GRADE 11

Functions 5 Reflection about the y – axis and average gradient and gradient of a curve at a point.

Answers

WEBSITE NOTES

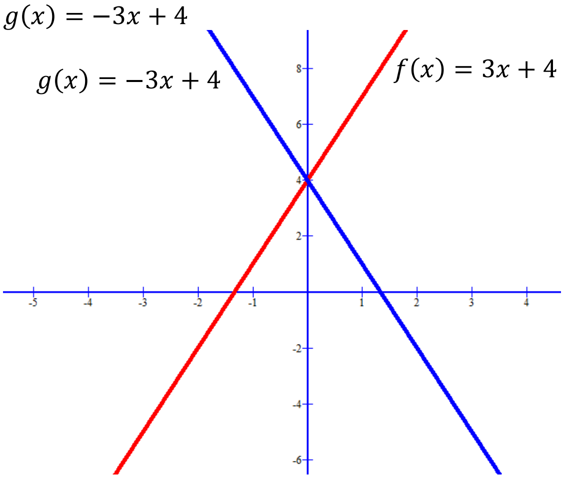
TOPIC:

- Reflection about the y – axis (All functions)
- Average gradient and gradient of a curve at a point.

REMEMBER THE FOLLOWING

Function change	Shift
f(x) + c	Shift the graph of f(x) up c units
f(x) - c	Shift the graph of f(x) down c units
f(x + c)	Shift the graph of f(x) left c units
f(x - c)	Shift the graph of f(x) right c units
-f (x)	Reflect the graph of f(x) about the x-axis
f (-x)	Reflect the graph of f(x) about the y-axis
f(c.x)	Compress the graph of f(x) horizontally by a factor of c.
c.f(x)	Stretch the graph of f(x) vertically by a factor of c.

Reflection about y-axis
Straight Line Graph
$f(x) = mx + c$
Example 1
$f(x) = 3x + 4$
Write down the equation g(x) which reflects f(x) about the y-axis (x becomes "negativized")
$f(-x) = 3(-x) + 4$
$g(x) = -3x + 4$



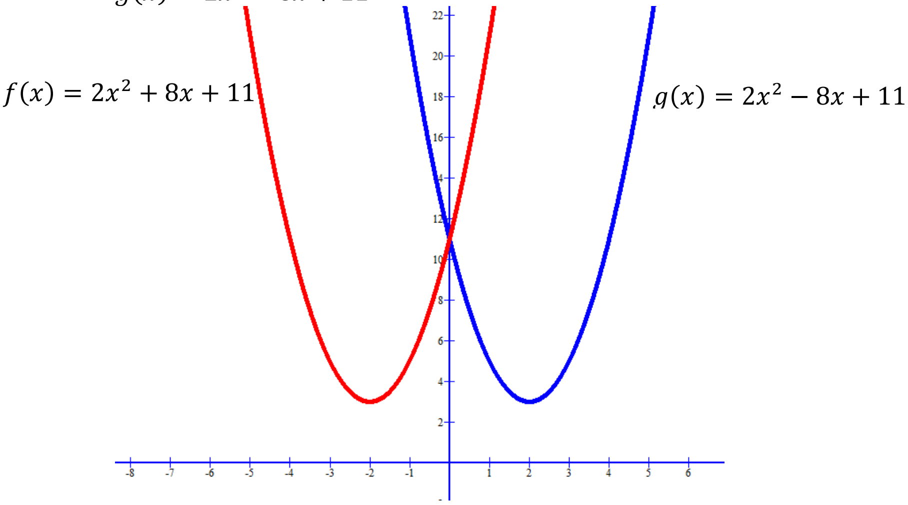
Parabola
$f(x) = a(x + p)^2 + q$ or $f(x) = ax^2 + bx + c$

Take note that the turning point is (-2 ;3)

Example 2
$f(x) = 2(x + 2)^2 + 3$ OR after multiplying out $f(x) = 2x^2 + 8x + 11$

Write down the equation g(x) which reflects f(x) about the y-axis (x becomes "negativized")

$f(-x) = 2(-x+2)^2 + 3$
$f(-x) = 2(-x+2)^2 + 3$
$g(x) = 2(x^2 - 4x + 4) + 3$
$g(x) = 2x^2 - 8x + 11$



Hyperbola

$f(x) = \frac{a}{x+p} + q$

Example 3

$f(x) = \frac{3}{x-1} - 1$

Remember the axis of symmetry is x=1 and y=-1

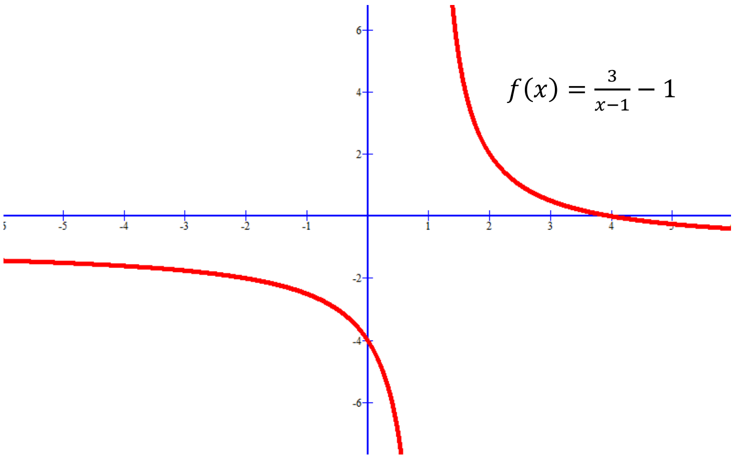
Write down the equation g(x) which reflects f(x) about the y-axis (x becomes "negativized")

