

## GRADE 11

### Functions 4 ANSWERS – Answers at the end

#### WEBSITE NOTES

##### TOPIC:

- Revise the effect of  $a$  and  $q$  and investigate the effect of  $p$  on the graphs of the functions defined by:

- $y = f(x) = ab^{x+p} + q; b > 0; b \neq 1$

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#### REMEMBER THE FOLLOWING

Function change	Shift
$f(x) + c$	Shift the graph of $f(x)$ up $c$ units
$f(x) - c$	Shift the graph of $f(x)$ down $c$ units
$f(x + c)$	Shift the graph of $f(x)$ left $c$ units
$f(x - c)$	Shift the graph of $f(x)$ right $c$ units
$-f(x)$	Reflect the graph of $f(x)$ about the $x$ -axis
$f(-x)$	Reflect the graph of $f(x)$ about the $y$ -axis
$f(c.x)$	Compress the graph of $f(x)$ horizontally by a factor of $c$ .
$c.f(x)$	Stretch the graph of $f(x)$ vertically by a factor of $c$ .

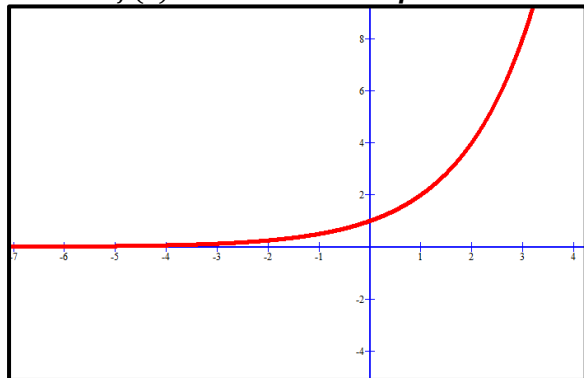
#### Exponential Graphs

##### Effect of $a$ , $p$ and $q$

$$f(x) = ab^{x+p} + q$$

##### Example 1

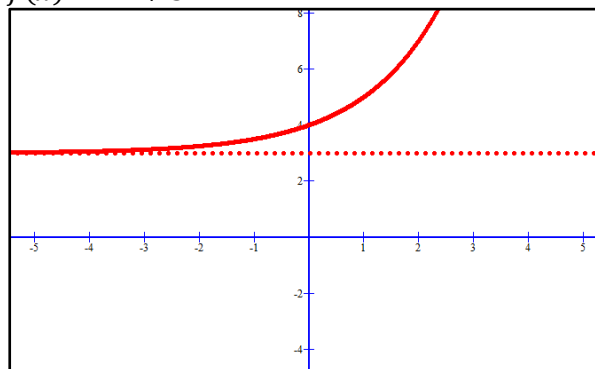
consider  $f(x) = 2^x$  for the examples that follow as well



- NO Vertical asymptote
- Y-Intercept is 1. (When  $x=0$ )
- The Horizontal asymptote  $y = 0$
- The graph increases from left to right therefore as  $x$  values increase so do the  $y$  values. We call this an increasing function.

##### Example 2

$$f(x) = 2^x + 3$$

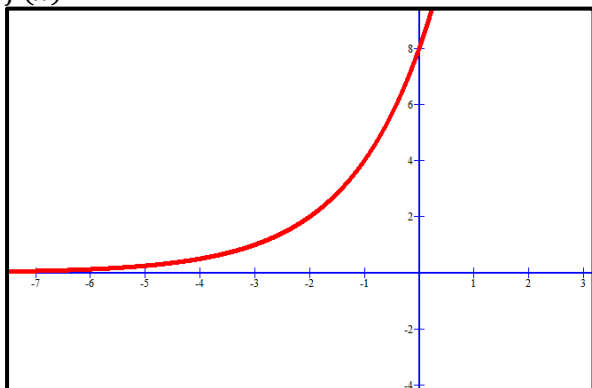


- NO Vertical asymptote
- Y-Intercept is 4. (When  $x=0$ )
- The Horizontal asymptote  $y = 3$ . (dotted line)
- The graph increases from left to right therefore as  $x$  values increase so do the  $y$  values. We call this an increasing function.

**The Graph has shifted up 3 units from the origin**

### Example 3

$$f(x) = 2^{x+3}$$



- NO Vertical asymptote
- Y-Intercept is 8. (When  $x=0$ )
- The Horizontal asymptote  $y = 0$ .
- The graph increases from left to right therefore as  $x$  values increase so do the  $y$  values. We call this an increasing function.

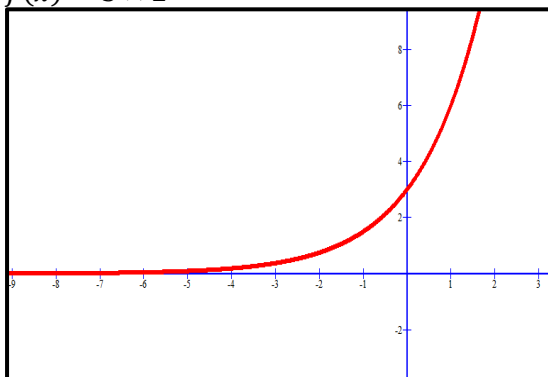
***The Graph has shifted left 3 units from the origin.***

#### SUMMARY SO FAR

$p$  changes will move the graph left or right.  
 $q$  changes will move the graph up or down.

