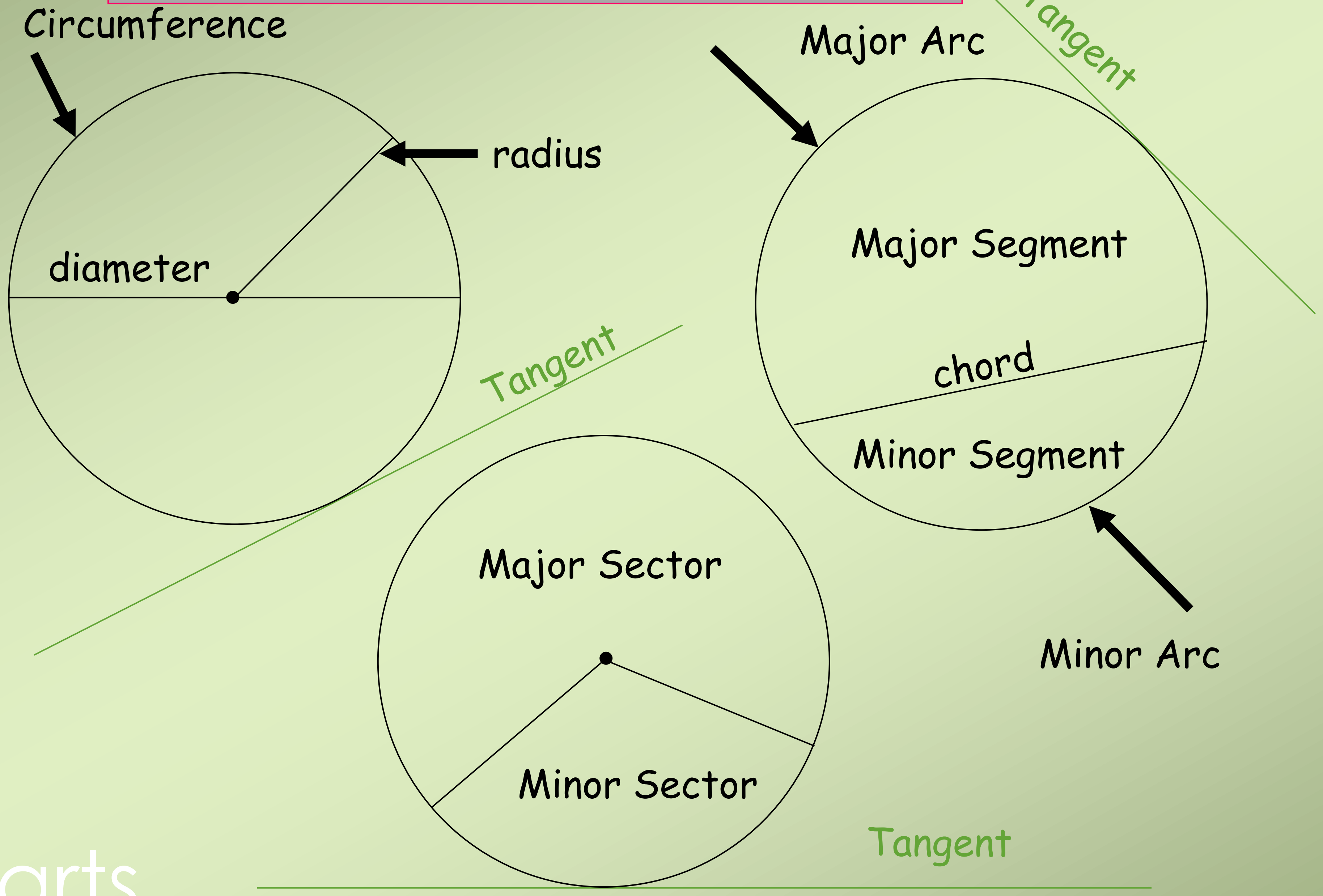
GEOMETRY GR 11

THEOREMS FOR GRADE 11 and GRADE 12 –PART 1

Parts of the Circle

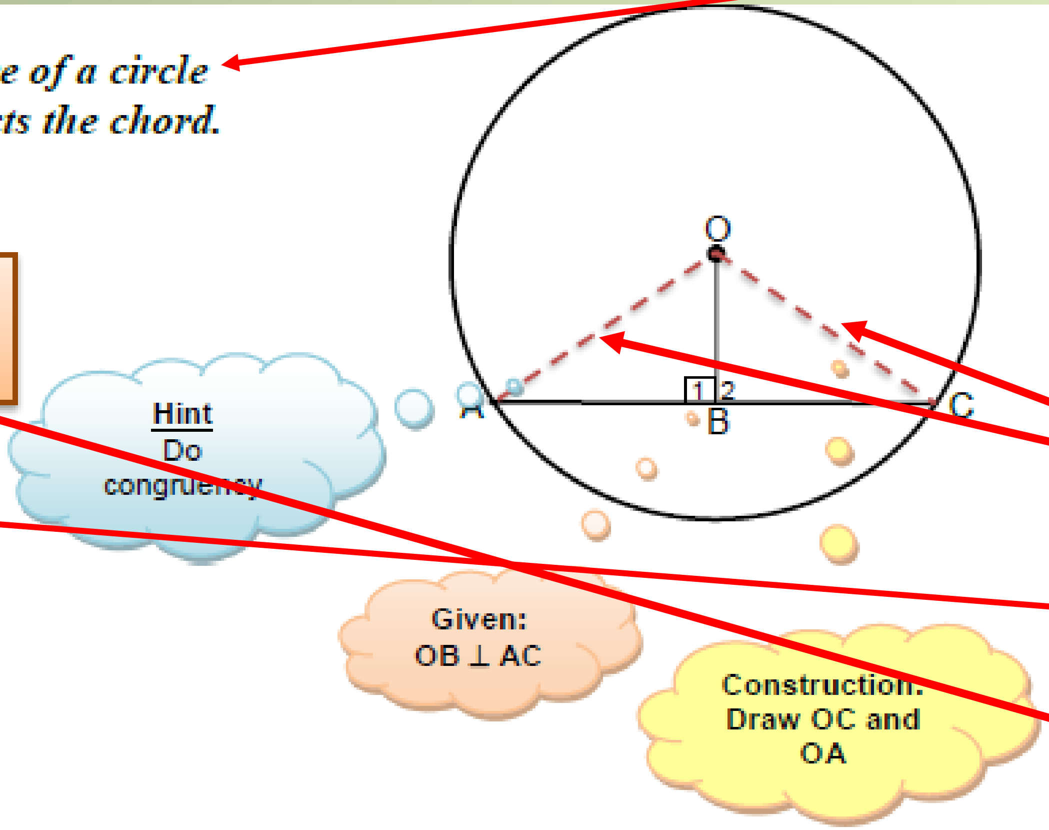


Parts

THEOREM 1

The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

STATEMENT OF THEOREM



line from centre ⊥ to chord

GIVEN: OB ⊥ AC

To Prove: AB = BC

Construction: Draw OA and OC

Proof:

