GRADE 12

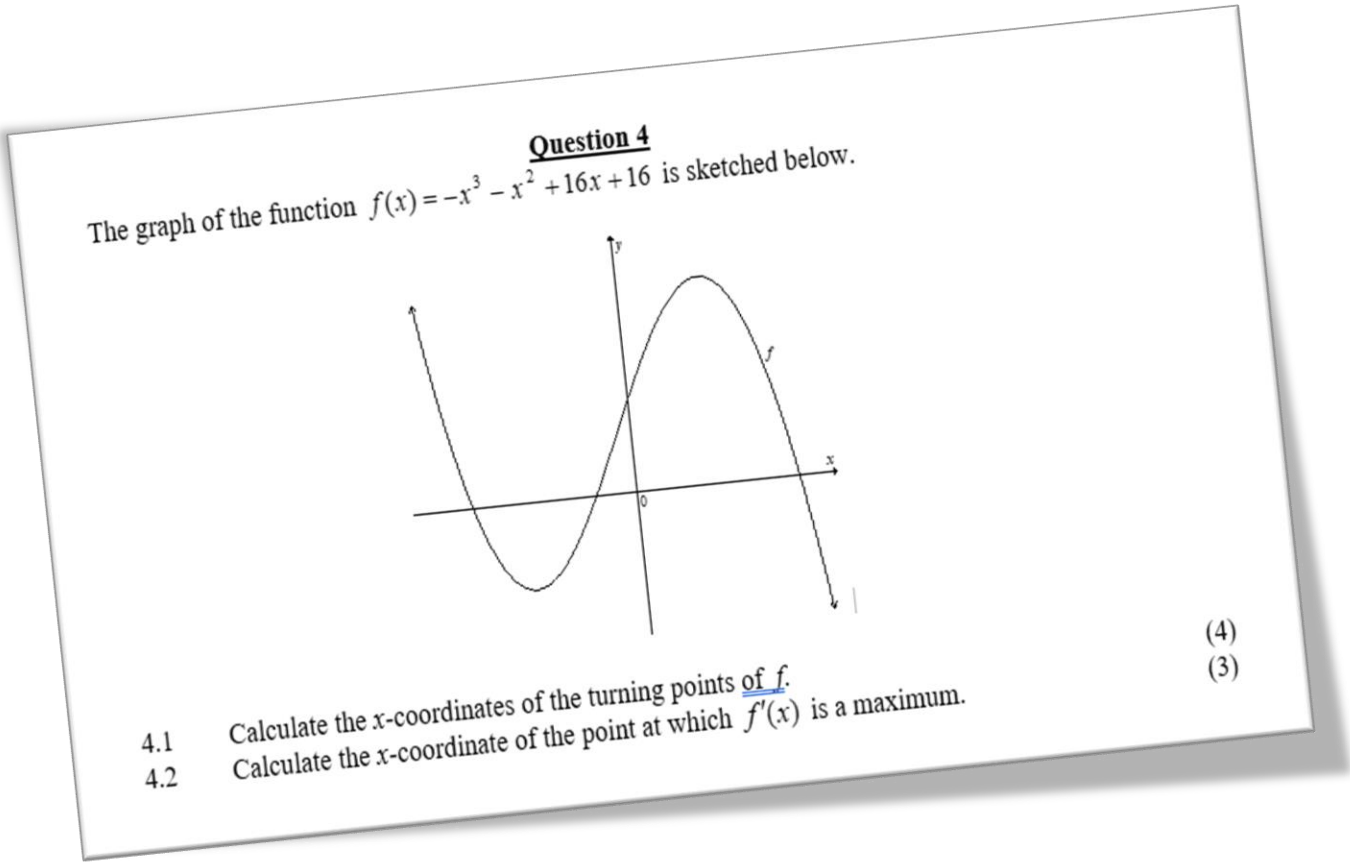
Calculus 6 – Revise Factorising Cubic functions and Sketching Cubic Functions.

WEBSITE NOTES

TOPIC:

- Cubic graphs

EXAM TYPE QUESTION



Question 4

The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.

4.1 Calculate the x-coordinates of the turning points of f. (4)

4.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

ANSWER

4.1 $f(x) = -x^3 - x^2 + 16x + 16$

The Turning Points will be where the Derivative (Gradient) = 0.

1. Work out the Derivative

   $f'(x) = -3x^2 - 2x + 16$

2. Equate the Derivative to 0

   $-3x^2 - 2x + 16 = 0$

3. Factorise

   $-(3x^2 + 2x - 16) = 0$

   $3x^2 + 2x - 16 = 0$

   $(3x + 8)(x - 2) = 0$

4. Solve for x

   $(3x + 8) = 0$ or $(x - 2) = 0$

   $x = -\frac{8}{3}$ or $x = 2$

   THEREFORE, THE X-COORDINATES OF THE TURNING POINT OF

   $f(x) = -x^3 - x^2 + 16x + 16$ is $x = -\frac{8}{3}$ AND $x = 2$

4.2 The maximum or minimum point of a parabola or cubic function is where the derivative = 0
For a cubic function there will be mostly a maximum and a minimum value because the derivative gives 2 answers. (
See above 4.1:

$x = -\frac{8}{3}$ will be a local minimum

$x = 2$ will be a local maximum

LOCAL because there are more than 1 turning point

NOW BACK TO THE QUESTION ASKED

A Parabola or Quadratic function can either have a maximum point or a minimum point. NOT BOTH.

Maximum or minimum point is where the graph turns. Where the derivative = 0.

$f'(x) = -3x^2 - 2x + 16$

1. Work out the second Derivative (Gradient)

   $f''(x) = -6x - 2$

2. Equate to 0

   $-6x - 2 = 0$

3. Solve for x

   $-6x - 2 = 0$

   $-6x = 2$

   $x = -\frac{2}{6} = -\frac{1}{3}$

Consider $f(x) = ax^2 + bx + c$

The above function will have a maximum if a is positive

The above function will have a minimum if a is negative

This will also be the x-coordinate of the point of infliction of f(x).

To determine the y-value substitute x= $-\frac{1}{3}$ for this example

The x-coordinate where $f'(x)$ is a maximum is $x = -\frac{1}{3}$