GRADE 12 FUNCTIONS PART 3

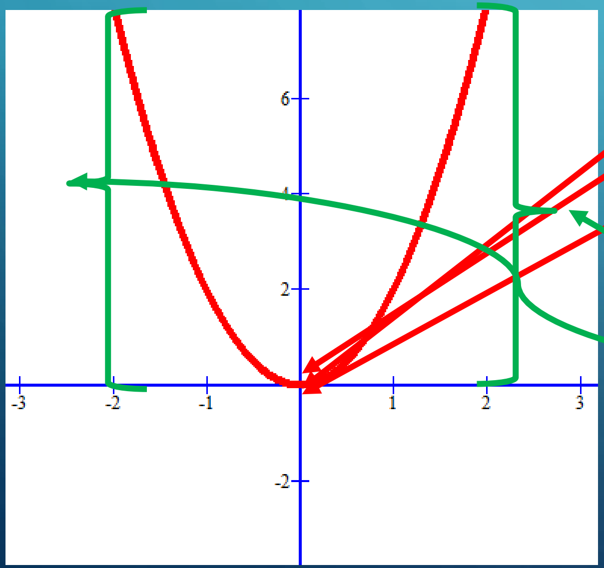
INVERSES OF:

- $y = ax^2$

LET US LOOK AT THE FUNCTION FIRST BEFORE THE INVERSE OF THE FUNCTION

$y = ax^2$

- **EXAMPLE**
- $y = 2x^2 \text{ or } f(x) = 2x^2$



Quadratic Graphs - PARABOLA

Domain

$x \in \mathbb{R}$

Range

$y \geq 0$

Y-Intercept (x=0)

(0;0)

X-Intercept (y=0)

(0; 0)

TURNING POINT

(0;0)

ASYMPTOTES

None

GRADIENT

Variable(changes at different points because it is a curve)

INCREASING OR DECREASING FUNCTION

Decreasing Function for values of $x < 0$

Increasing Function for values of $x > 0$

LET US NOW LOOK AT THE INVERSE FUNCTION

REPRESENTS a y-value still as f(x) did.

IMPORTANT

- $f^{-1}(x)$ *is the inverse function notation of the inverse of* $f(x)$
- X BECOMES Y and Y BECOMES X
- THIS MEANS THE FOLLOWING :
  - DOMAIN OF THE FUNCTION BECOMES THE RANGE OF THE INVERSE FUNCTION (and vice versa)
  - THE Y-INTERCEPT OF THE FUNCTION BECOMES X-INTERCEPT (and vice versa)
  - THE LINE OF SYMMETRY BETWEEN THE FUNCTION AND THE INVERSE FUNCTION IS Y=X

THE FUNCTION NEEDS TO BE IN THE FORM OF Y=...... OR $f^{-1}(x) = \cdots$.

LET US NOW LOOK AT THE INVERSE FUNCTION

LET US LOOK AT AN EXAMPLE TO ILLUSTRATE THE CONCEPT:

$$y = 2x^2 \text{ or } f(x) = 2x^2$$

We use $y = 2x^2$

1. **X becomes Y and Y becomes X**

$x = 2y^2$

Change to the form of y = ...

$x = 2y^2$

$\frac{x}{2} = y^2$

$\pm\sqrt{\frac{x}{2}} = y$

$y = \pm\sqrt{\frac{x}{2}}$ ← INVERSE FUNCTION

$\therefore f^{-1}(x) = \pm\sqrt{\frac{x}{2}}$

REMEMBER X (and everything associated with X) becomes Y ((and everything associated with Y) and VICE VERSA

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(0;0)

X-Intercept (y=0)

(0; 0)

TURNING POINT

(0;0)