In Δ OAB and Δ OCB

(i) OA = OC [radii]
(ii) OB = OB [common]
(iii) $\hat{B}_1 = \hat{B}_2 = 90^\circ$ [given; OB⊥ AC]

∴ Δ OAB ≡ ΔOCB [hyp; 90˚∠;side]
AB = BC [from congruency]

This theorem needs to learnt "off by heart". If the Question asks you to prove the theorem, this is the proof.

REMEMBER THE CONSTRUCTIONS.

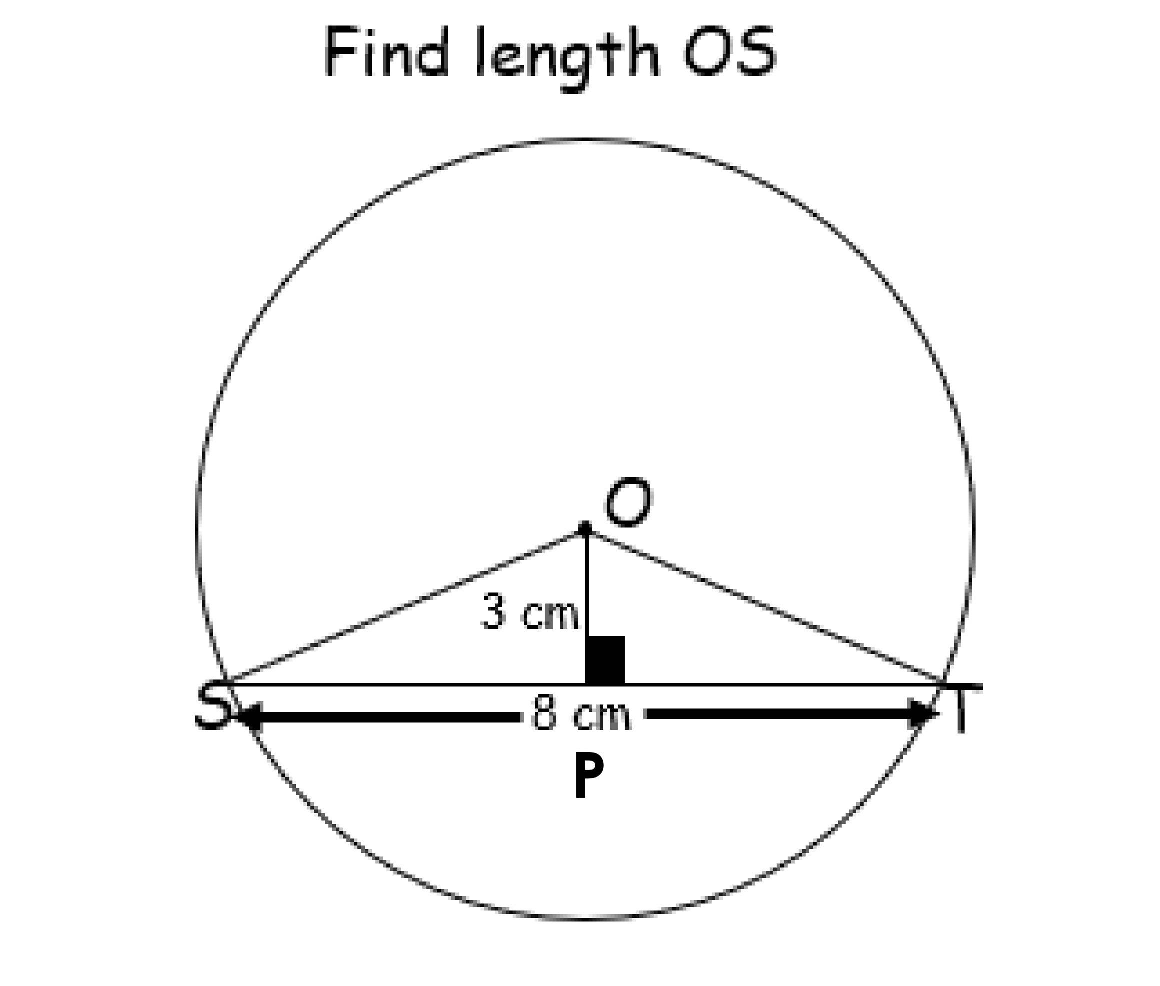
REMEMBER GIVEN INFO.

