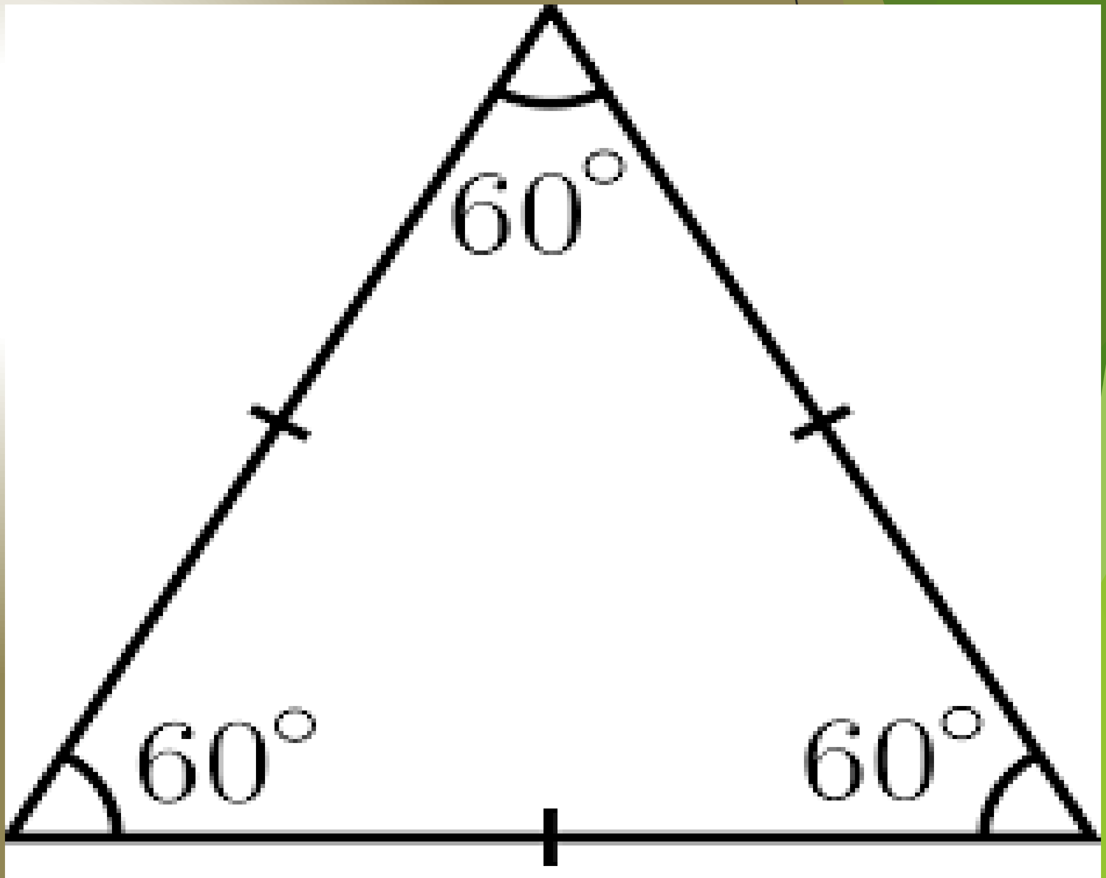
CHAPTER 3:
SHAPE AND SPACE GEOMETRY

3.1 Geometry of 2D shapes
3.2 Geometry of 3D Objects
3.3 Geometry of straight lines
3.4 Transformation geometry
3.5 Construction of Geometric Figures

