

GRADE 12

Functions REVISION Reflection about the y – axis and average gradient and gradient of a curve at a point.

WEBSITE NOTES

TOPIC:

- Reflection about the y – axis (All functions)
- Average gradient and gradient of a curve at a point.

REMEMBER THE FOLLOWING

Function change	Shift
$f(x) + c$	Shift the graph of $f(x)$ up c units
$f(x) - c$	Shift the graph of $f(x)$ down c units
$f(x + c)$	Shift the graph of $f(x)$ left c units
$f(x - c)$	Shift the graph of $f(x)$ right c units
$-f(x)$	Reflect the graph of $f(x)$ about the x-axis
$f(-x)$	Reflect the graph of $f(x)$ about the y-axis
$f(cx)$	Compress the graph of $f(x)$ horizontally by a factor of c .
$c.f(x)$	Stretch the graph of $f(x)$ vertically by a factor of c .

Reflection about y-axis

Straight Line Graph

$$f(x) = mx + c$$

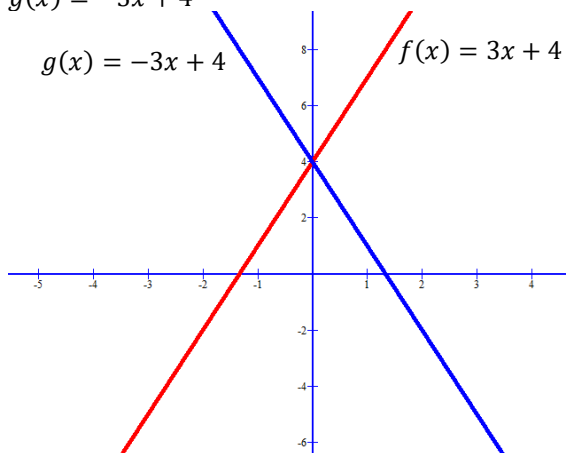
Example 1

$$f(x) = 3x + 4$$

Write down the equation $g(x)$ which reflects $f(x)$ about the y-axis (x becomes “negativized”)

$$f(-x) = 3(-x) + 4$$

$$g(x) = -3x + 4$$



Parabola

$$f(x) = a(x + p)^2 + q \quad \text{or} \quad f(x) = ax^2 + bx + c$$

Example 2

$$f(x) = 2(x + 2)^2 + 3 \quad \text{OR after multiplying out} \quad f(x) = 2x^2 + 8x + 11$$

Write down the equation $g(x)$ which reflects $f(x)$ about the y-axis (x becomes “negativized”)

$$f(-x) = 2(-x + 2)^2 + 3$$

$$f(-x) = 2(-x + 2)^2 + 3$$

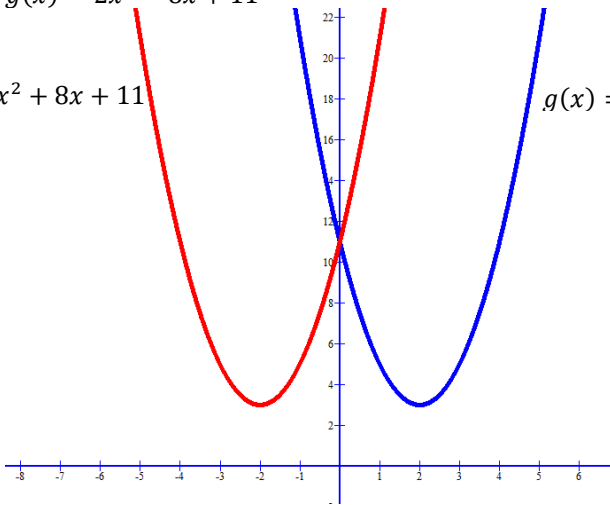
Take note that the turning point is $(-2 ; 3)$

$$g(x) = 2(x^2 - 4x + 4) + 3$$

$$g(x) = 2x^2 - 8x + 11$$

$$f(x) = 2x^2 + 8x + 11$$

$$g(x) = 2x^2 - 8x + 11$$



Hyperbola

$$f(x) = \frac{a}{x+p} + q$$

Example 3

$$f(x) = \frac{3}{x-1} - 1$$

Remember the axis of symmetry is $x=1$
and $y=-1$

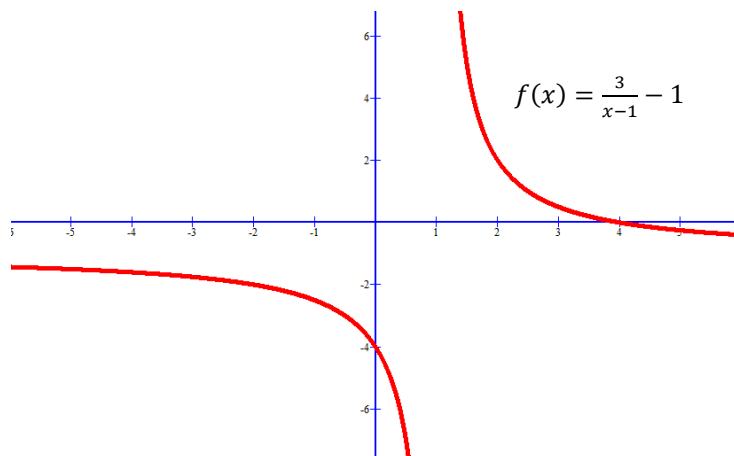
Write down the equation $g(x)$ which reflects $f(x)$ about the y -axis (x becomes "negativized")