This is used when you do applications of the theorem as a reason

Congruency is used to prove.

The proof won't change. Learn it and if you are asked it, then you can get the marks.

Example of an application



OP = 3cm (Given)
ST = 8cm (Given)
OP ⊥ ST (Given)
SP = 4cm = PT (Line from centre O ⊥chord ST)
USING PYTHAGORUS YOU CAN NOW WORK OUT OS.

$$OS^2 = OP^2 + SP^2$$
$$OS^2 = 3^2 + 4^2$$
$$OS^2 = 9 + 16$$
$$OS^2 = 25$$
$$OS = 5$$

NOTE THAT YOU DO NOT PROVE THE THEREOM IN THIS EXAMPLE BECAUSE THE QUESTION WAS ASKING TO WORK OUT A SIDE. WE USE THE THEOREM TO HELP US WORK OUT THE SIDE, THAT IS WHY THE REASON IS INDICATED.

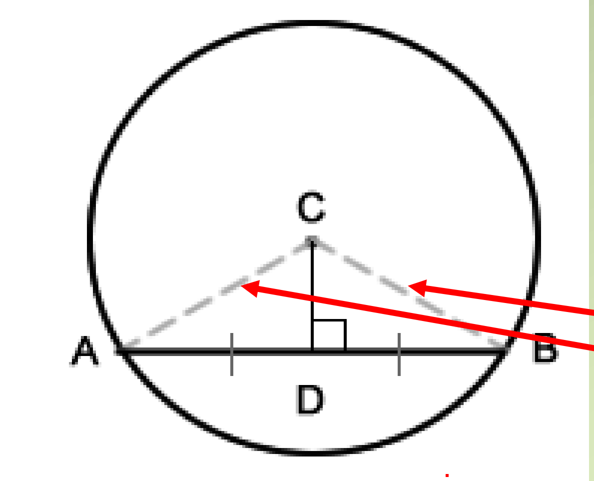
SP = 4cm = PT (Line from centre O ⊥chord ST)

This is an application of the theorem.

Theorem 2

STATEMENT OF THEOREM

The perpendicular bisector of a chord passes through the centre of the circle



This theorem needs to learnt "off by heart". If the Question asks you to prove the theorem, this is the proof.

REMEMBER THE CONSTRUCTIONS.

The perpendicular bisector of a chord contains the centre of the circle

Statement	Reason
In ΔCAD and ΔCBD,	
AD = DB	Given
∠CDA = ∠CDB	CD ⊥ to AB
CD = CD	Common
ΔCDA and ΔCDB	SAS
CA = CB	Congruent triangles
C is center of circle	The center of the circle is the only point within the circle that has points on the circumference equal distance from it.