CLASSIFY 2D-SHAPES

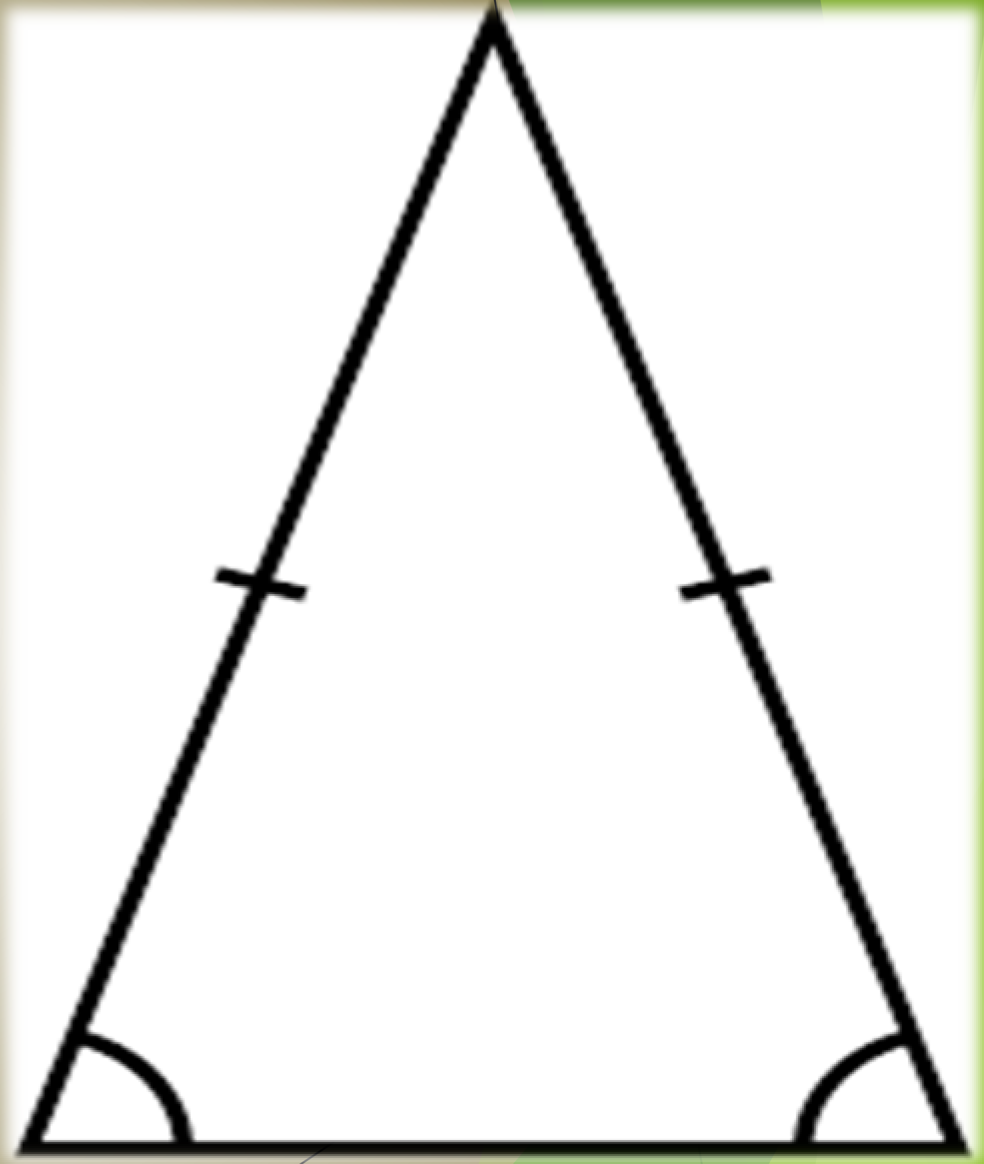
1.Equilateral Triangles

∗all sides equal
∗all angles = 60°



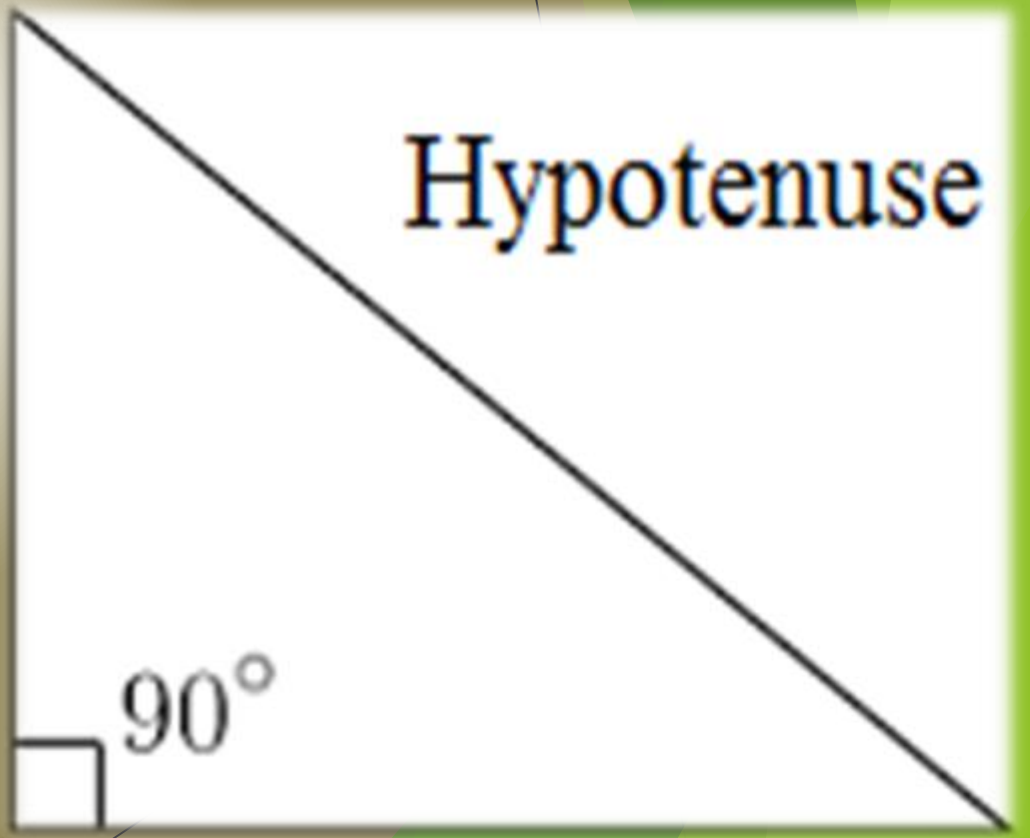
2. Isosceles Triangles

* 2 sides equal
* Base angles of equal sides are equal



3. Right - angled Triangles

* 1 angle = 90˚
* The side opposite the 90˚ is the largest side & called the hypotenuse