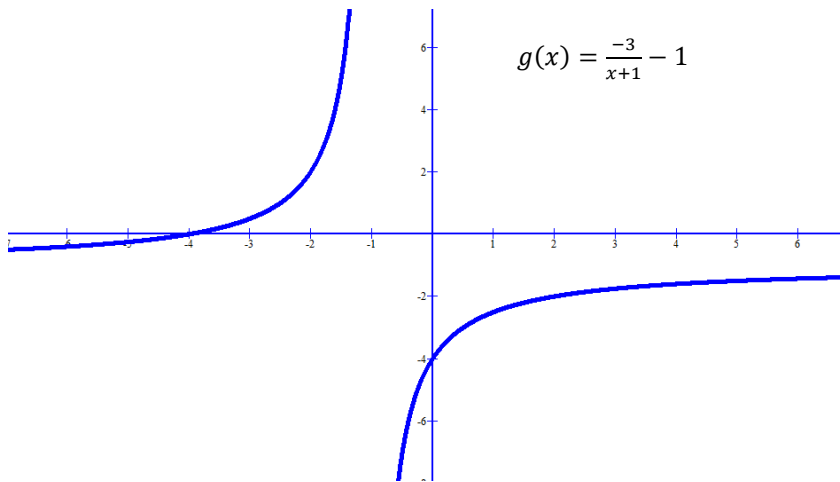
$$f(-x) = \frac{3}{(-x)-1} - 1$$

$$g(x) = \frac{3}{-(x+1)} - 1$$

$$g(x) = \frac{-3}{x+1} - 1$$

Remember the axis of symmetry is now
 $x=-1$ and $y=-1$





Exponential Graphs

$f(x) = ab^{x+p} + q$

Asymptote is $y = 3$

Example 4

$f(x) = 5 \cdot 2^{x-2} + 3$

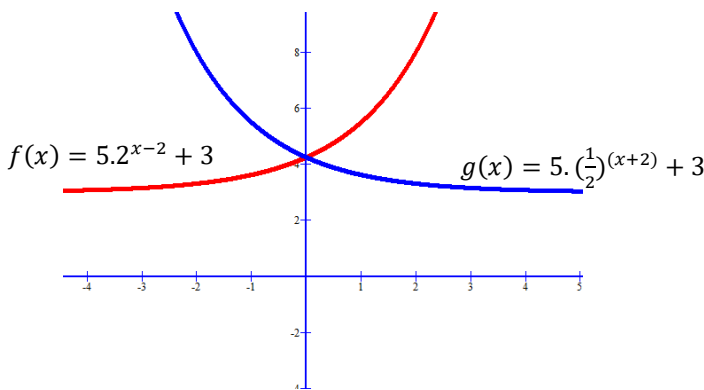
Write down the equation $g(x)$ which reflects $f(x)$ about the y-axis (x becomes “negativized”)

$f(-x) = 5 \cdot 2^{(-x)-2} + 3$

$g(x) = 5 \cdot 2^{-(x+2)} + 3$

Asymptote is $y = 3$

$g(x) = 5 \cdot \left(\frac{1}{2}\right)^{(x+2)} + 3$



Example 5 (Try yourself)

Consider $f(x) = \frac{4}{x+2} - 1$

- a. Write down the asymptotes
- b. Write down the equation $g(x)$ which reflects $f(x)$ about the y-axis
- c. **Try the following question Determine the symmetry lines**

AVERAGE GRADIENT BETWEEN TWO POINTS

$m = \frac{y_2 - y_1}{x_2 - x_1}$

A curve does not have a specific gradient like a straight-line graph. A curve can have an average gradient.

REMEMBER y_2 IS ALSO $f(x_2)$ and y_1 IS ALSO $f(x_1)$

Example 1

Consider $f(x) = x^2 + 7x + 10$. Determine the average gradient between the points $x=2$ and $x=-1$

Answer

1. First work out the y value at $x=2$ and $x=-1$

LET $x_1=2$ and $x_2=-1$

$$f(2) = (2)^2 + 7(2) + 10.$$

$$f(2) = 4 + 14 + 10$$

$$f(2) = 28 = y_1$$

$$f(-1) = (-1)^2 + 7(-1) + 10$$

$$f(-1) = 1 - 7 + 10$$

$$f(-1) = 4 = y_2$$

2. Use the gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4 - 28}{-1 - 2}$$

$$m = \frac{-24}{-3} = 8$$

The average gradient between $x=2$ and $x=-1$ for $f(x)$ is 8.

Example 2 (Try yourself)

Determine the average gradient of the graph of $y = 5x^2 - 4$ between:

a) $x = 1$ and $x = 3$

b) $x = 2$ and $x = 3$

Example 3 (Try yourself)

Determine the average gradient of the graph of $g(x) = \frac{4}{x-3} - 1$ between:

a) $x = -1$ and $x = 0$