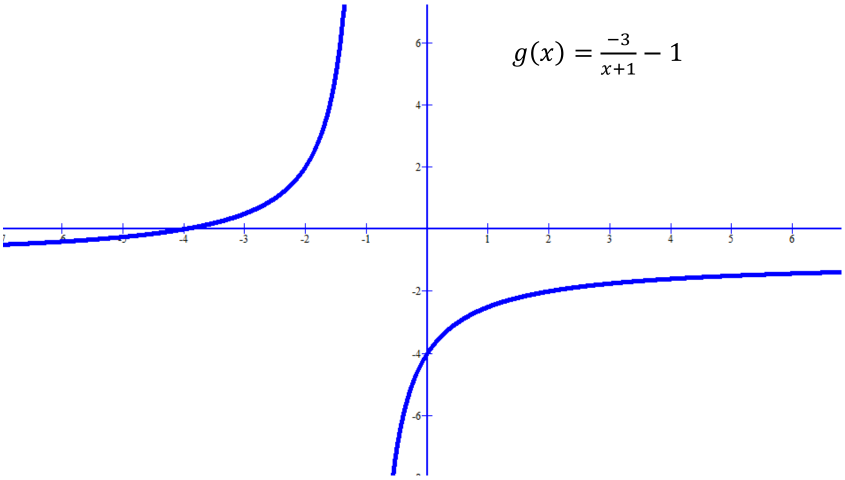
$f(-x) = \frac{3}{(-x)-1} - 1$

$g(x) = \frac{3}{-(x+1)} - 1$

$g(x) = \frac{-3}{x+1} - 1$

Remember the axis of symmetry is now x=-1 and y=-1





Exponential Graphs

$f(x) = ab^{x+p} + q$

Asymptote is y = 3

Example 4

$f(x) = 5.2^{x-2} + 3$

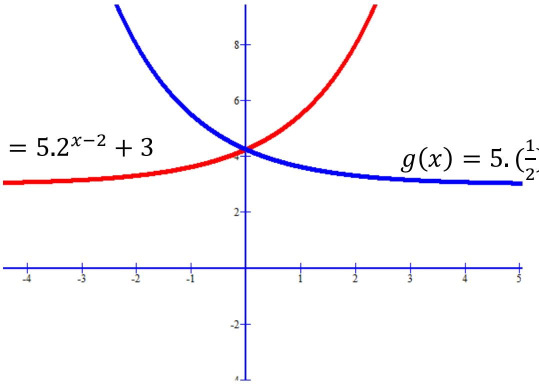
Write down the equation g(x) which reflects f(x) about the y-axis (x becomes "negativized")

$f(-x) = 5.2^{(-x)-2} + 3$

$g(x) = 5.2^{-(x+2)} + 3$

$g(x) = 5.(\frac{1}{2})^{(x+2)} + 3$

Asymptote is y = 3



Example 5 (Try yourself)

Consider $f(x) = \frac{4}{x+2} - 1$

a. Write down the asymptotes
b. Write down the equation g(x) which reflects f(x) about the y-axis
c. ***Try the following question Determine the symmetry lines***

Answers

a. x = -2 and y = -1
b. $f(x) = \frac{4}{-x+2} - 1$

   $f(-x) = \frac{4}{-(x-2)} - 1$

   $g(x) = \frac{-4}{x-2} - 1$
c. Symmetry Line 1: y = x + c
   -1 = -2 +c
   1=c
   y = x + 1

Symmetry Line 2: y = - x + c
-1 = -(-2) + c
-1 = 2 + c
-3 = c
y = -x -3

AVERAGE GRADIENT BETWEEN TWO POINTS

A curve does not have a specific gradient like a straight-line graph. A curve can have an average gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

REMEMBER y_2 IS ALSO $f(x_2)$ and y_1 IS ALSO $f(x_1)$

Example 1

Consider $f(x) = x^2 + 7x + 10$. Determine the average gradient between the points x=2 and x=-1

Answer

1. **First work out the y value at x=2 and x=-1**

   LET x_1=2 and x_2=-1

   $f(2) = (2)^2 + 7(2) + 10.$
   $f(2) = 4 + 14 + 10$
   $f(2) = 28 = y_1$

   $f(-1) = (-1)^2 + 7(-1) + 10$
   $f(-1) = 1 - 7 + 10$
   $f(-1) = 4 = y_2$

2. **Use the gradient formula**

   $m = \frac{y_2 - y_1}{x_2 - x_1}$
   $m = \frac{4-28}{-1-2}$
   $m = \frac{-24}{-3} = 8$

   The average gradient between x=2 and x=-1 for f(x) is 8.

Example 2 (Try yourself)

Determine the average gradient of the graph of $y = 5x^2 - 4$ between:

a) $x = 1$ and $x = 3$

b) $x = 2$ and $x = 3$

Answers

a) The y-values at $x = 1$ and $x = 3$ are 1 and 41

$m = 20$

The y-values at $x = 2$ and $x = 3$ are 16 and 41

$m = 25$

Example 3 (Try yourself)

Determine the average gradient of the graph of $g(x) = \frac{4}{x-3} - 1$ between:

a) $x = -1$ and $x = 0$

Answers

The y-values at $x = -1$ and $x = 0$ are -2 and $-\frac{7}{3}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-\frac{7}{3} - (-2)}{0 - (-1)}$

$m = -\frac{1}{3}$