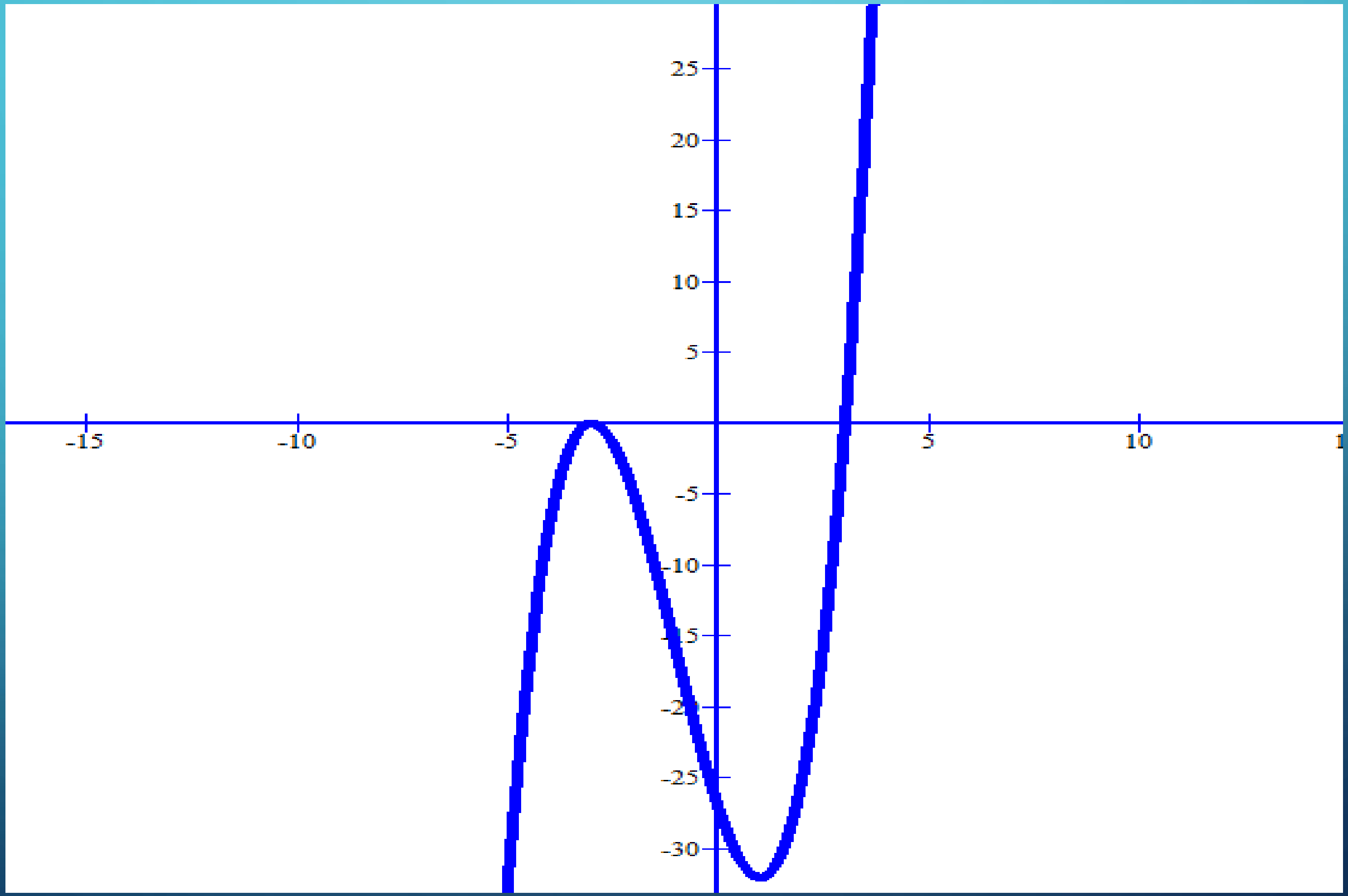
