GRADE 12
Calculus 4– Second derivative and concavity.
WEBSITE NOTES

TOPIC:

- Second derivative and concavity.

Example 1

Consider $f(x) = x^3 + 3x^2 - 9x - 27$

Work out the second derivative of the above and thus the point of infliction (Concavity- concaving up or concaving down).

There is a point on a cubic graph where the concavity changes. Point of infliction is used to help draw the cubic graph. We are just working out the point now.

1. WORK OUT THE DERIVATIVE USING THE RULES
   $f(x) = x^3 + 3x^2 - 9x - 27$
   $f'(x) = 3x^2 + 6x - 9$
2. WORK OUT THE DERIVATIVE OF THE DERIVATIVE USING THE RULES (SECOND DERIVATIVE)
   $f'(x) = 3x^2 + 6x - 9$
   $f''(x) = 6x + 6$
3. EQUATE THE SECOND DERIVATIVE TO 0
   $f''(x) = 6x + 6$
   $0 = 6x + 6$
   $x = -1$
4. SUBSTITUTE x = -1 INTO THE ORIGINAL EQUATION
   $f(x) = x^3 + 3x^2 - 9x - 27$
   $f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) - 27$
   $f(-1) = -16$
   $f(-1) = -16$ is therefore the point of inflection because at $x = -1$ is where there is a change in concavity.
   The point is (-1; -16)

Example 2 (Try yourself)

1. Work out the point of infliction for the following
   a. $f(x) = x^3 - 5x^2 - 8x + 12$
   b. $f(x) = x^3 - 9x^2 + 27x + 37$

Answers

a. $f(x) = x^3 - 5x^2 - 8x + 12$
   $f'(x) = 3x^2 - 10x - 8$
   $f''(x) = 6x - 10$
   $6x - 10 = 0$
   $x = \frac{10}{6} = \frac{5}{3}$
   SUBSTITUTE BACK (Use your calculator)
   $y = -\frac{286}{27}$
   $(\frac{5}{3}; -\frac{286}{27})$ is point of infliction

b. $x = 3$
   $y = 64$
   (3;64) is the point of infliction