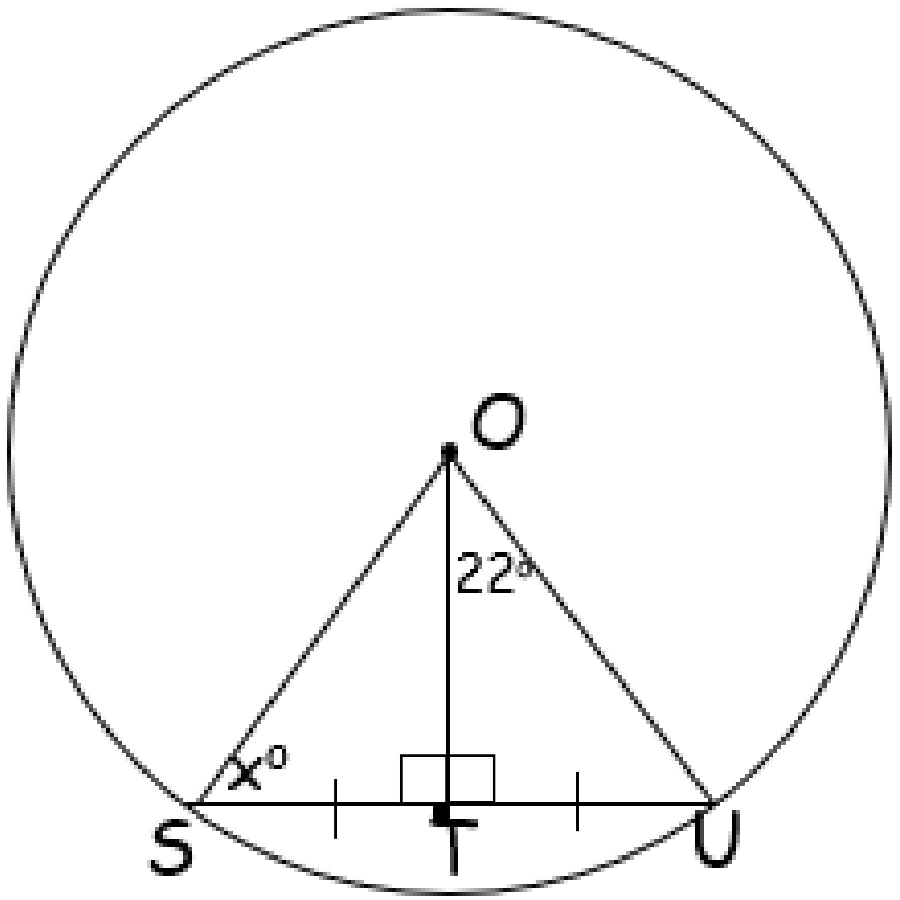
REMEMBER GIVEN INFO.

Congruency is used to prove.

The proof won't change. Learn it and if you are asked it, then you can get the marks.

Example of an application

1. Is O the centre of circle below?
2. Determine angle x.



1.

In $\triangle SOT$ *and* $\triangle UOT$

1. OT = OT (Given)
2. $S\hat{T}O = U\hat{T}O$ (Given)
3. TS = TU (Given)

$\therefore \triangle SOT \equiv \triangle UOT$ (SAS)

SO = UO (Congruency)

$\therefore$ O is the centre of the circle.

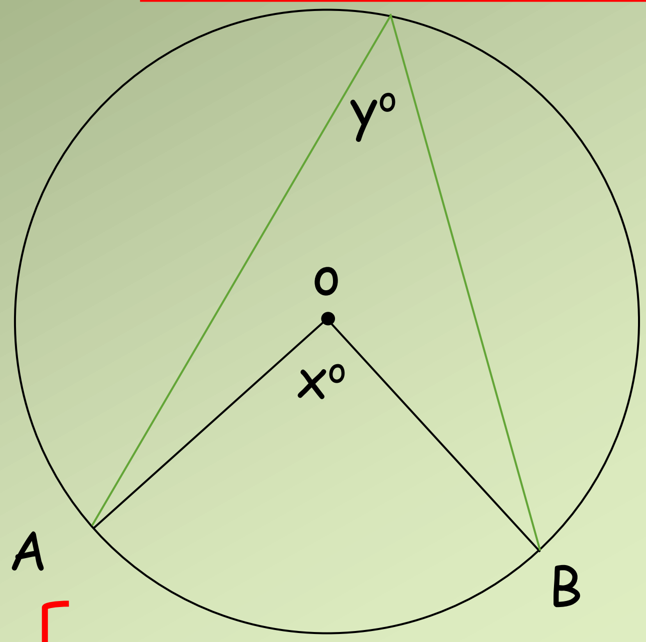
2.

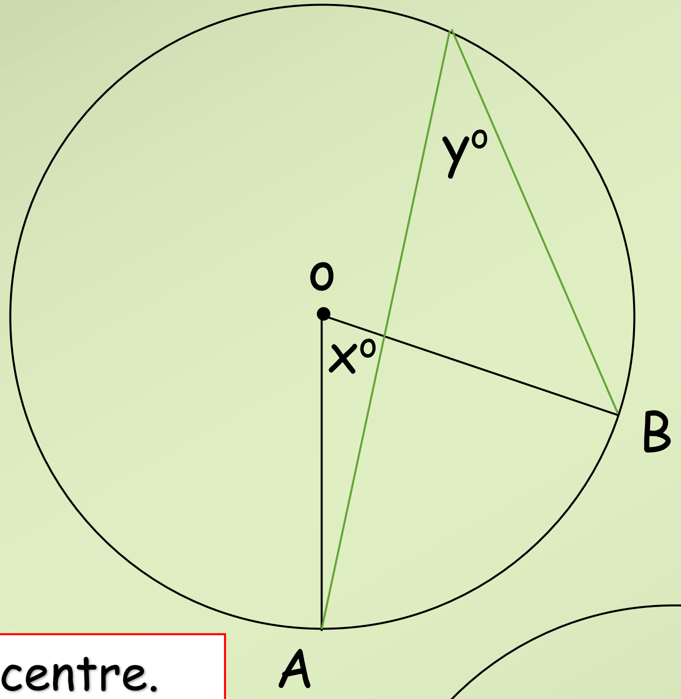
$S\hat{O}T = 22^0$ $(\triangle SOT \equiv \triangle UOT)$