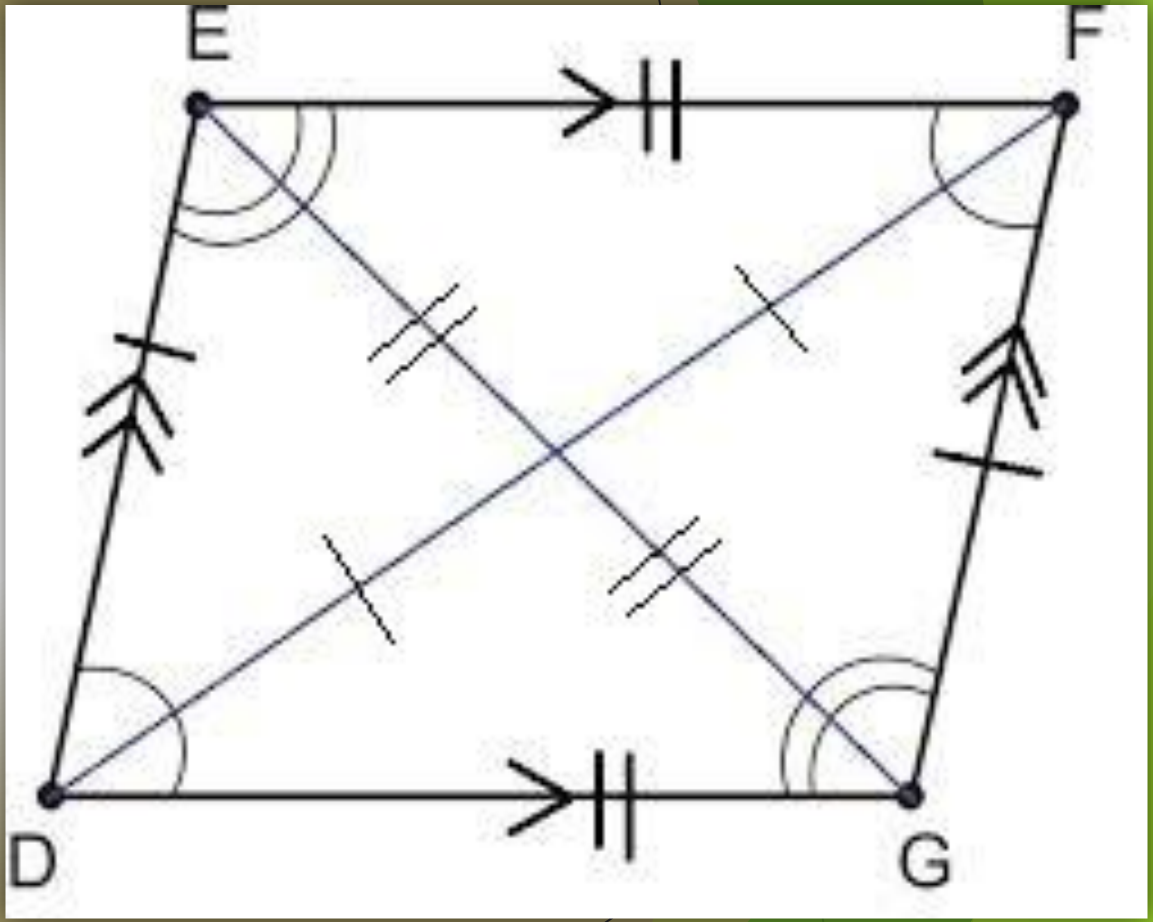


Classifying Triangles Song

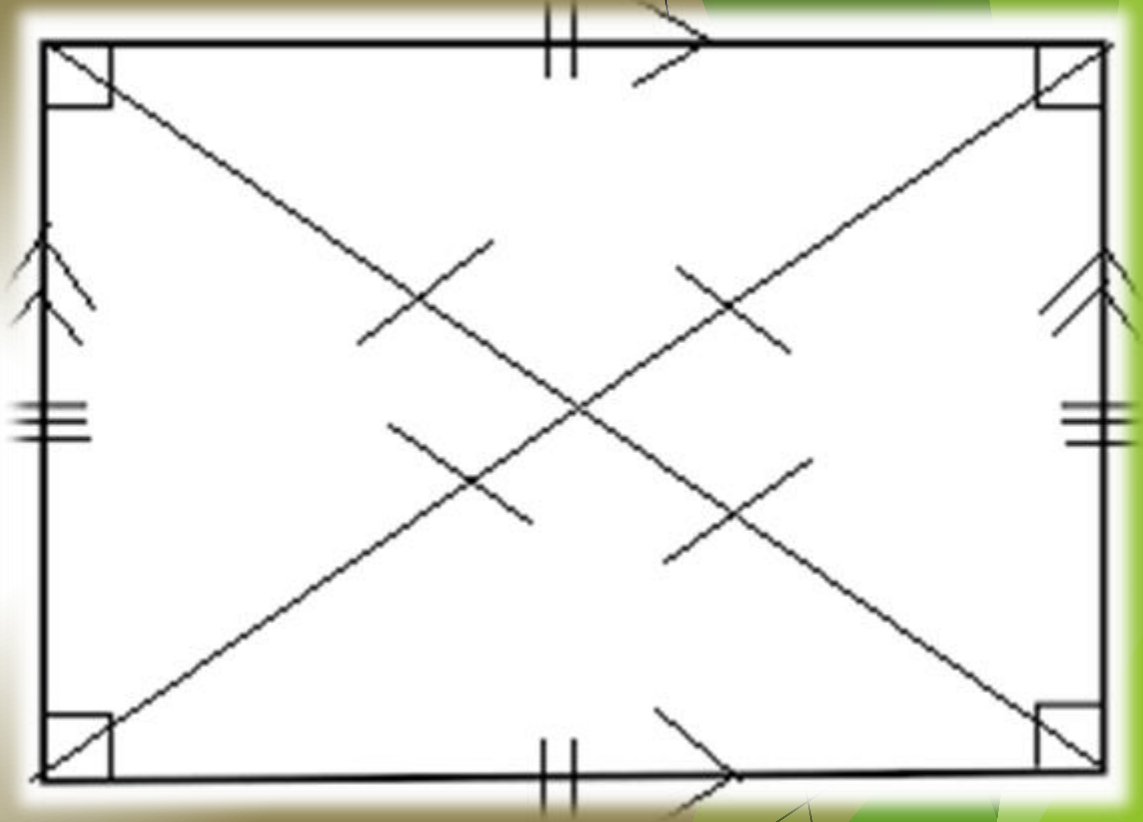
CLASSIFY 2D-SHAPES: Quadrilaterals

1.Parallelogram

*Opposite sides parallel
*Opposite sides equal
*Opposite angles equal
*Diagonals bisect each other