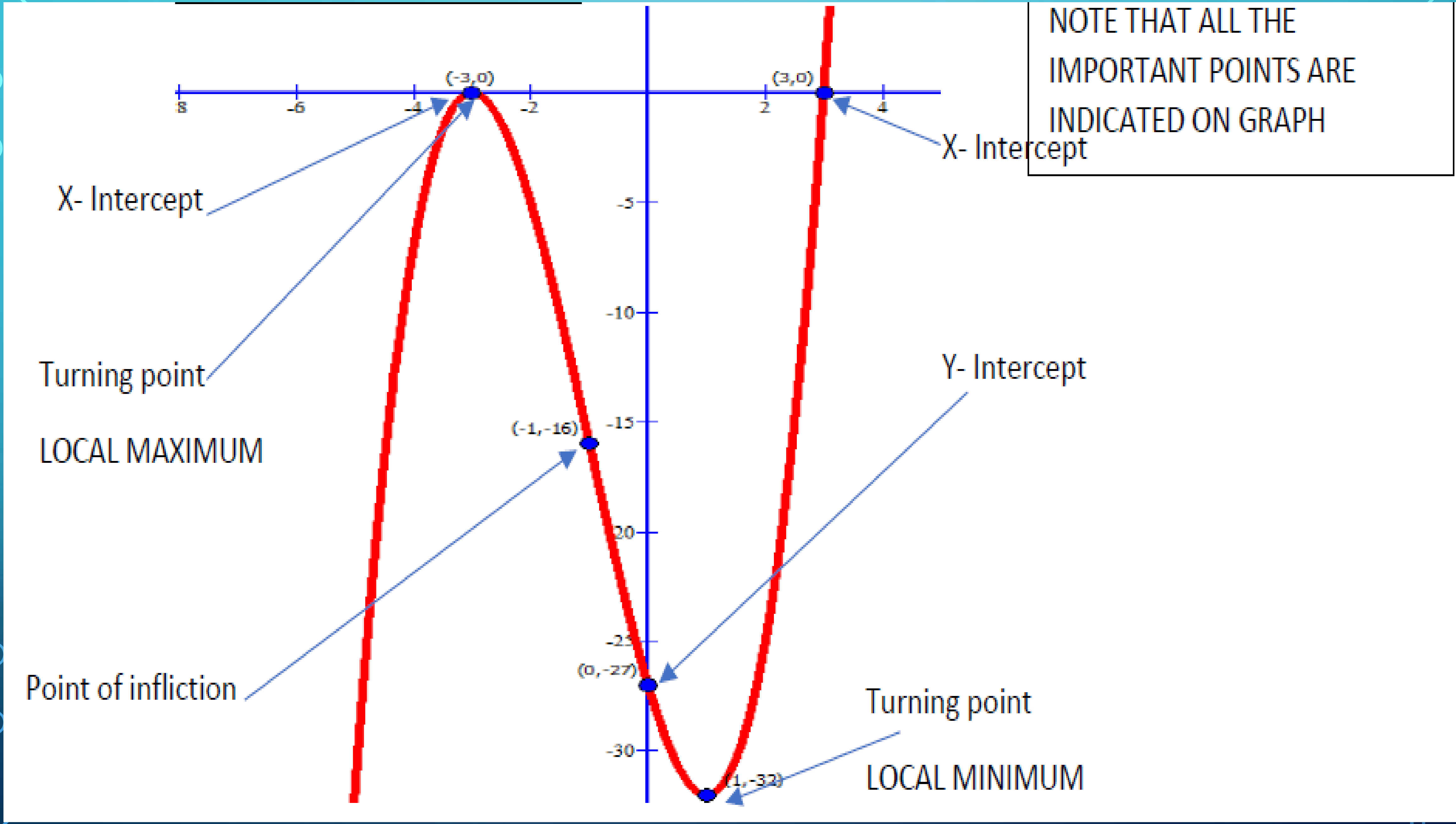
CUBIC GRAPHS AND THE DERIVATIVE