ASYMPTOTES

None

GRADIENT

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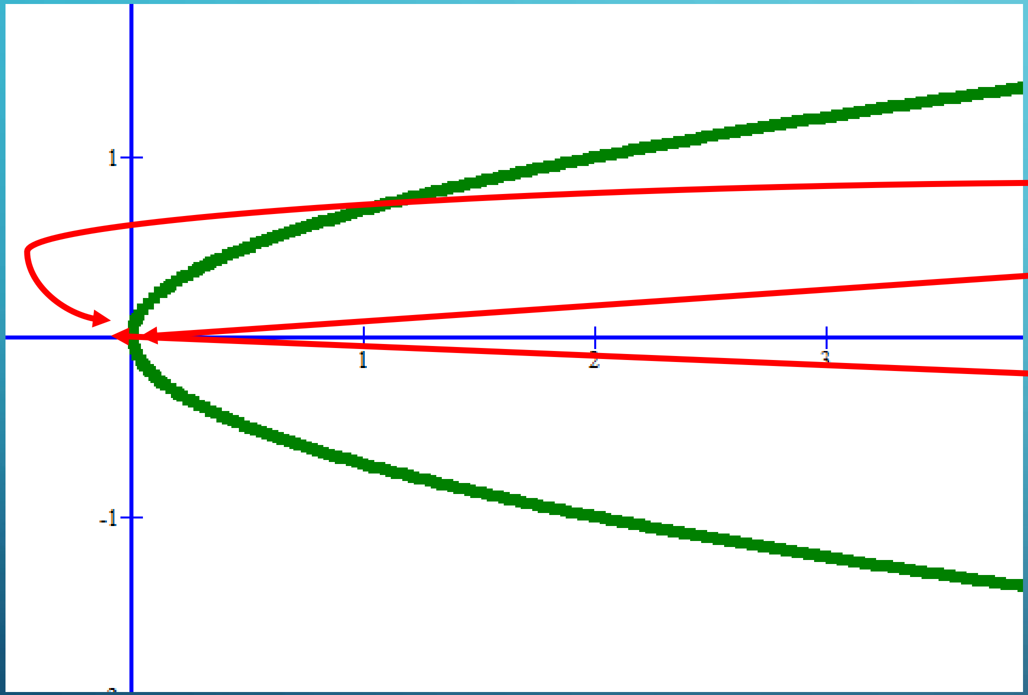
LET US NOW LOOK AT THE INVERSE OF THE FUNCTION GRAPH



Quadratic Graphs - PARABOLA

Domain

$x \geq 0$

Range

$y \in \mathbb{R}$

Y-Intercept (x=0)

(0;0)

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(0; 0)

TURNING POINT

(0;0)

ASYMPTOTES

None

GRADIENT

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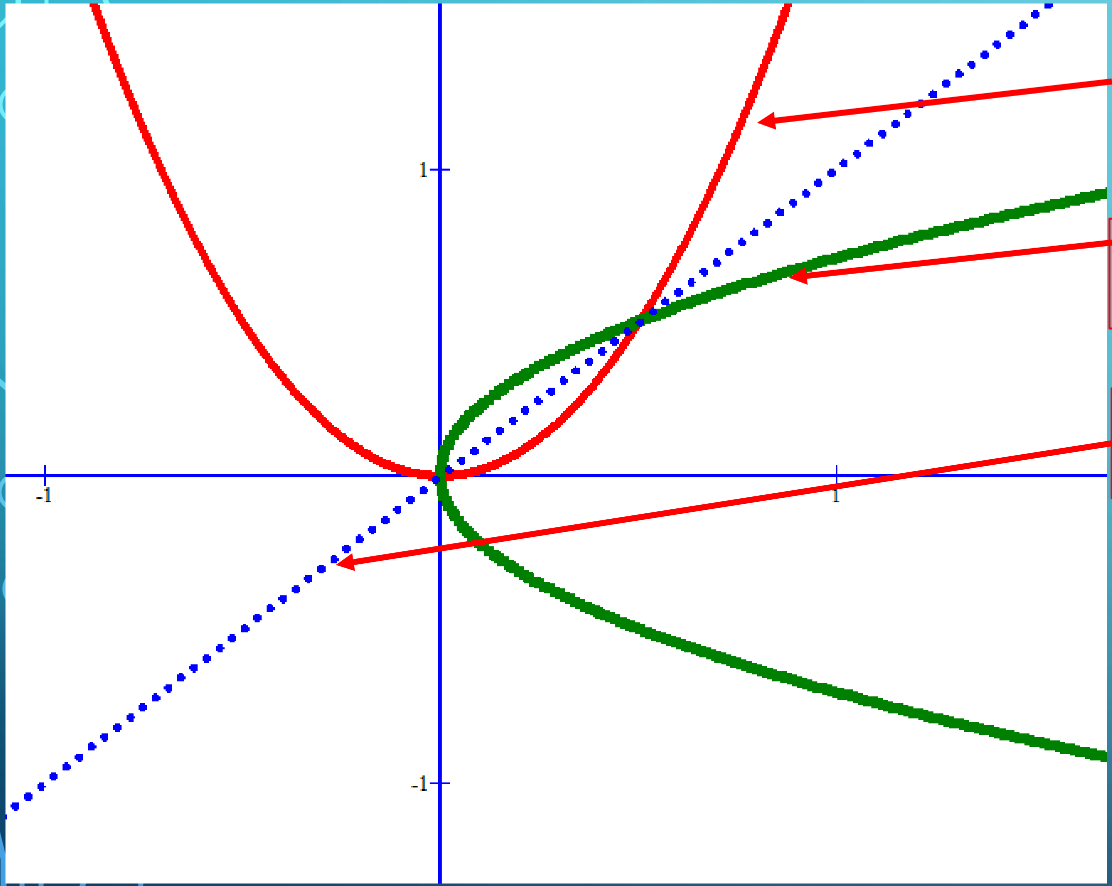
INVERSE IS NOT A FUNCTION (Vertical Line Test)

