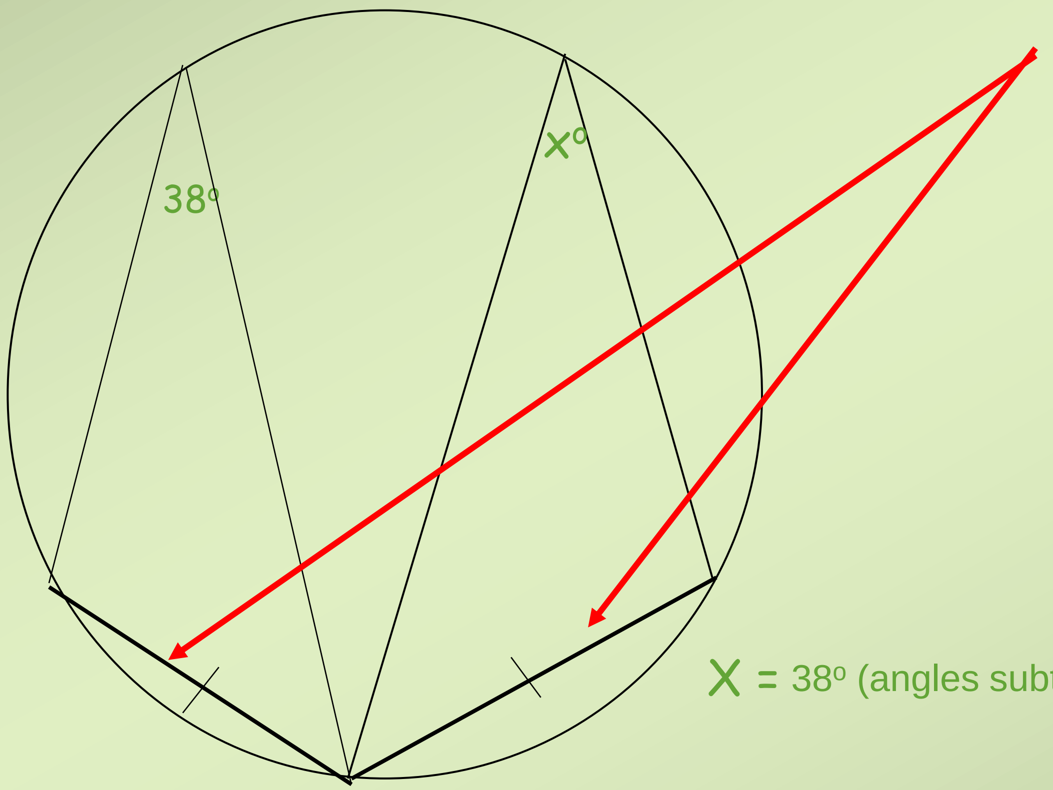


Angle x = 30° (angle subtended by same chord or arc)

Angle y = 40° (angle subtended by same chord or arc)

Theorem 4 Examples	Angles subtended by a chord of the same length in the same segment are equal. (Special case of theorem 4)

Find the unknown angle



If the chords are equal in length and subtend angles at the circumference, those angles are equal.

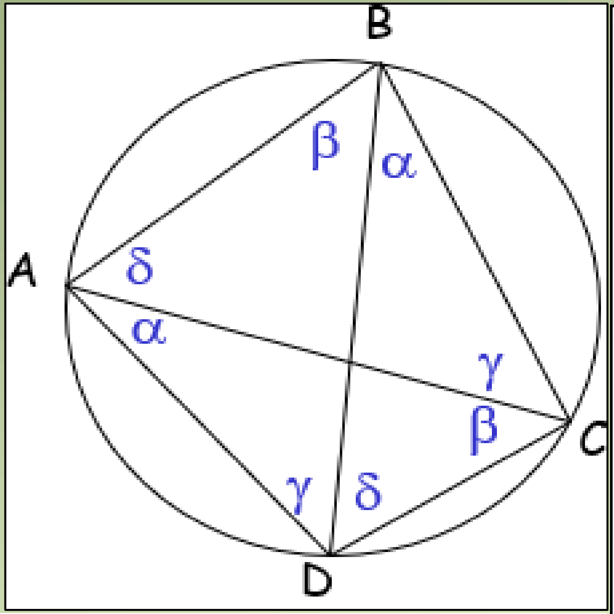
IT IS A SPECIAL CASE OF THE THEOREM 4.

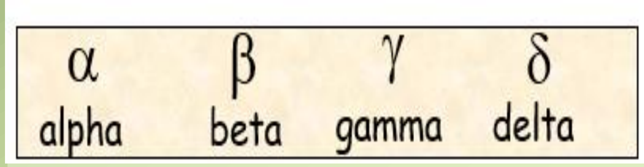
Angle subtended by same chord or arc.

