GRADE 11
Functions 4
WEBSITE NOTES

TOPIC:
- Revise the effect of *a* and *q* and investigate the effect of p on the graphs of the functions defined by:
  - $y = f(x) = ab^{x+p} + q; \quad b>0 \ ; \ b \neq 1$

REMEMBER THE FOLLOWING

Function change	Shift
f(x) + c	Shift the graph of f(x) up c units
f(x) - c	Shift the graph of f(x) down c units
f(x + c)	Shift the graph of f(x) left c units
f(x - c)	Shift the graph of f(x) right c units
-f (x)	Reflect the graph of f(x) about the x-axis
f (-x)	Reflect the graph of f(x) about the y-axis
f(c.x)	Compress the graph of f(x) horizontally by a factor of c.
c.f(x)	Stretch the graph of f(x) vertically by a factor of c.

Exponential Graphs
Effect of a, p and q

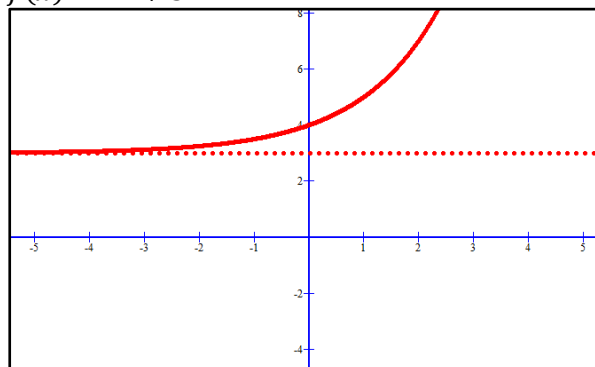$f(x) = ab^{x+p} + q$
Example 1
$consider \ f(x) = 2^x$ **for the examples that follow as well**
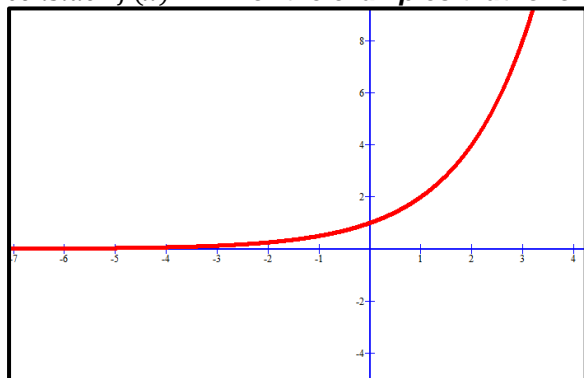
- NO Vertical asymptote
- Y-Intercept is 1. (When x=0)
- The Horizontal asymptote y = 0
- The graph increases from left to right therefore as x values increase so do the y values. We call this an increasing function.
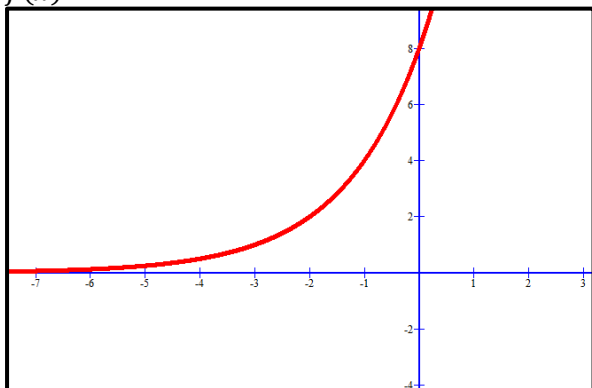
Example 2
$f(x) = 2^x + 3$

- NO Vertical asymptote
- Y-Intercept is 4. (When x=0)
- The Horizontal asymptote y = 3. (dotted line)
- The graph increases from left to right therefore as x values increase so do the y values. We call this an increasing function.
The Graph has shifted up 3 units from the origin

Example 3

$f(x) = 2^{x+3}$

- NO Vertical asymptote
- Y-Intercept is 8. (When x=0)
- The Horizontal asymptote y = 0.
- The graph increases from left to right therefore as x values increase so do the y values. We call this an increasing function.

The Graph has shifted left 3 units from the origin.

SUMMARY SO FAR
p changes will move the graph left or right.
q changes will move the graph up or down.

