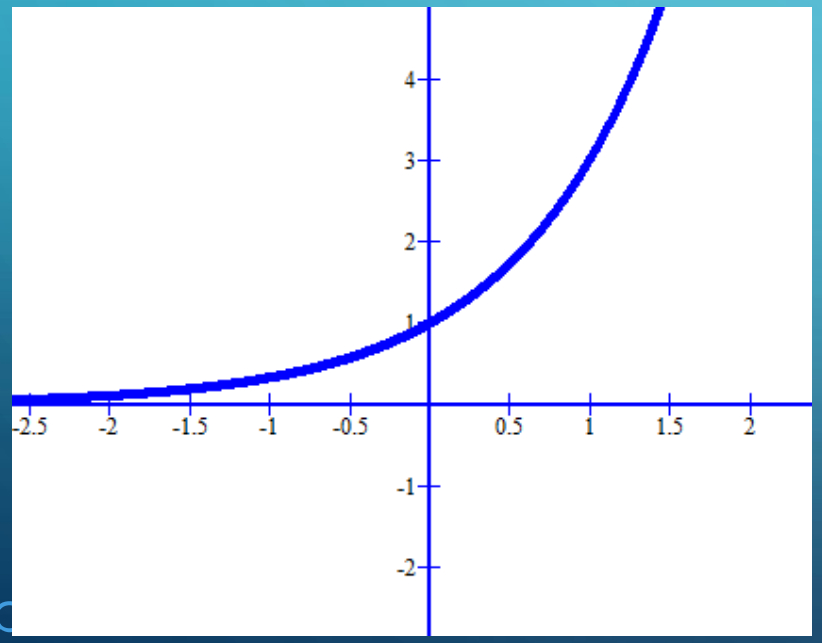
# GRADE 12 FUNCTIONS PART 4

## LOG AND EXPONENTIAL FUNCTIONS

# GRAPH OF $y = b^x$ - EXPONENTIAL GRAPH

- Example

$y = 3^x$



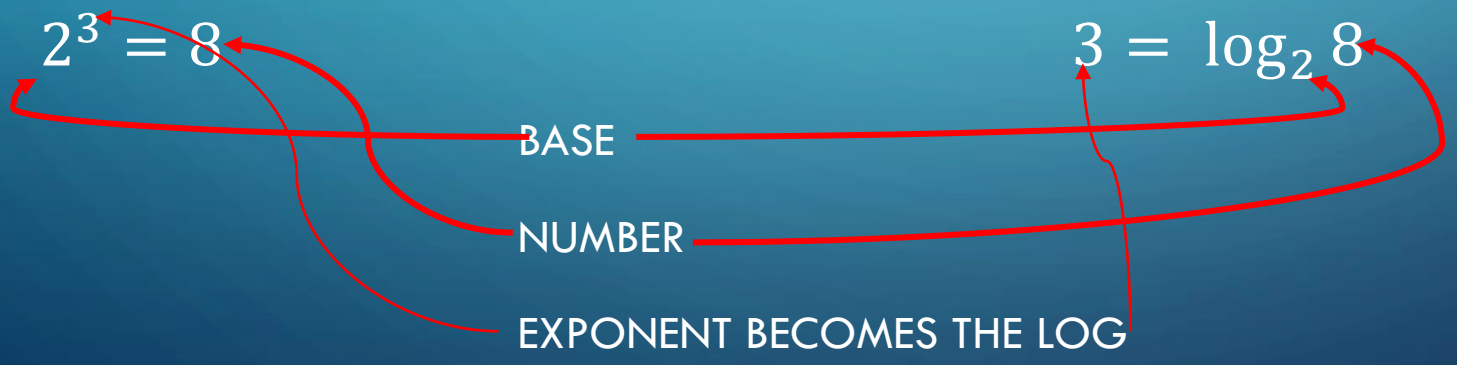

- Domain: $x \in \mathbb{R}$
- Range: $y > 0$
- Increasing function (As x increases so does y)
- Y-Intercept (when x=0): (0;1)
- X-Intercept: (No x-intercept)
- Asymptote: y=0 (x-axis)