TO SEE THE RELATIONSHIP BETWEEN THE DERIVATIVE AND THE CUBIC GRAPH

CONSIDER $f(x) = x^3+3x^2-9x-27$



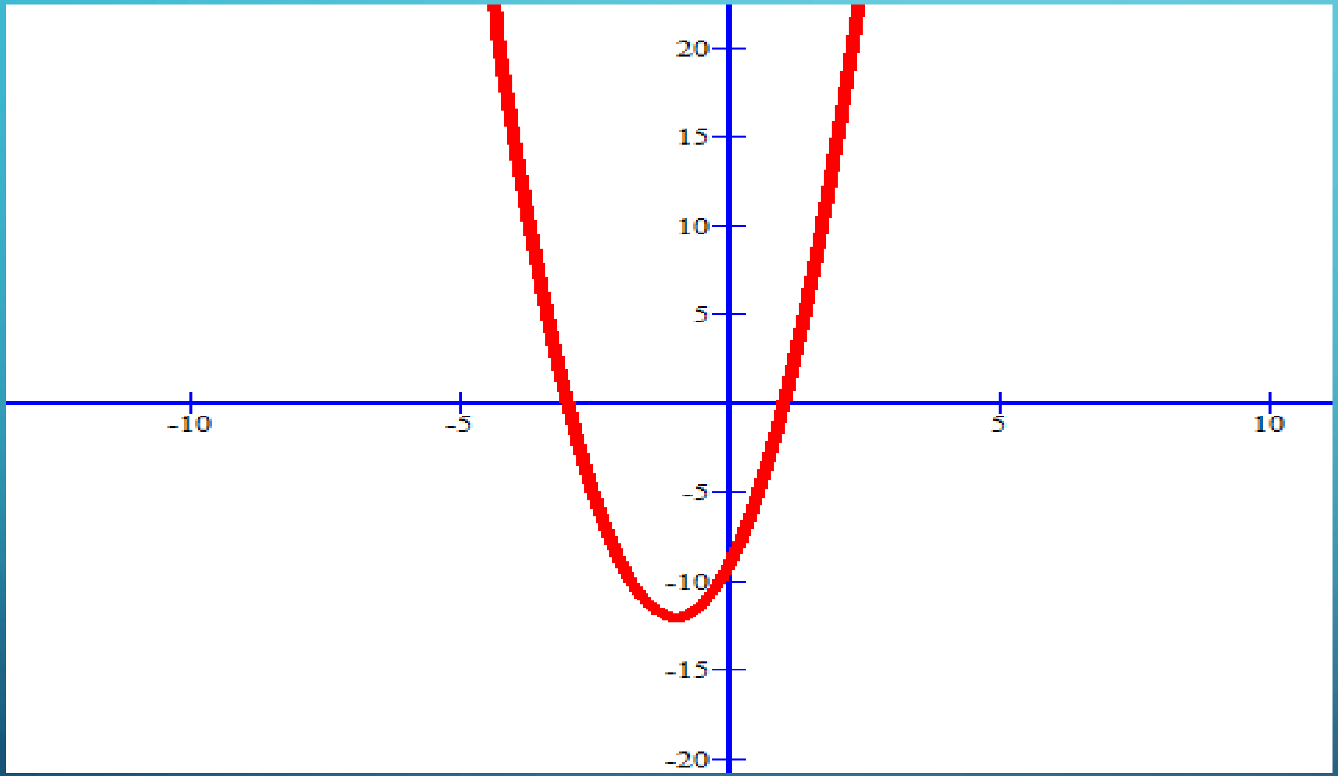


NOTE THAT ALL THE IMPORTANT POINTS ARE INDICATED ON GRAPH
(-3,0)
(3,0)
X- Intercept
X- Intercept
Turning point
LOCAL MAXIMUM
(-1,-16)
Point of infliction
Y- Intercept
(0,-27)
Turning point
LOCAL MINIMUM
(1,-32)
-8
-6
-4
-2
2
4
-5
-10
-15
-20
-25
-30