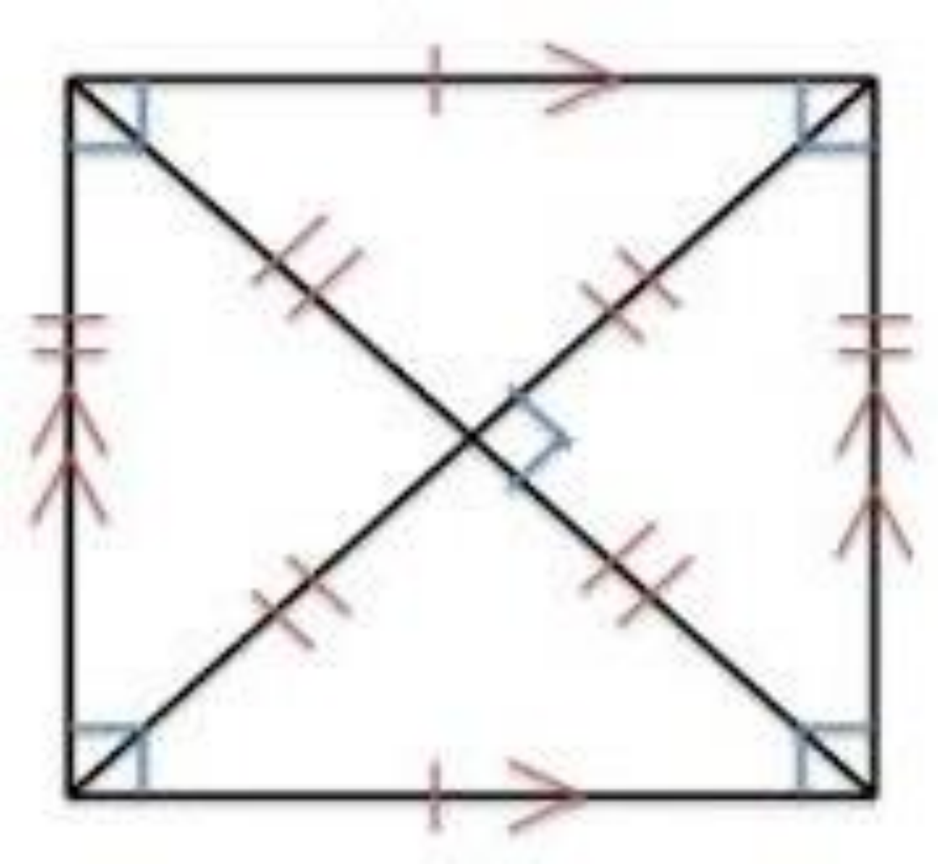
2. Rectangles



- *Opposite sides parallel
- *Opposite sides equal
- *All 4 angles equal = 90˚
- *Diagonals equal
- *Diagonals bisect each other