# CHANGING EXPONENTIAL FUNCTION TO A LOG FUNCTION

- An Exponential Expression can be changed into a Log expression and vice versa

Example

$2^3 = 8$ CHANGED INTO A LOG - $3 = \log_2 8$



# TRY THE FOLLOWING:

- **Exercise 1**

Rewrite the following in LOG form:

a. $2^4 = 16$

b. $5^2 = 25$

c. $3^2 = 9$

d. $3^3 = 27$

e. $2^5 = 32$

f. $10^2 = 100$

# ANSWERS TO EXERCISE 1

- **Exercise 1**

Rewrite the following in LOG form:

a. $2^4 = 16$
b. $5^2 = 25$
c. $3^2 = 9$
d. $3^3 = 27$
e. $2^5 = 32$
f. $10^2 = 100$

**ANSWERS**

a. $2 = \log_5 25$
b. $3 = \log_2 9$
c. $3 = \log_3 27$
d. $5 = \log_2 32$
e. $2 = \log_{10} 100$

***READING THE LOG NOTATION***

$2 = \log_5 25$

Reads "2 is equal to LOG 25 base *5*"

THEREFORE TO GENERALISE :

$y = \log_a x$

Reads "y is equal to LOG x base a"

# TRY THE FOLLOWING:

**Exercise 1**

Rewrite the following in EXPONENTIAL form:

a. $2 = \log_6 36$

b. $6 = \log_2 64$

c. $2 = \log_7 49$

d. $y = \log_2 x$

# ANSWERS TO EXERCISE 2

**Exercise 1**

Rewrite the following in EXPONENTIAL form:

a. $2 = \log_6 36$

b. $6 = \log_2 64$

c. $2 = \log_7 49$

d. $y = \log_2 x$

**ANSWERS**

a. $6^2 = 36$

b. $2^6 = 64$

c. $7^2 = 49$

d. $2^y = x$

# INVERSE OF $y = a^x$

- The inverse of $y = a^x$ (EXPONENTIAL FUNCTION) is $x = a^y$ (REMEMBER x becomes y and y becomes x)
- To make y the subject of the inverse we use the LOG function.
- THEREFORE $x = a^y$ becomes $y = \log_a x$
- If $\mathrm{f(x)} = a^x$ then the inverse is $f^{-1}(x) = \log_a x$

# EXAMPLE INVOLVING GRAPHS

Example

If $f(x) = 3^x$

a. Determine $f^{-1}\ (INVERSE)$ in the form of y = .....

b. Sketch the graphs of $f^{-1}(x)$ and $f(x)$ and $y = x$ on the same set of axes

c. Determine the Domain and Range of $f^{-1}(x)$ and $f(x)$

# ANSWER TO EXAMPLE INVOLVING GRAPHS

a. $f(x) = 3^x$

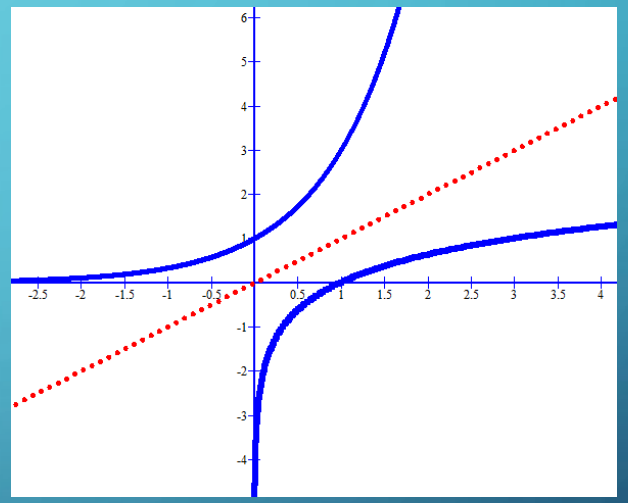
$y = 3^x$

$x = 3^y$

$y = \log_3 x$

$\therefore f^{-1}(x) = \log_3 x$

b.




c.

Domain of f(x): $x \in \mathbb{R}$

Range of f(x) : $y > 0$

Domain of $f^{-1}(x)$: $x > 0$

Range of $f^{-1}(x)$: $y \in \mathbb{R}$

REMEMBER THAT EVERYTHING TO DO WITH x BECOMES y AND EVERYTHING TO DO WITH y BECOMES x. THIS APPLIES TO THE DOMAIN AND RANGE AS WELL.

