**GRADE 12**
**Calculus 2– Differential Rules**
**WEBSITE NOTES**

**TOPIC:**

- Rules of differentiation.

---

**NOTE:** The notation we use for the derivative of $y = f(x)$ is

$f'(x)$ or $y'$ or $\frac{dy}{dx}$ or $D_x[f(x)]$.

When we find the derivative of a function, we say we **differentiate** the function.

**NOTE!!!**

**If the question asks you to find the derivative but does not say USING FIRST PRINCIPLES, then you use the DIFFERENTIATION RULES as below.**

**Rules**

**1.** If $f(x) = b$ then $f'(x) = 0$
where $b$ is a constant

**Example 1**

If $h(x) = 12$, then $h'(x) = 0$

**2.** If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

**Example 2**

If $k(x) = x^5$, then $k'(x) = 5x^4$

**3.** $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

**Example 3**

If $f(x) = x^5 + x^4$, then $\frac{d}{dx}f(x) = 5x^4 + 4x^3$

**NOTE!!!**

$\frac{d}{dx}f(x)$

Is the same as saying – the derivative of the function f(x) or f '(x)

4. $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$

**Example 4**

If $f(x) = 3x^5$ then

$\frac{d}{dx}f(x) = 3 \times \frac{d}{dx}f(x)\ (x^5) = 3 \times 5x^4 = 15x^4$

***Sometimes you may need to use distributive law in order to get the function in standard form. It must be in a standard form before you differentiate*.**

***Multiply out first using FOIL***

**Example 5**

Determine $f'(x)$ if $f(x) = (3x + 2)(x - 5)$

**Solution**

$f(x) = 3x^2 - 13x - 10$

$\therefore f'(x) = 6x - 13$

***Sometimes you may need to change roots into exponents before doing differentiation.***

**Example 6**

$\sqrt{x} = x^{\frac{1}{2}}$

so $\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}}$

- $\frac{dy}{dx}$ *as the derivative of y with respect to x*
- $\frac{d}{dx}\sqrt{x}$ *as the derivative of* $\sqrt{x}$ *with respect to x*
- $\frac{d}{dx}f(x)$ *as the derivative of* $f(x)$ *with respect to x*

**REMEMBER**

**NOTE:** The notation we use for the derivative of $y = f(x)$ is

$f'(x)$ or $y'$ or $\frac{dy}{dx}$ or $D_x[f(x)]$.

When we find the derivative of a function, we say we **differentiate** the function.

## Example 7 (Try Yourself)

***You need to find the Derivative of each question using the rules and not first principles because it does not say first principles.***

***Remember STANDARD FORM and ROOTS into EXPONENTS first***

**a)** Evaluate $D_x[(x^3 - 3)^2]$

**b)** Find $f'(x)$ if $f(x) = \sqrt[3]{x}$

**c)** Find $\frac{d}{dx}\sqrt[3]{x^5}$

**d)** Differentiate $f(x)$ if $f(x) = \sqrt{x^4}$

**e)** Find $f'(x)$ if $f(x) = \sqrt{16x^3}$

**[11]**

## ANSWERS

| | |
|---|---|
| **a)** $D_x[(x^3 - 3)^2]$<br>$= D_x[x^6 - 6x^3 + 9]$ ✓<br>$= 6x^5 - 18x^2$ ✓ ✓ (3) | First multiply out<br><br>Apply the rules of differentiation |
| **b)** $\sqrt[3]{x} = x^{\frac{1}{3}}$ so $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$ ✓ ✓ (2) | **c)** $\sqrt[3]{x^5} = x^{\frac{5}{3}}$ so $\frac{d}{dx}(\sqrt[3]{x^5}) = \frac{5}{3}x^{\frac{2}{3}}$ ✓ ✓ (2) |
| **d)** $\sqrt{x^4} = x^{\frac{4}{2}} = x^2$ ✓<br>so $f'(x) = 2x^1 = 2x$ ✓ (2) | **e)** $f(x) = \sqrt{16x^3} = 4(x^3)^{\frac{1}{2}} = 4x^{\frac{3}{2}}$ ✓<br>So $f'(x) = \frac{3}{2}.4.x^{\frac{3}{2}-1} = 6x^{\frac{1}{2}}$ ✓ (2)<br>You can write the answer as $6\sqrt{x}$ or as $6x^{\frac{1}{2}}$<br>**[11]** |