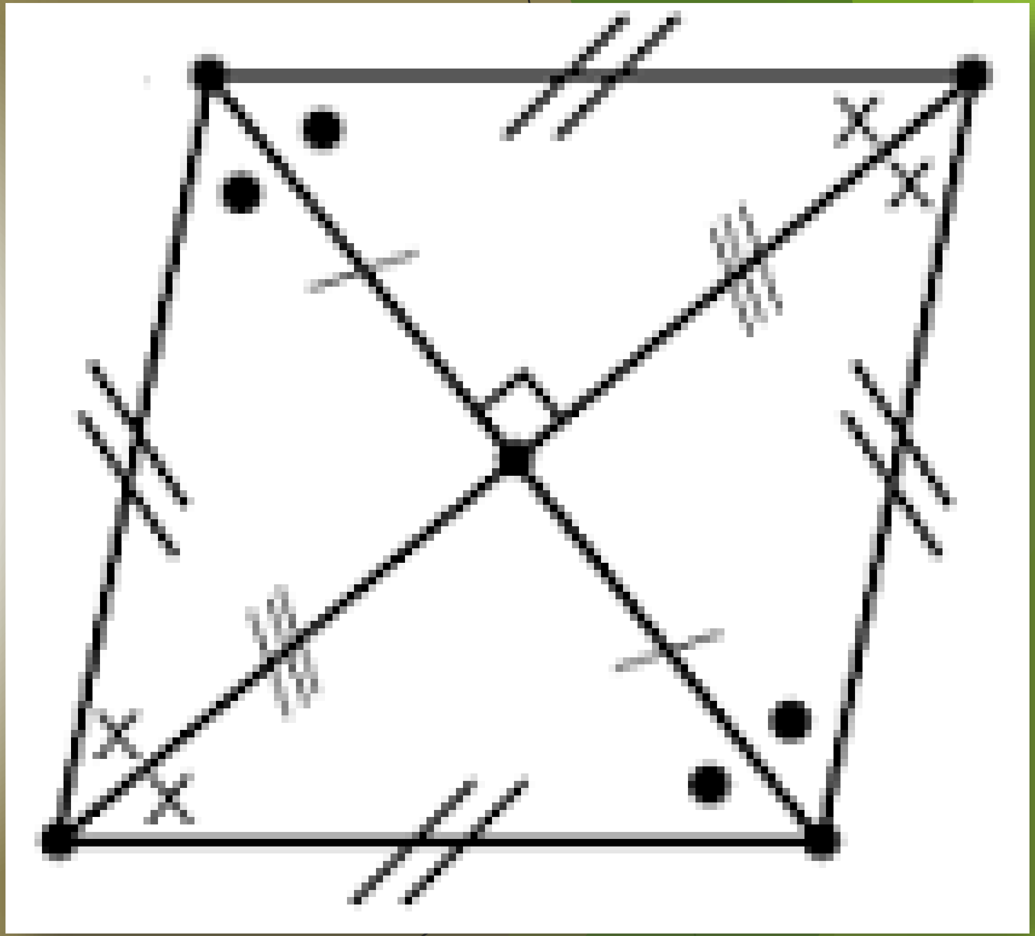
3. Square

* Opposite sides parallel
* Opposite sides equal
* All 4 angles equal = 90˚
* Diagonals bisect each other at 90˚