NOW COMPARE TO DERIVATIVE GRAPH OF THE SAME FUNCTION

- $f(x) = x^3+3x^2-9x-27$
- Derivative is
- $f'(x) = 3x^2+6x-9$

NOW SKETCH f' (x) = $3x^2+6x-9$

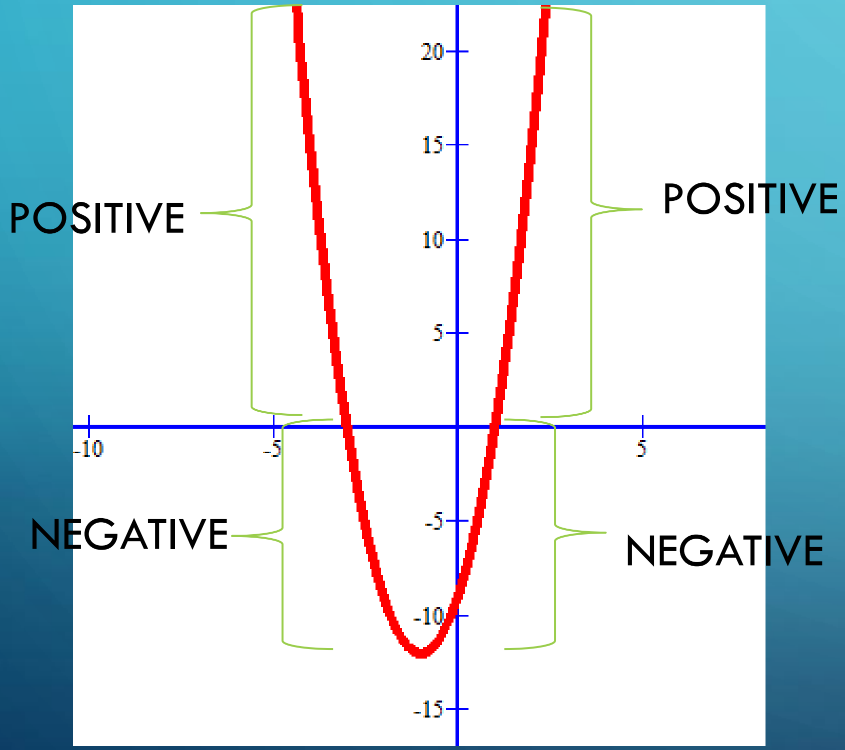


- The graph will cut the x-axis at (-3;0) and (1;0)
- The graph will cut the y-axis at (0;-9)
- REMEMBER THIS GRAPH IS ILLUSTRATING THE DERIVATIVE OF THE ORIGINAL F(X) GRAPH.