# SKETCHING THE GRAPHS

To Sketch the Exponential and Log Graph, use a table.

$$f(x) = 3^x$$

- FOR THE FUNCTION: CHOOSE ANY X-Values. As long as there is negatives and positives and 0.
- Substitute into the function to get the y-value.
- Plot the points and draw a freehand curve between the points

| | | | | | | |
|---|---|---|---|---|---|---|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| $f(x) = 3^x$ | $3^{-2} = \frac{1}{9}$ | $3^{-1} = \frac{1}{3}$ | $3^0 = 1$ | $3^1 = 3$ | $3^2 = 9$ | $3^3 = 27$ |
| POINTS TO PLOT | $(-2; \frac{1}{9})$ | $(-1; \frac{1}{3})$ | (0;1) | (1;3) | (2;9) | (3;27) |

FOR THE INVERSE: CHOOSE ANY y-Values. As long as there is negatives and positives and 0

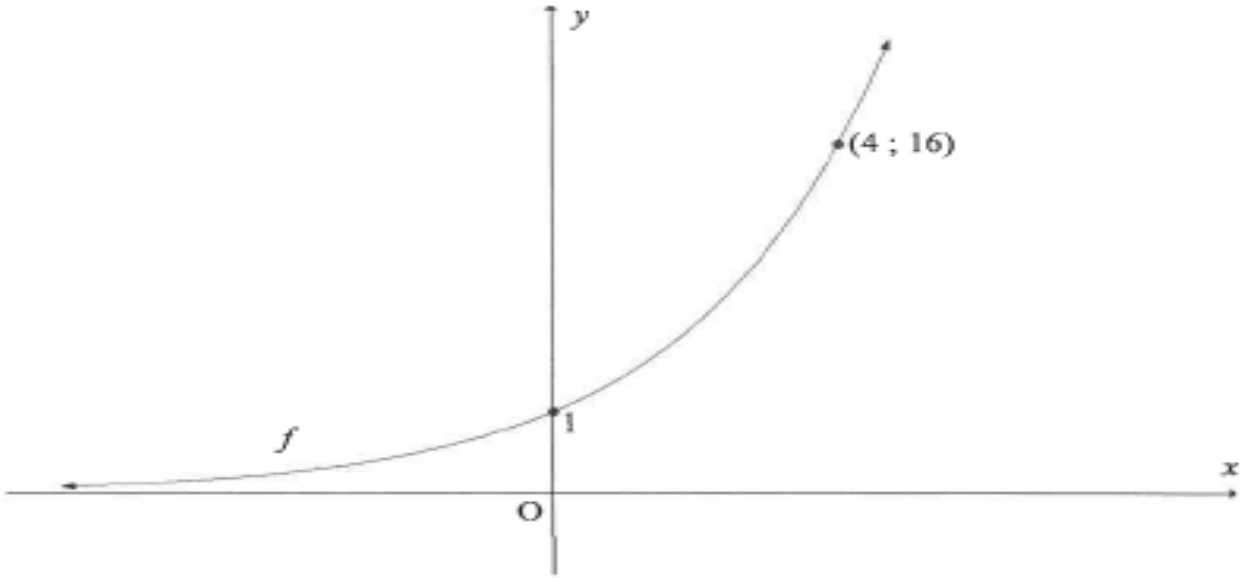
| | | | | | | |
|---|---|---|---|---|---|---|
| y | -2 | -1 | 0 | 1 | 2 | 3 |
| $x = 3^y (which\ is\ y = \log_3 x)$ | $3^{-2} = \frac{1}{9}$ | $3^{-1} = \frac{1}{3}$ | $3^0 = 1$ | $3^1 = 3$ | $3^2 = 9$ | $3^3 = 27$ |
| POINTS TO PLOT | $(\frac{1}{9}; -2)$ | $(\frac{1}{3}; -1)$ | (1;0) | (3;1) | (9;2) | (27;3) |

Notice that the x and y coordinates swap around from the function to the inverse.

# PAST PAPER QUESTION

**QUESTION 5**

Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $(4 ; 16)$ lies on $f$.




5.1 Determine the value of $k$. (2)

5.2 Graph $g$ is obtained by reflecting graph $f$ about the line $y = x$. Determine the equation of $g$ in the form $y = \ldots$ (2)

5.3 Sketch the graph $g$. Indicate on your graph the coordinates of two points on $g$. (4)