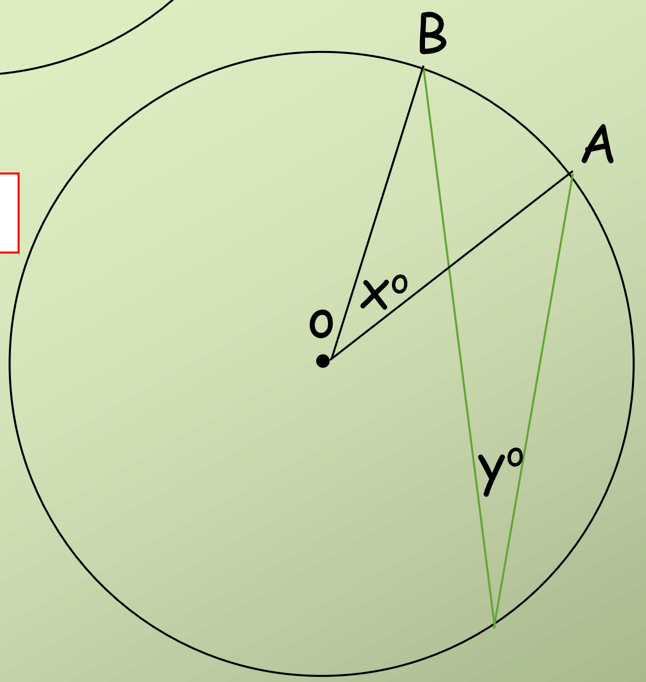
$90^0 + 22^0 + x = 180^0$ (Angles of $\triangle$ SOT supplementary)

$\therefore x = 68^0$

Terminology to understand next Theorem







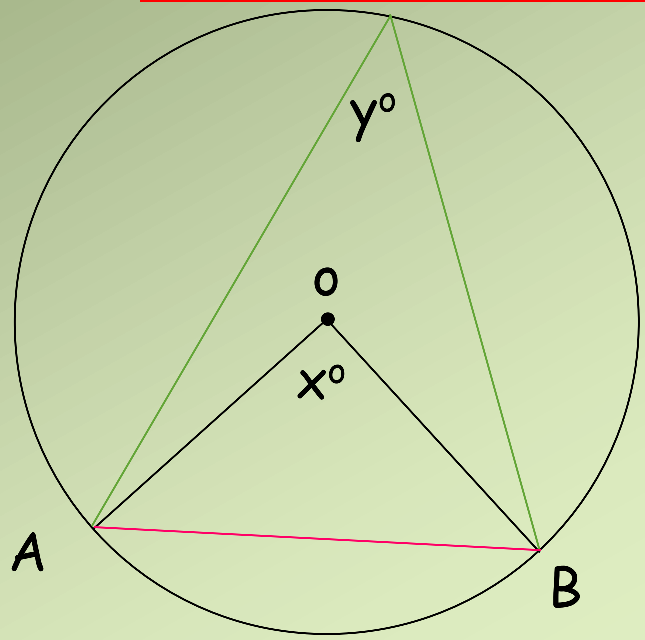
FOR ALL THREE DIAGRAMS

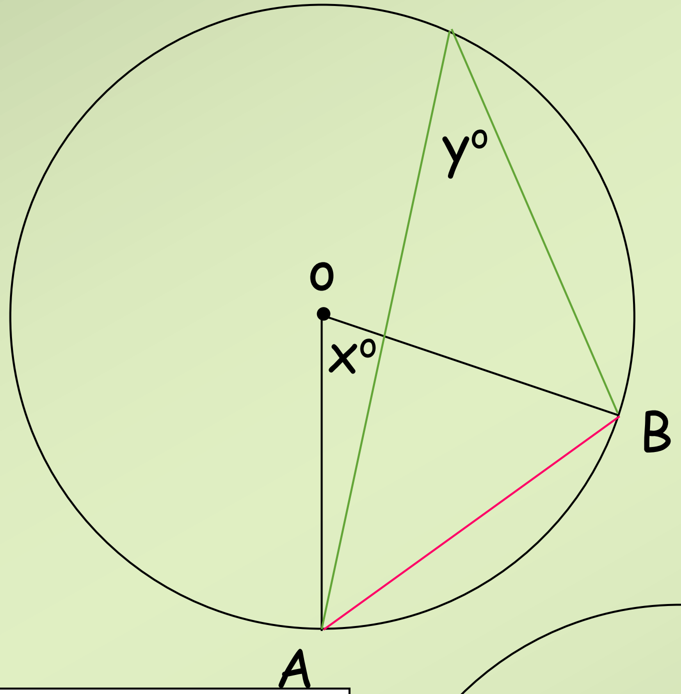
Arc AB subtends angle x at the centre.