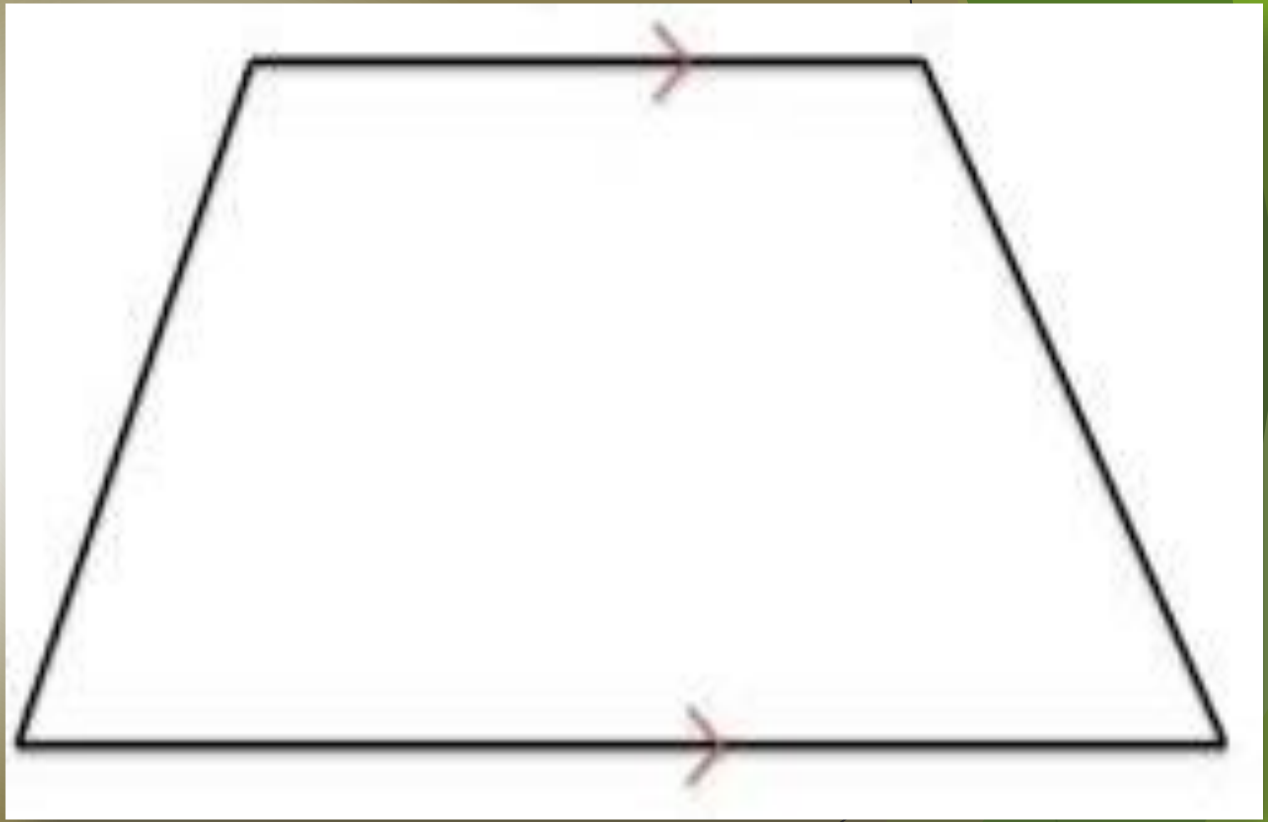
4. Rhombus

* Opposite sides parallel
* All 4 sides equal
* Opposite angles equal
* Diagonals bisect each other at 90°
* Diagonals bisect the angles of vertices