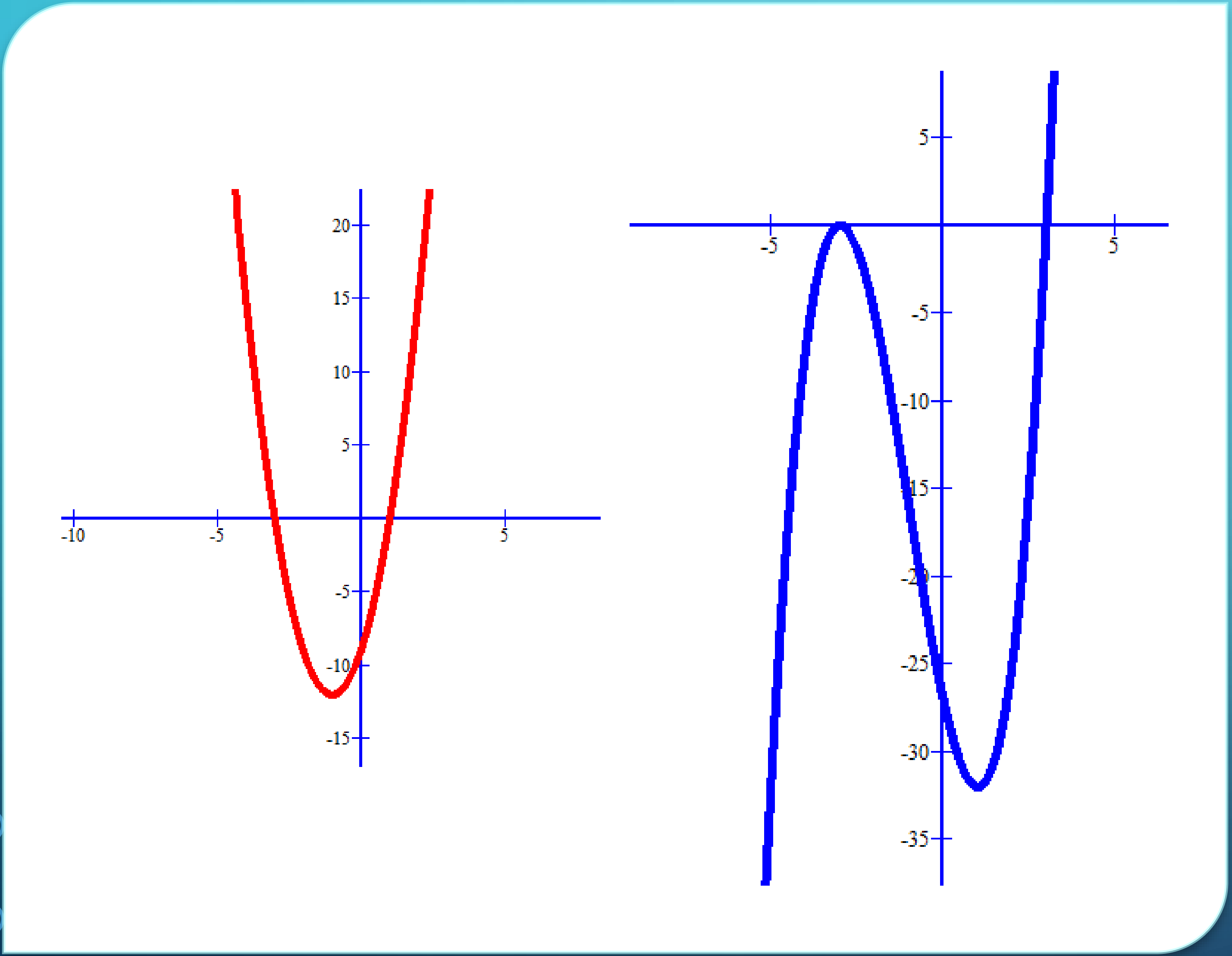
BREAK DOWN THE DERIVATIVE GRAPH - f'(x)

- The graph represents the derivative of the f(x) graph.
- Derivative is the gradient of a graph at a point.
- The f'(x) graph (GRADIENT OF ORIGINAL f(x) GRAPH) is positive above the x-axis.
- The f'(x) graph (GRADIENT OF ORIGINAL f(x) GRAPH) is negative below the x-axis.

The f'(x) graph (gradient of original f(x) graph) is positive above the x-axis.
The f'(x) graph (gradient of original f(x) GRAPH) is negative below the x-axis.