COMPARE THE FUNCTION TO THE INVERSE GRAPH



PARABOLA

INVERSE OF PARABOLA

Axis of Symmetry (y=x)

EXAMPLES

- Example 1

If f(x) $= -2x^2$, Determine the following:

a. The inverse of the function in the form of y = ….
b. Determine the Domain and Range of f(x).
c. Determine the Domain and Range of $f^{-1}(x)$.
d. Sketch f(x) and $f^{-1}(x)$ on the same set of axes. Also include the y=x (axis of symmetry) as well.

EXAMPLES

- <u>Example 1</u>

If f(x) = $-2x^2$, Determine the following:

a. The inverse of the function in the form of y = ….

ANSWER

a. $y = -2x^2$

$x = -2y^2$ x becomes y and y becomes x

$\frac{x}{-2} = y^2$

$\pm\sqrt{-\frac{x}{2}} = y$

$y = \pm\sqrt{-\frac{x}{2}}$

EXAMPLES

- **Example 1**

If f(x) = $-2x^2$, Determine the following:

b. Determine the Domain and Range of f(x).

Answer

b.

Domain

$x \in \mathbb{R}$

Range

$y \geq 0$

EXAMPLES

- Example 1

If f(x) $= -2x^2$, Determine the following:

c. Determine the Domain and Range of $f^{-1}(x)$.

Answer

c.