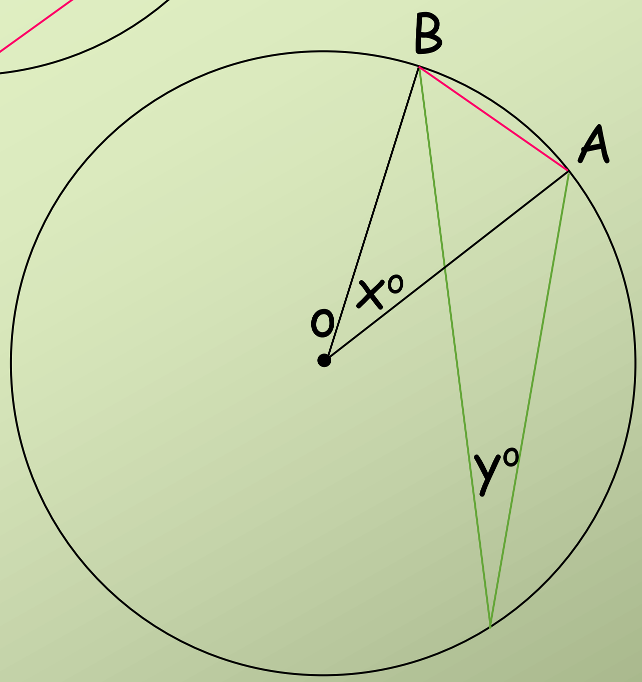
Arc AB subtends angle y at the circumference.

HINT:
Put left finger on A
Put right finger on B
Move along lines to centre O where your fingers will meet. (Angle at Centre)
Put left finger on A
Put right finger on B
Move along lines to circumference where your fingers will meet. (Angle at Circumference)

Terminology to understand next Theorem







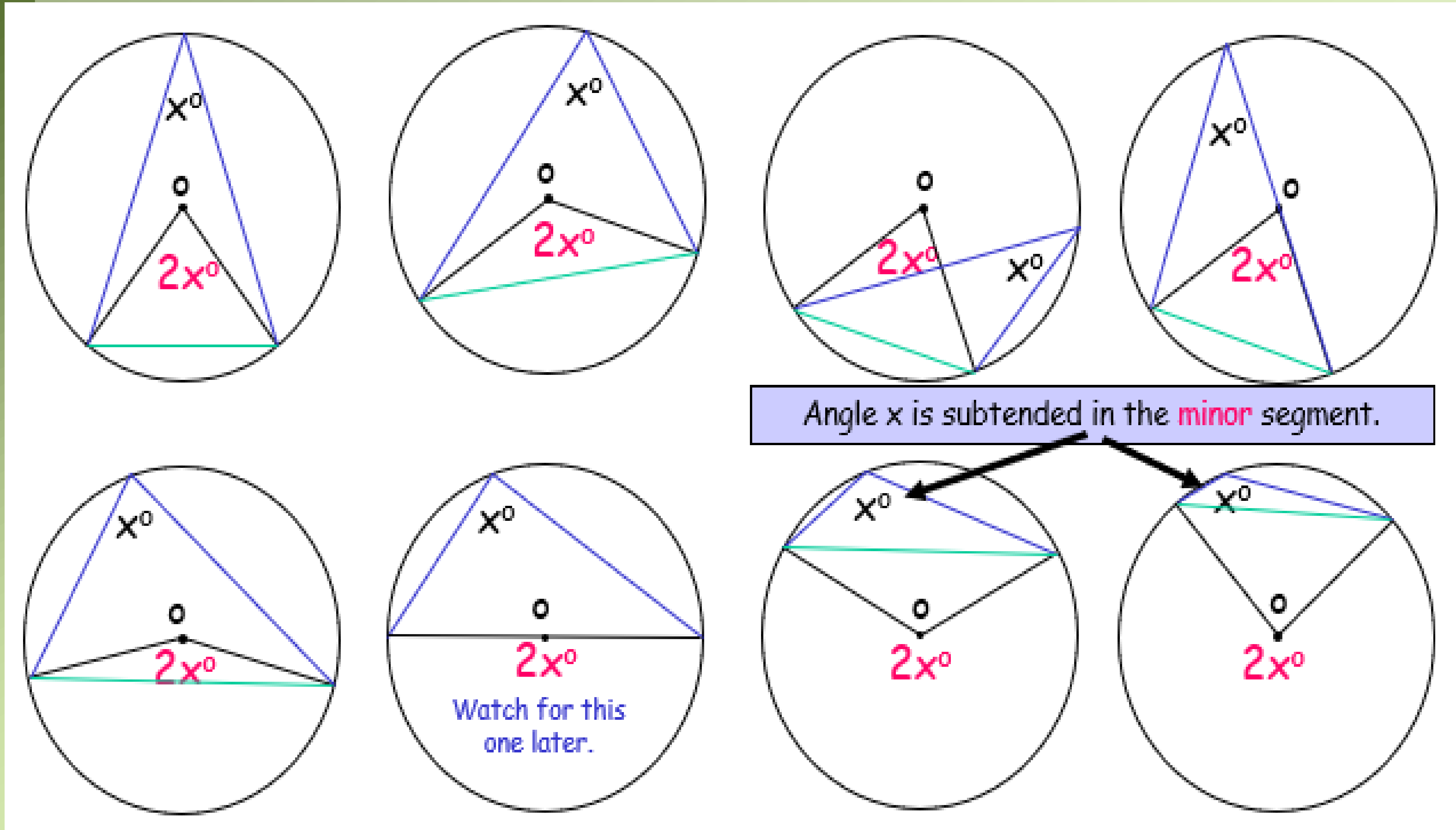
FOR ALL THREE DIAGRAMS

Chord AB subtends angle x at the centre.