### Example 4

$$f(x) = 3 \times 2^x$$

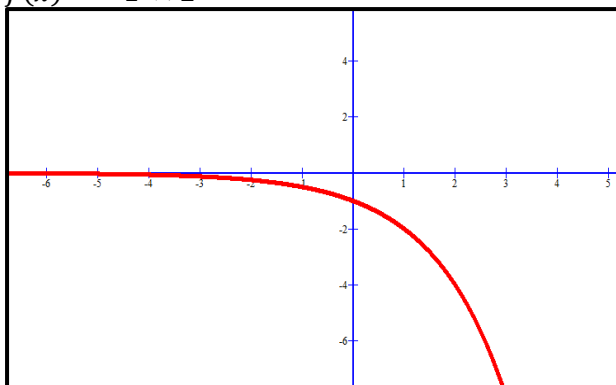


- NO Vertical asymptote
- Y-Intercept is 3. (When  $x=0$ )
- The Horizontal asymptote  $y = 0$ .
- The graph increases from left to right therefore as  $x$  values increase so do the  $y$  values. We call this an increasing function.

***The Graph has become steeper because the value of  $a$  has increased. Remember we are comparing to the graph  $f(x) = 2^x$  and  $a$  was 1. ( $f(x) = 1 \times 2^x$  is the same as  $f(x) = 2^x$ )***

### Example 5

$$f(x) = -1 \times 2^x$$

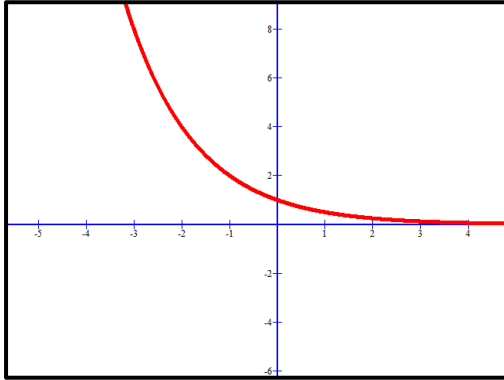


- NO Vertical asymptote
- Y-Intercept is -1. (When  $x=0$ )
- The Horizontal asymptote  $y = 0$ .
- The graph decreases from left to right therefore as  $x$  values increase and the  $y$  values decrease. We call this a decreasing function.

***The Graph has reflected about the  $x$ -axis because the value of  $a$  is negative. Remember we are comparing to the graph  $f(x) = 2^x$  and  $a$  was 1. ( $f(x) = 1 \times 2^x$  is the same as  $f(x) = 2^x$ )***

### Example 6

$$f(x) = \frac{1}{2} \times 1^x$$

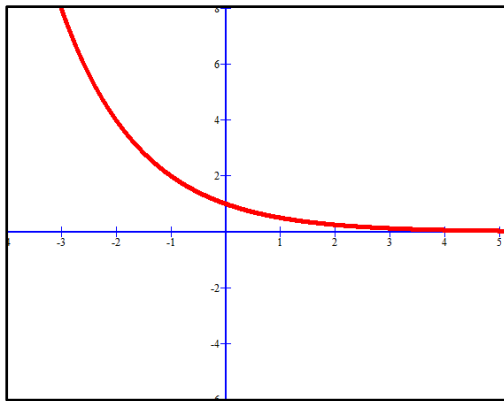


- NO Vertical asymptote
- Y-Intercept is 1. (When x=0)
- The Horizontal asymptote y = 0.
- The graph decreases from left to right therefore as x values increase and the y values decrease. We call this a decreasing function.

**The Graph has reflected about the y-axis because the value of a is negative. We are comparing to the graph  $f(x) = 1^x$  and a is 1. ( $f(x) = 1 \times 1^x$  is the same as  $f(x) = 1^x$ )**

### Example 7

$$f(x) = 2^{-x}$$



- NO Vertical asymptote
- Y-Intercept is 1. (When x=0)
- The Horizontal asymptote y = 0.
- The graph decreases from left to right therefore as x values increase and the y values decrease. We call this a decreasing function.

**The Graph has reflected about the y-axis because the value of a is negative. We are comparing to the graph  $f(x) = 2^x$  and a is 1. ( $f(x) = 1 \times 2^x$  is the same as  $f(x) = 2^x$ )**

NOTE

$$f(x) = \frac{1}{2} \times 1^x \text{ is the same as } f(x) = 2^{-x}$$

WHY???

$$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x = \frac{1}{2} \times 1^x$$

### SUMMARY SO FAR

p changes will move the graph left or right.

q changes will move the graph up or down.

If a is positive and bigger number value, then the graph becomes steeper.

If a is more negative, then the graph is reflected about the x-axis

If a is fraction, then the graph is reflected about the y-axis

### Example 8 (Try yourself)

For the following Functions determine the following

- SHIFT
- ASYMPTOTE
- Y-INTERCEPT

1.  $f(x) = 5.2^{x-2} + 3$

Answer:

- Shift  
2 units right and 3 units up
- Asymptote  
 $y = 3$
- Y-Intercept  
(When  $x=0$ )  
 $(0; 4 \frac{1}{4})$

2.  $f(x) = -4.3^{x+2} - 1$

- Shift  
2 units left and 1 unit down
- Asymptote

- $y = -1$   
C. Y-Intercept  
(0;37)
3.  $f(x) = 2.5^{x+1} - 2$   
A. Shift  
1 unit left and 2 units down  
B. Asymptote  
 $y = -2$   
C. Y-Intercept  
(0;8)
4.  $f(x) = 3 \cdot \frac{1}{2}^x + 1$   
A. Shift  
0 units left and 1 unit up  
B. Asymptote  
 $y = 1$   
C. Y-Intercept  
(0;4)