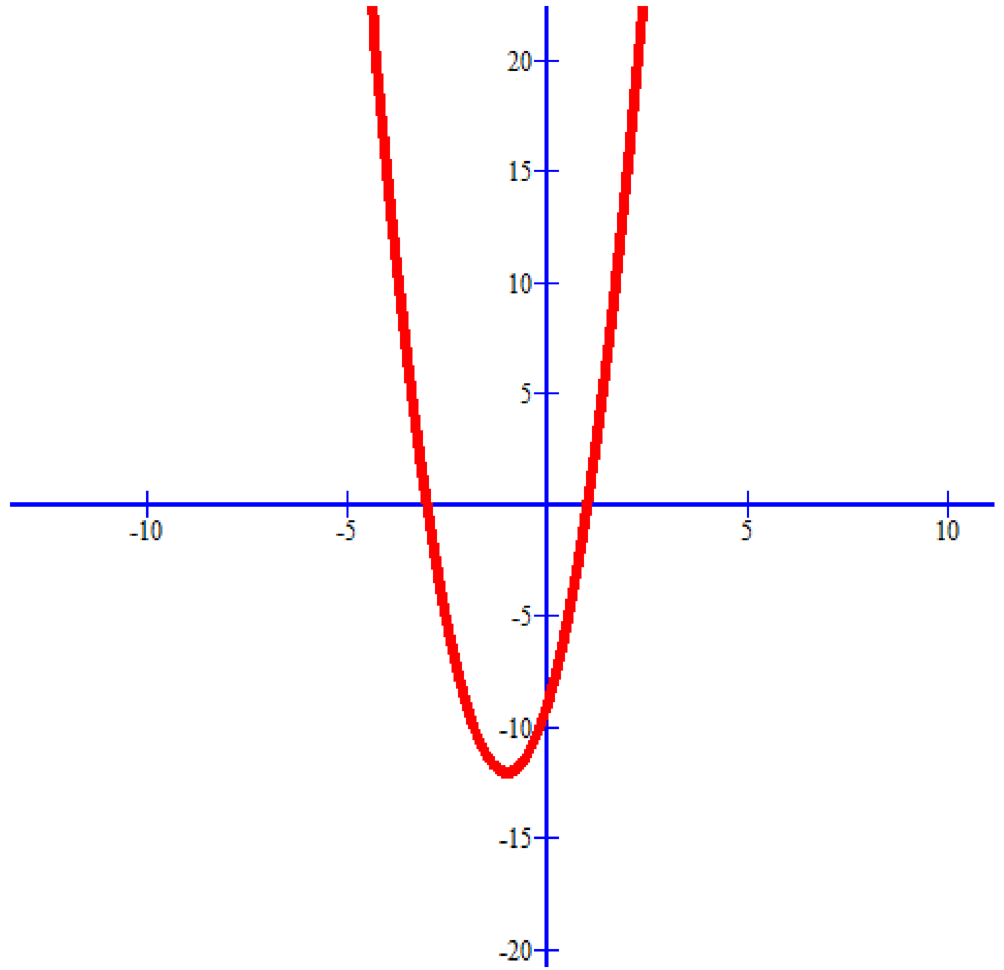


- At x=-3 the graph cuts the x-axis, therefore is 0 at that point. This means derivative of original f(x) is 0.
- At x=1 the graph cuts the x-axis again, therefore is 0 at this point as well. This means derivative of original f(x) is 0.
- Points x <-3, will be positive (above the x-axis)
- Points -3 < x < 1, will be negative (below the x-axis)
- Points x>1, will be positive (above the x-axis)