Chord AB subtends angle y at the circumference.

HINT:
Put left finger on A
Put right finger on B
Move along lines to centre O where your fingers will meet. (Angle at Centre)
Put left finger on A
Put right finger on B
Move along lines to circumference where your fingers will meet. (Angle at Circumference)

Example of Angle at Centre and Angle at Circumference. 2x is at centre and x is at centre.



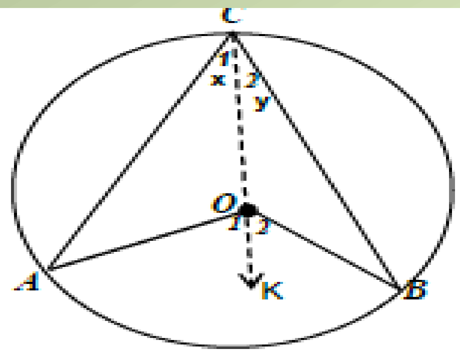
YOU DON'T NEED TO WORK OUT x and 2x. THIS IS JUST TO ILLUSTRATE THE CONCEPT OF ANGLE AT CENTRE AND ANGLE AT CIRCUMFERENCE

THEOREM 3

STATEMENT OF THEOREM

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).

∠ at centre = 2 ×∠ at circumference



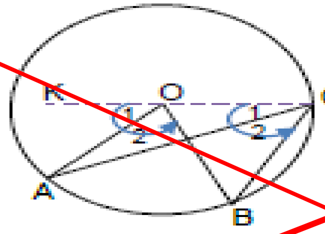


Diagram B

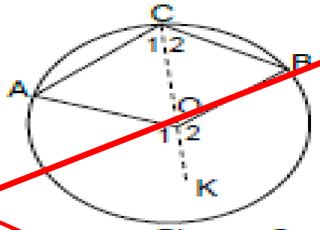


Diagram C

Given: Circle with centre O and arc AB subtended $A\hat{O}B$ at the centre and $A\hat{C}B$ on the circumference.

To Prove: $B\hat{O}C = 2B\hat{A}C$

Construction: Draw AO extended.

Proof:

In diagram A, B and C:

Let $\hat{A}_1 = x$ and $\hat{A}_2 = y$

OA = OB [radii]

$\hat{A}_1 = \hat{B} = x$ [∠s opp equal sides]

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext ∠ of Δ]

$\therefore \hat{O}_1 = 2x$

$\therefore \hat{O}_1 = 2\hat{A}_1$

Similarly $\hat{O}_2 = 2y$

$\therefore \hat{O}_2 = 2\hat{A}_2$

Hint: ext ∠ of Δ isosceles Δ

In diagrammes A and C:

$\therefore \hat{O}_1 + \hat{O}_2 = 2x + 2y$

$= 2(x + y)$

$A\hat{O}B = 2A\hat{C}B$

In diagram B :

$\hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

$A\hat{O}B = 2A\hat{C}B$

This Proof has three different diagrams that could be given- maybe all or maybe one or two.

Proof is the same for all three diagrams up to a point.

For Diagram A and C

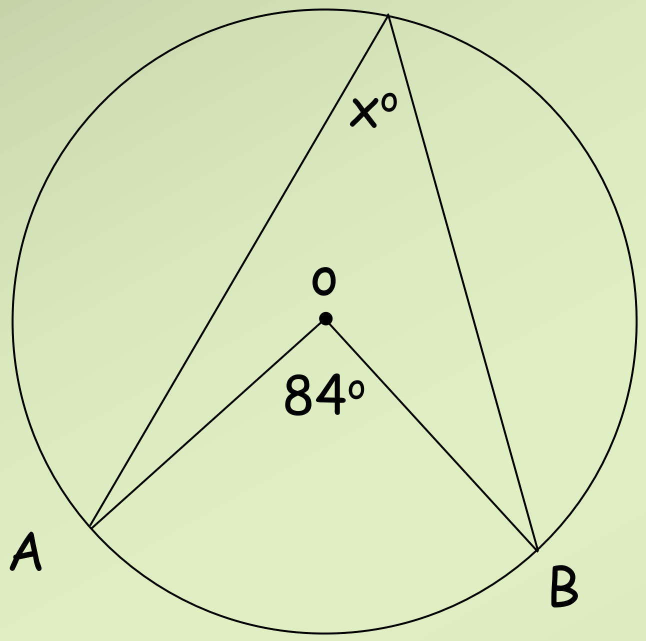
For Diagram B

This is used when you do applications of the theorem as a reason

Example Application Questions

Find the unknown angles giving reasons for your answers.