IMPORTANT TO REMEMBER THIS SPECIAL CASE

X = 38º (angles subtended by equal chords)

Theorem 5





Opposite angles of cyclic quad are supplementary (adds up to 180^0)

Given: Cyclic Quad ABCD

Proof

Construction: Draw BD and AC

USING CHORD DC

$D\hat{B}C = D\hat{A}C = \alpha$ (angle subtended by same chord or arc)

USING CHORD AD

$A\hat{B}D = A\hat{C}D = \beta$ (angle subtended by same chord or arc)

USING CHORD AB

$B\hat{D}A = B\hat{C}A = \gamma$ (angle subtended by same chord or arc)

USING CHORD BC

$B\hat{A}C = B\hat{D}C = \delta$ (angle subtended by same chord or arc)

$D\hat{B}C + D\hat{A}C + A\hat{B}D + A\hat{C}D + B\hat{D}A + B\hat{C}A + B\hat{A}C + B\hat{D}C = 360^0$ (Angles of Quad $= 360^0$)

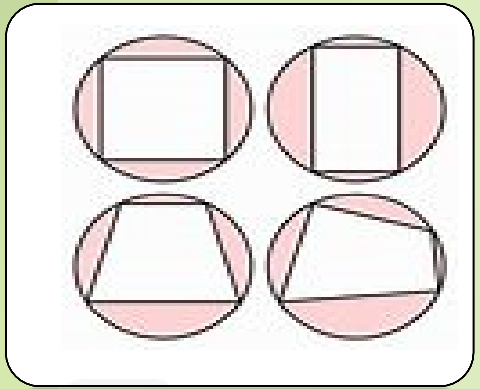
$\alpha + \alpha + \beta + \beta + \gamma + \gamma + \delta + \delta = 360^0$

$2\alpha + 2\beta + 2\gamma + 2\delta = 360^0$

$2(\alpha + \beta + \gamma + \delta) = 360^0$

$\alpha + \beta + \gamma + \delta = 180^0$

$\therefore B\hat{A}D + B\hat{C}D = 180^0$ and $A\hat{B}C + A\hat{D}C = 180^0$

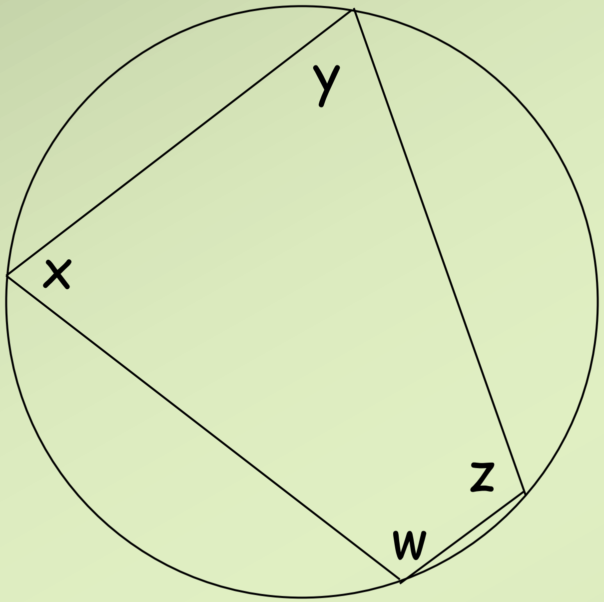


A cyclic quadrilateral is a quadrilateral whose vertices all lie on a single circle

Theorem 5 Examples

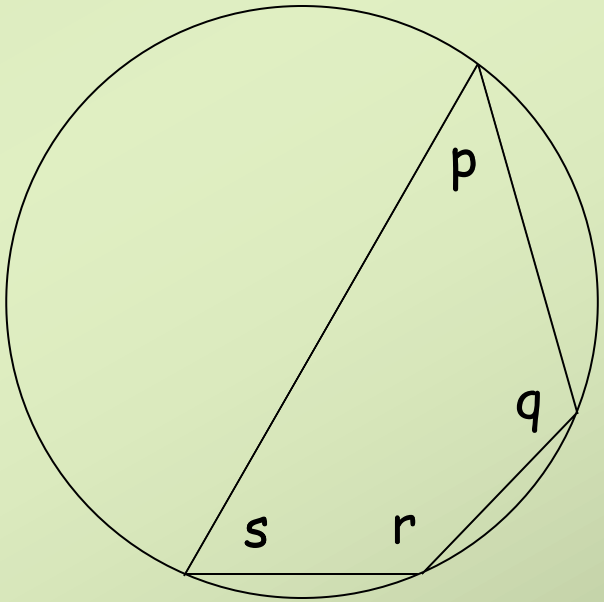
Cyclic Quadrilateral Theorem.

The opposite angles of a cyclic quadrilateral are supplementary. (They sum to 180°)



Angles y + w = 180° (Opp angles of Cyclic Quad Supp.)

Angles x + z = 180° (Opp angles of Cyclic Quad Supp.)



Angles p + r = 180° (Opp angles of Cyclic Quad Supp.)

Angles q + s = 180° (Opp angles of Cyclic Quad Supp.)