Domain

$x \geq 0$

Range

$y \in \mathbb{R}$

EXAMPLES

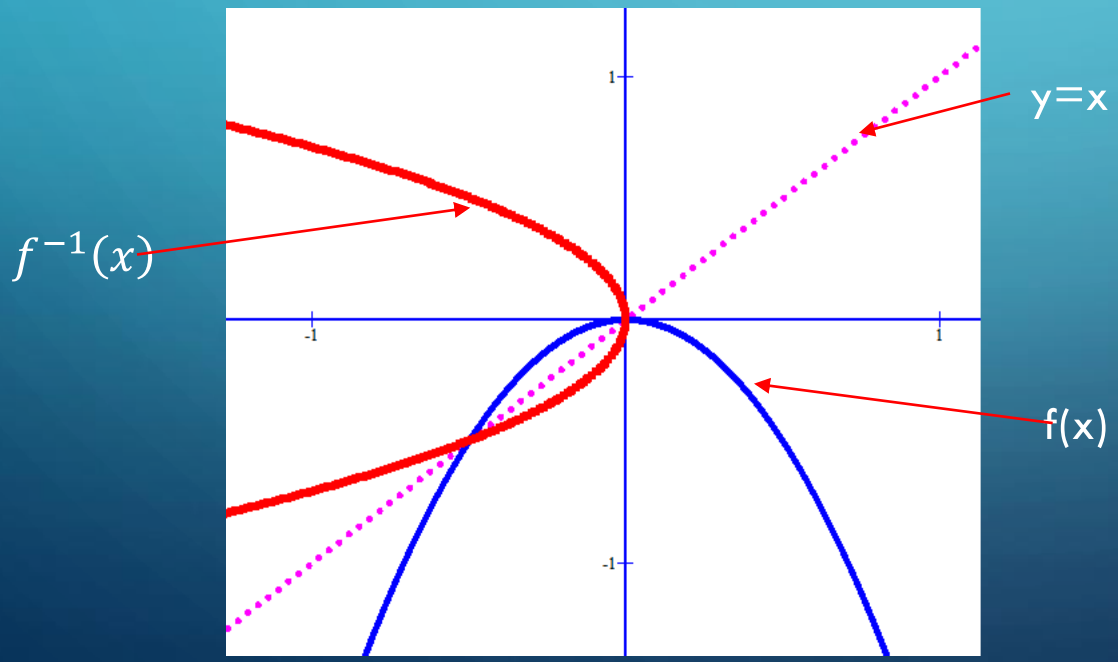
- Example 1

If f(x) $= -2x^2$, Determine the following:

d. Sketch f(x) and $f^{-1}(x)$ on the same set of axes. Also include the y=x (axis of symmetry) as well.

<u>Answer</u>

d.



EXAMPLES

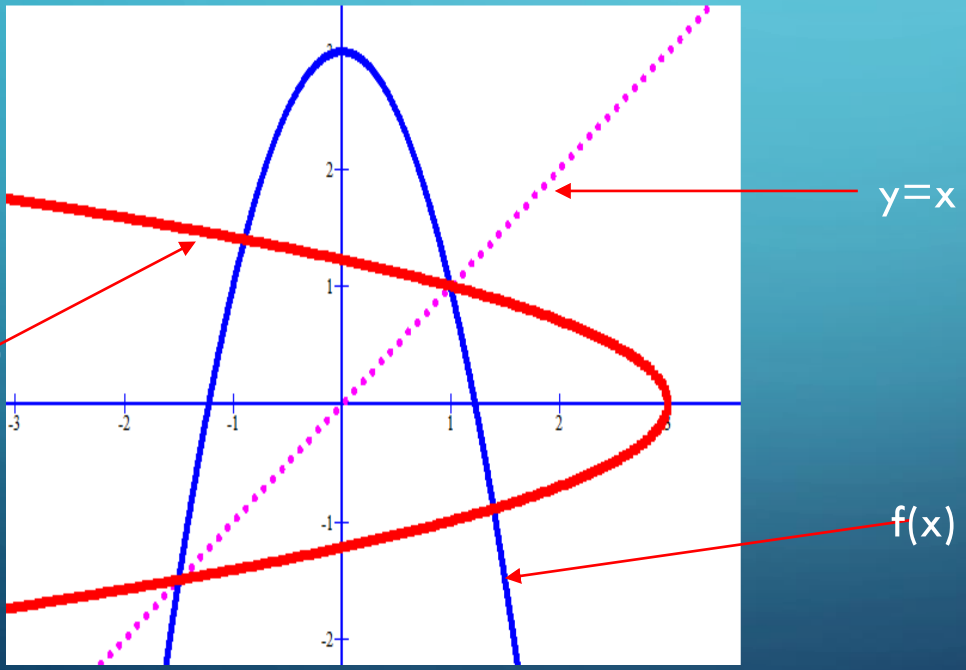
- **Example 2**

If f(x) = $-2x^2 + 3$, Determine the following:

a. Sketch f(x) and $f^{-1}(x)$ on the same set of axes. Also include the y=x (axis of symmetry) as well.

Answer

a.



$f^{-1}(x)$

$y = -2x^2 + 3$ *x becomes y and y becomes x*

$x = -2y^2 + 3$

$\frac{x}{-2} - 3 = y^2$

$\pm\sqrt{-\frac{x}{2} - 3} = y$

$y = \pm\sqrt{-\frac{x}{2} - 3}$

$\therefore f^{-1}(x) = \pm\sqrt{-\frac{x}{2} - 3}$