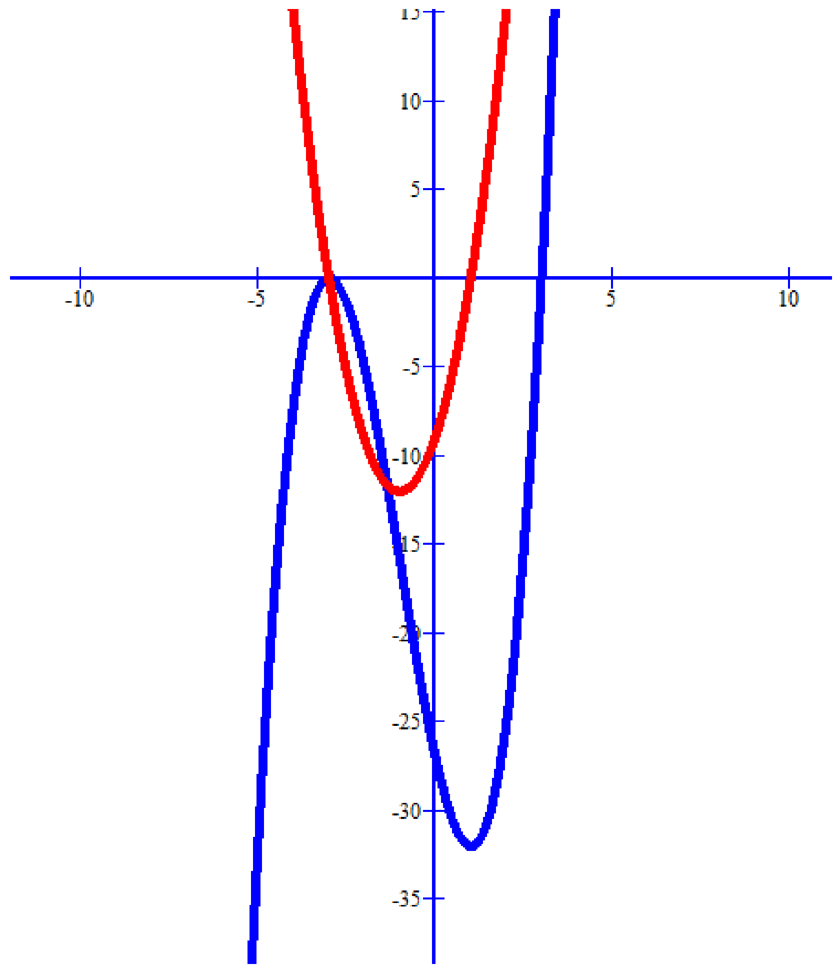


- THEREFORE
- The original graph will have a positive gradient for x < -3
- The original graph will have a negative gradient for -3 <x <1
- The original graph will have a positive gradient for x > 1

TURNING POINT OF f'(x)



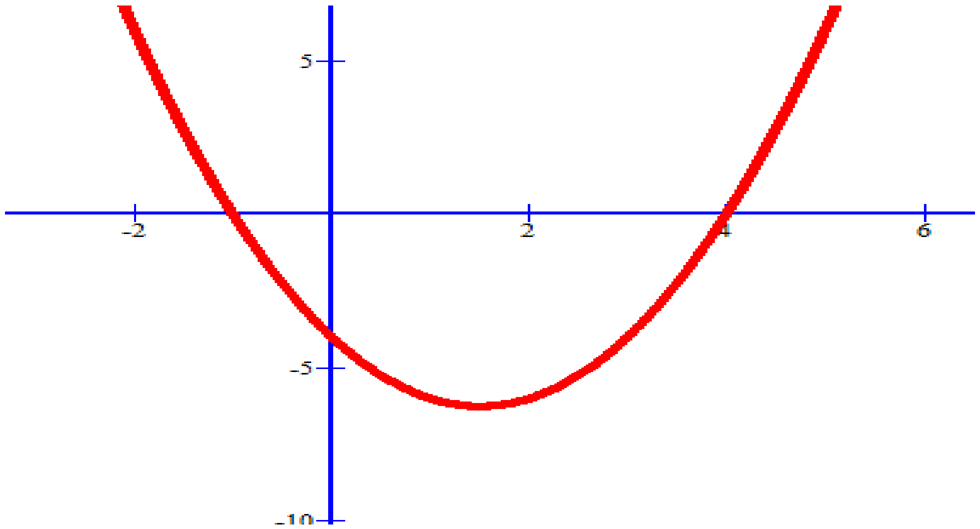
- The graph of f'(x) (Derivative of f(x)) turns where x=-1. Where the This can be further explained by working out the second derivative of f(x). Derivative (Gradient) of the Derivative (Gradient) is 0
  - $f'(x) = 3x^2+6x-9$
  - $f''(x) = 6x+6$
  - $f''(x) = 6x+6 = 0$
  - $x = -1$
- This would then be the x-value of the point of infliction of f(x)