Example 4

$f(x) = 3 \times 2^x$

- NO Vertical asymptote
- Y-Intercept is 3. (When x=0)
- The Horizontal asymptote y = 0.
- The graph increases from left to right therefore as x values increase so do the y values. We call this an increasing function.
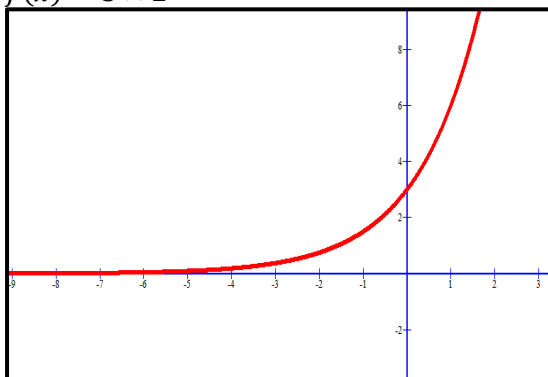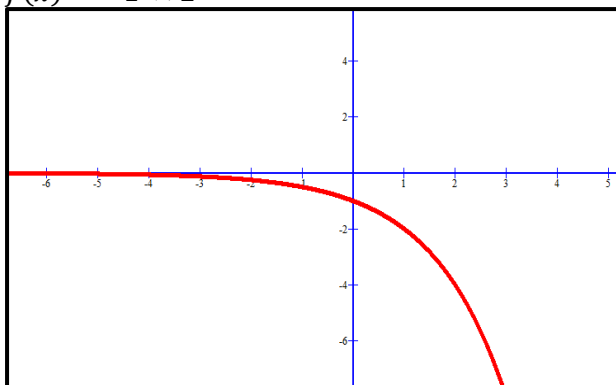
The Graph has become steeper because the value of a has increased. Remember we are comparing to the graph $f(x) = 2^x$ ***and a was 1.*** $(f(x) = 1 \times 2^x$ ***is the same as*** $f(x) = 2^x$)

Example 5

$f(x) = -1 \times 2^x$

- NO Vertical asymptote
- Y-Intercept is -1. (When x=0)
- The Horizontal asymptote y = 0.
- The graph decreases from left to right therefore as x values increase and the y values decrease. We call this a decreasing function.

The Graph has reflected about the x-axis because the value of a is negative. Remember we are comparing to the graph $f(x) = 2^x$ ***and a was 1.*** $(f(x) = 1 \times 2^x$ ***is the same as*** $f(x) = 2^x$)
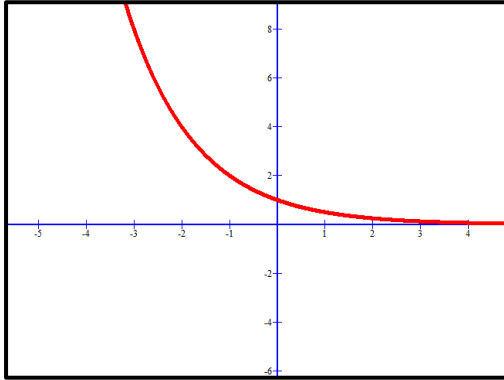
Example 6

$$f(x) = \frac{1}{2} \times 1^x$$

- NO Vertical asymptote
- Y-Intercept is 1. (When x=0)
- The Horizontal asymptote y = 0.
- The graph decreases from left to right therefore as x values increase and the y values decrease. We call this a decreasing function.

The Graph has reflected about the y-axis because the value of a is negative. We are comparing to the graph $f(x) = 1^x$ *and a is 1. (*$f(x) = 1 \times 1^x$ *is the same as* $f(x) = 1^x$ *)*

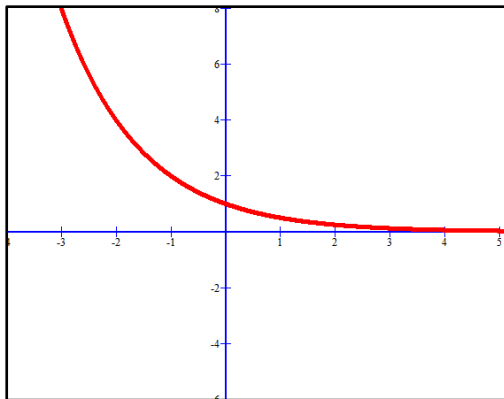## Example 7

$$f(x) = 2^{-x}$$

- NO Vertical asymptote
- Y-Intercept is 1. (When x=0)
- The Horizontal asymptote y = 0.
- The graph decreases from left to right therefore as x values increase and the y values decrease. We call this a decreasing function.

The Graph has reflected about the y-axis because the value of a is negative. We are comparing to the graph $f(x) = 2^x$ *and a is 1. (*$f(x) = 1 \times 2^x$ *is the same as* $f(x) = 2^x$ *)*

NOTE

$f(x) = \frac{1}{2} \times 1^x$ is the same as $f(x) = 2^{-x}$

WHY???

$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x = \frac{1}{2} \times 1^x$

SUMMARY SO FAR
p changes will move the graph left or right.
q changes will move the graph up or down.
If a is positive and bigger number value, then the graph becomes steeper.
If a is more negative, then the graph is reflected about the x-axis
If a is fraction, then the graph is reflected about the y-axis

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Example 8 (Try yourself)
For the following Functions determine the following
- A. SHIFT
- B. ASYMPTOTE
- C. Y-INTERCEPT
    1. $f(x) = 5.2^{x-2} + 3$
       Answer:
       A. Shift
          2 units right and 3 units up
       B. Asymptote
          y =3
       C. Y-Intercept
          (When x=0)
          4 ¼
    2. $f(x) = -4.3^{x+2} - 1$
    3. $f(x) = 2.5^{x+1} - 2$
    4. $f(x) = 3.\frac{1}{2}^x + 1$