## Example 8 (Try Yourself)

## June 2015 Exam Paper 1

***REMEMBER THE DERIVATIVE IS THE GRADIENT AT A POINT.***

***TO WORK OUT A TANGENT OF A GRAPH YOU NEED TO WORK THE GRADIENT OUT FIRST.***

**QUESTION 8**

8.1 If $f(x) = \frac{4}{x}$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 5x^2 + 5x + 2$ (2)

8.2.2 $D_x\left[\sqrt[3]{x^2} - \frac{1}{2}x\right]$ (3)

8.3 Given: $p(x) = x^3 + 2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of $p$ to have a negative gradient. (3)

**[13]**

## Answers

| | | |
|---|---|---|
| 8.1 | $f(x+h)=\frac{4}{x+h}$<br>$f(x+h)-f(x)=\frac{4}{x+h}-\frac{4}{x}$<br>$=\frac{4x\text{-}4(x+h)}{x(x+h)}$<br>$=\frac{4x-4x-4h}{x(x+h)}$<br>$=\frac{-4h}{x(x+h)}$<br>$\frac{f(x+h)-f(x)}{h}=\frac{\frac{-4h}{x(x+h)}}{h}$<br>$=\frac{-4h}{xh(x+h)}$<br>$\frac{-4}{x(x+h)}$<br>$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$<br>$=\lim_{h\to 0}\frac{-4}{x(x+h)}$<br>$=\frac{-4}{x^2}$ | ✓subst. into formula<br>✓ $\frac{4x-4(x+h)}{x(x+h)}$<br>✓ $\frac{-4}{x(x+h)}$<br>✓formula<br>✓answer<br>(5) |
| 8.2.1 | $y=5x^2+5x+2$<br>$\frac{dy}{dx}=10x+5$ | ✓ $10x$<br>✓ 5<br>(2) |
| 8.2.2 | $D_x\left[\sqrt[3]{x^2}-\frac{1}{2}x\right]$<br>$=D_x\left[x^{\frac{2}{3}}-\frac{1}{2}x\right]$<br>$=\frac{2}{3}x^{\frac{-1}{3}}-\frac{1}{2}$ | ✓ $x^{\frac{2}{3}}$<br>✓ $\frac{2}{3}x^{\frac{-1}{3}}$<br>✓ $-\frac{1}{2}$<br>(3) |
| 8.3 | $p(x)=x^3+2x$<br>$p'(x)=3x^2+2$<br>$3x^2\geq 0$ or / *of* $x^2\geq 0$ for all/*vir alle* $x\in\mathbf{R}$<br>$\therefore 3x^2+2\geq 2>0$ for all/*vir alle* $x\in\mathbf{R}$<br>i.e. $p'(x)>0$ for all/*vir alle* $x\in\mathbf{R}$<br>i.e. all tangents to $p$ have gradient greater than (or equal to) 2. Thus there is no tangent to $p$ that has negative gradient.<br><br>*Alle raaklyne aan p sal dus 'n gradiënt groter (of gelyk aan) 2 hê. Daar sal dus geen raaklyn aan p wees met 'n negatiewe gradiënt nie.* | ✓ $p'(x)=3x^2+2$<br>✓states & justifies / *noem en verduidelik* $p'(x)>0$<br>✓linking derivative to gradient of tangent/*verband tussen gradiënt en afgeleide*<br>(3)<br>[13] |