THEREFORE PUTTING IT ALL TOGETHER

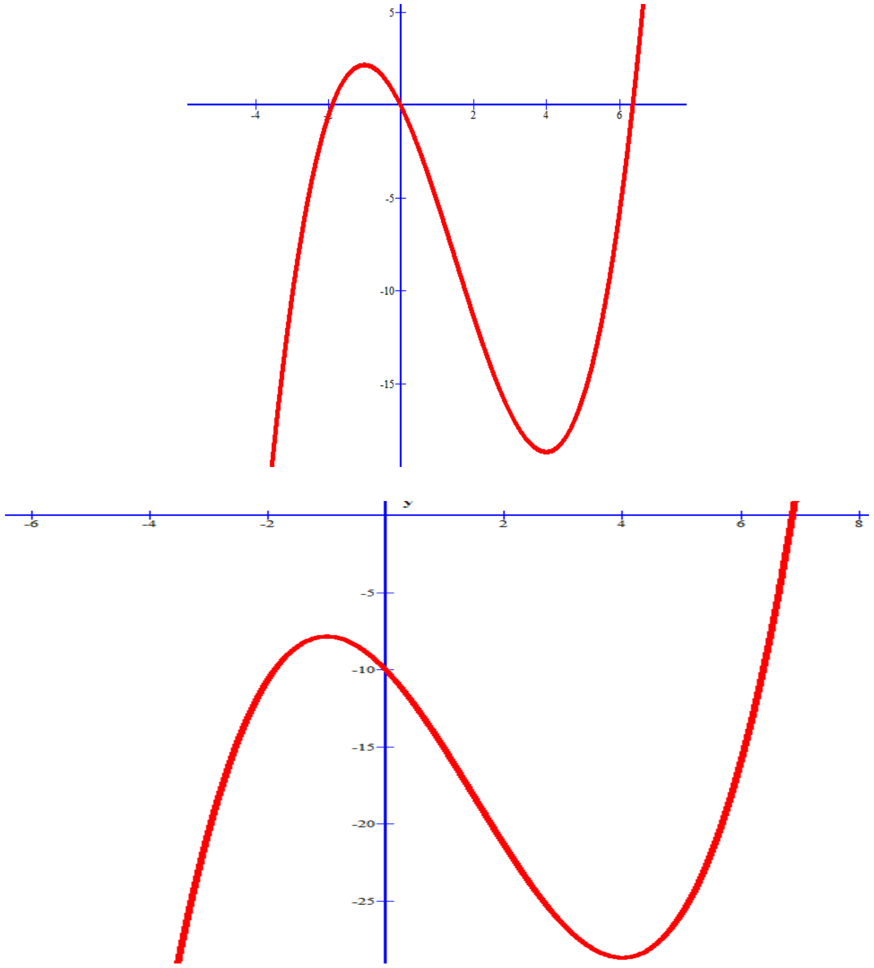
- The RED graph is the Derivative – f'(x).
- The BLUE graph is the original cubic graph – f(x).
- The Cubic graph f(x) has a positive gradient (going up) for $x < -3$ and $x > 1$.
- The Cubic graph f(x) has a negative gradient (going down) for $-3 < x < 1$.
- The point of infliction will be where x=-1. This is where the f'(x) turns or where the derivative of the derivative is 0. f"(x) =0.
- To obtain the y-value of the point of infliction substitute x=-1 into the original f(x).

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



EXAMPLE TO TRY

1. For what x-values will the f(x) graph have negative gradient?
2. For what x-values will the f(x) graph have positive gradient?
3. Sketch a rough sketch of f(x).



ANSWER

- 1. $-1 < x < 4$
- 2. $x < -1$ AND $x > 4$
- 3.

When sketching the graph in this example there are things, we do not know but we were asked to draw a rough sketch. We know the gradient, so we know the shape and thus the turning points at x-values. We cannot know for certain what the y-value will be where the graph turns. I have two possible sketches. There are more possible sketches with the information given in this example. If you were given more info you could have drawn a more precise rough sketch.