1



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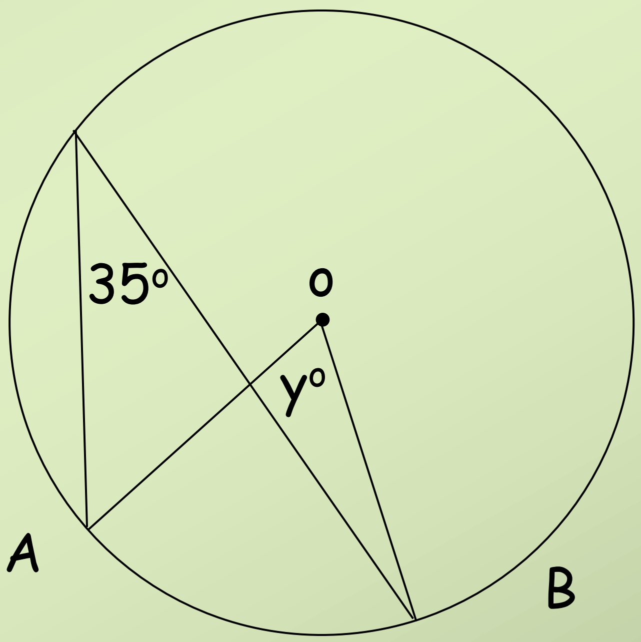
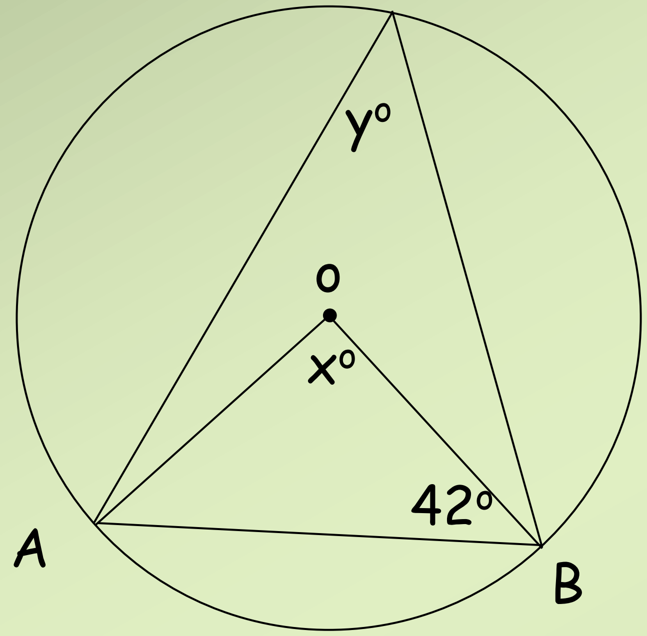


angle x =	42° (Angle at the centre = 2 x angle at circum).
angle y =	70° (Angle at the centre = 2 x angle at circum).

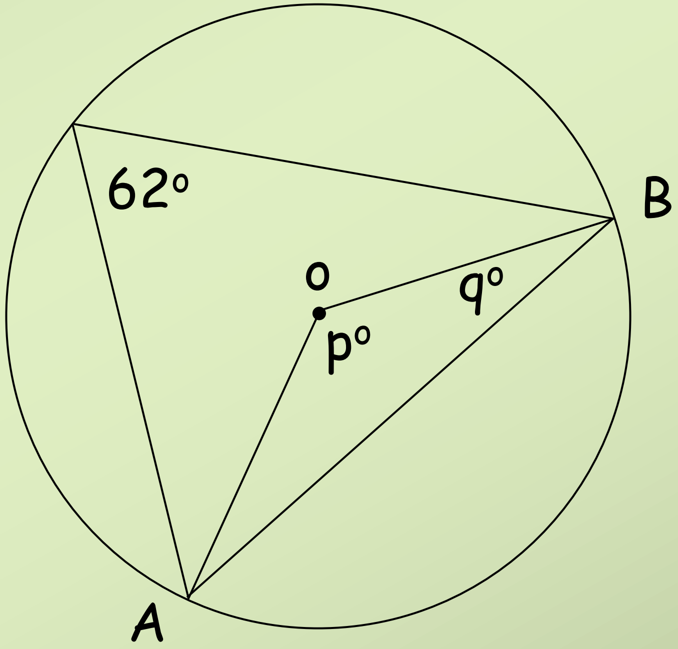
Example Application Questions

Find the unknown angles giving reasons for your answers.

3



4



3.
$O\hat{A}B = 42^0$ $(OA = OB\ RADIUS\ -$
$ANGLES\ OPP\ EQUAL\ SIDES)$
x+ $O\hat{A}B$+ $O\hat{B}A = 180^0$ (ANGLES OF TRIANGLE
SUPPLEMENTARY
x + 42^0+ 42^0= 180^0
x = 96^0
y = 48^0 (Angle at the centre = 2 x angle at circum).

4.
p = 124^0 (Angle at the centre = 2 x angle at circum).

$O\hat{A}B = q = O\hat{B}A$ $(OA = OB\ RADIUS\ -$
$ANGLES\ OPP\ EQUAL\ SIDES)$
$124^0 + q$+ q $= 180^0$ (ANGLES OF TRIANGLE SUPPLEMENTARY
124^0 + 2q= 180^0
q=(180 – 124)÷2
∴q =28^0