

GRADE 12

Calculus 4– Second derivative and concavity.

WEBSITE NOTES

TOPIC:

- Second derivative and concavity.

Example 1

Consider $f(x) = x^3 + 3x^2 - 9x - 27$

Work out the second derivative of the above and thus the point of inflection (Concavity- concaving up or concaving down).

There is a point on a cubic graph where the concavity changes. Point of inflection is used to help draw the cubic graph. We are just working out the point now.

1. **WORK OUT THE DERIVATIVE USING THE RULES**

$$f(x) = x^3 + 3x^2 - 9x - 27$$

$$f'(x) = 3x^2 + 6x - 9$$

2. **WORK OUT THE DERIVATIVE OF THE DERIVATIVE USING THE RULES (SECOND DERIVATIVE)**

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

3. **EQUATE THE SECOND DERIVATIVE TO 0**

$$f''(x) = 6x + 6$$

$$0 = 6x + 6$$

$$x = -1$$

4. **SUBSTITUTE $x = -1$ INTO THE ORIGINAL EQUATION**

$$f(x) = x^3 + 3x^2 - 9x - 27$$

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) - 27$$

$$f(-1) = -16$$

$f(-1) = -16$ is therefore the point of inflection because at $x = -1$ is where there is a change in concavity.

The point is $(-1; -16)$

Example 2 (Try yourself)

1. Work out the point of inflection for the following

a. $f(x) = x^3 - 5x^2 - 8x + 12$

b. $f(x) = x^3 - 9x^2 + 27x + 37$