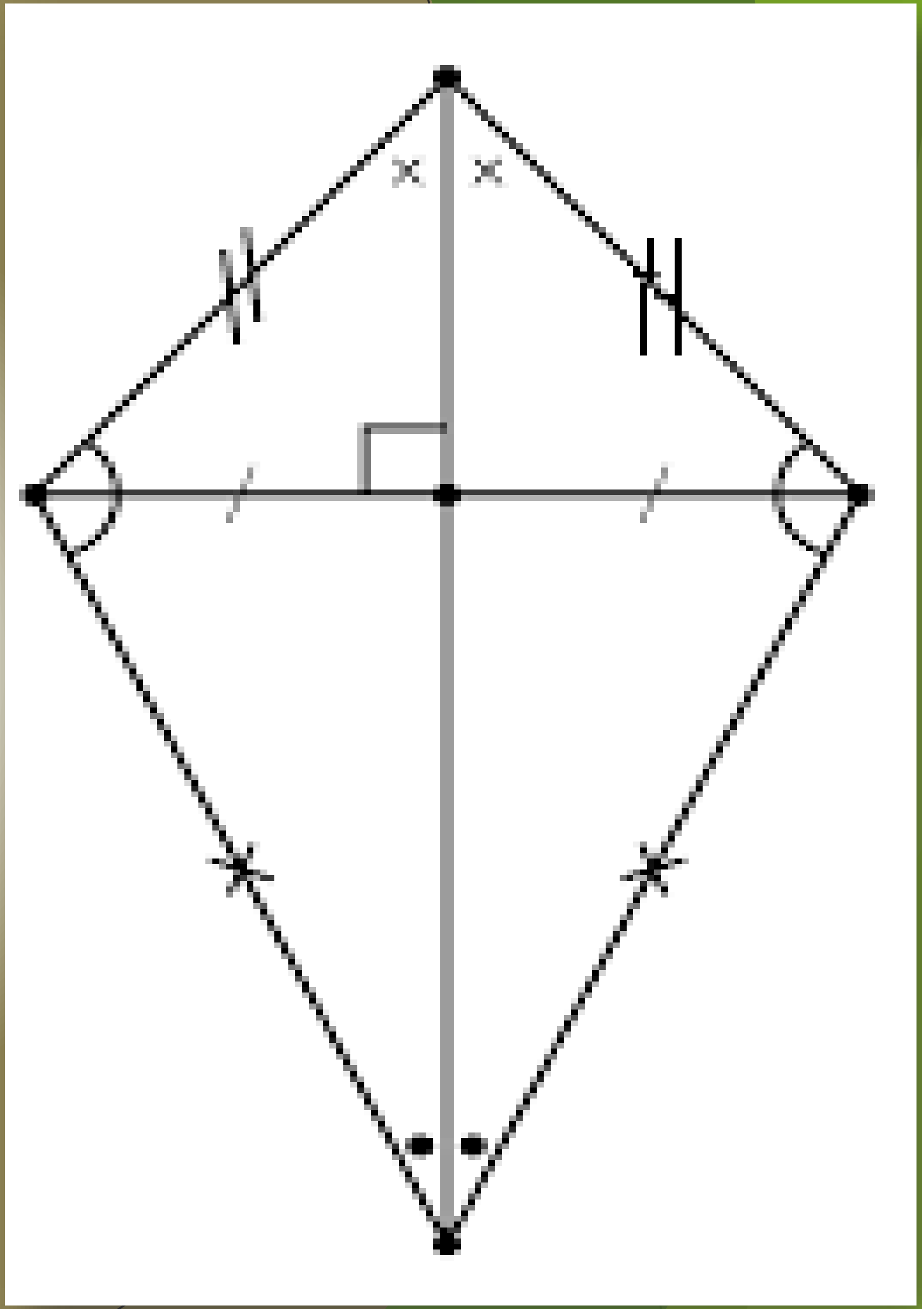
5. Trapezium

*One pairs of opposite sides parallel



6. Kite

* Two pairs of adjacent sides equal
* Equal angles opposite line of symmetry



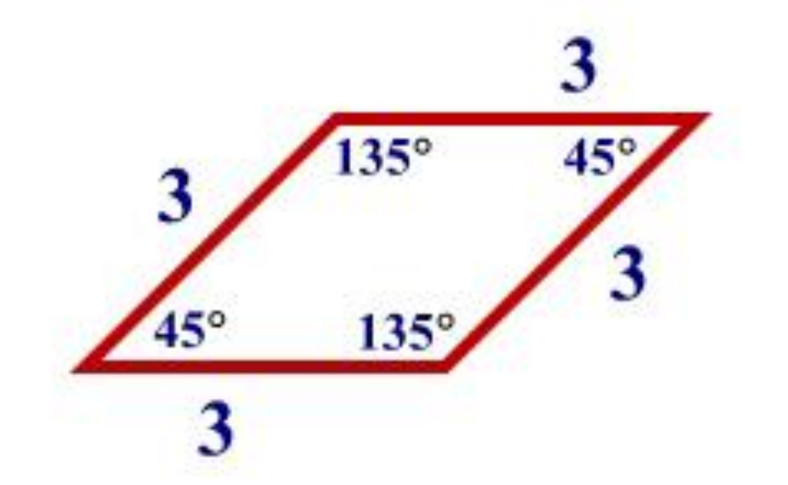
Quadrilateral song

EXERCISE

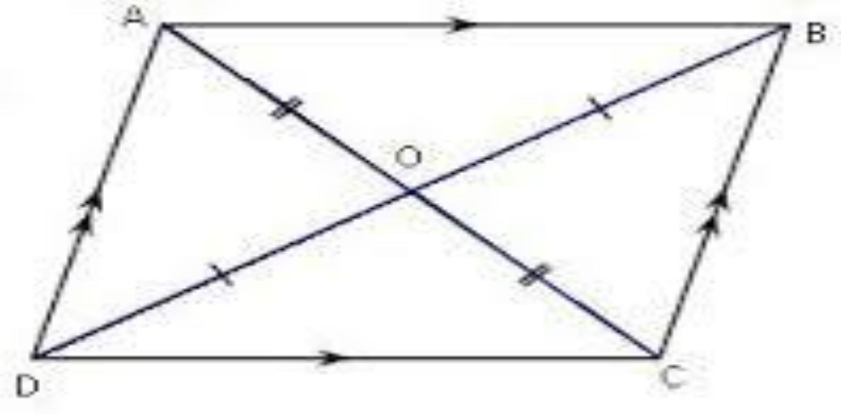
Quadrilateral Proofs

Identify the following shapes (be specific) and give reasons for your answers

1.

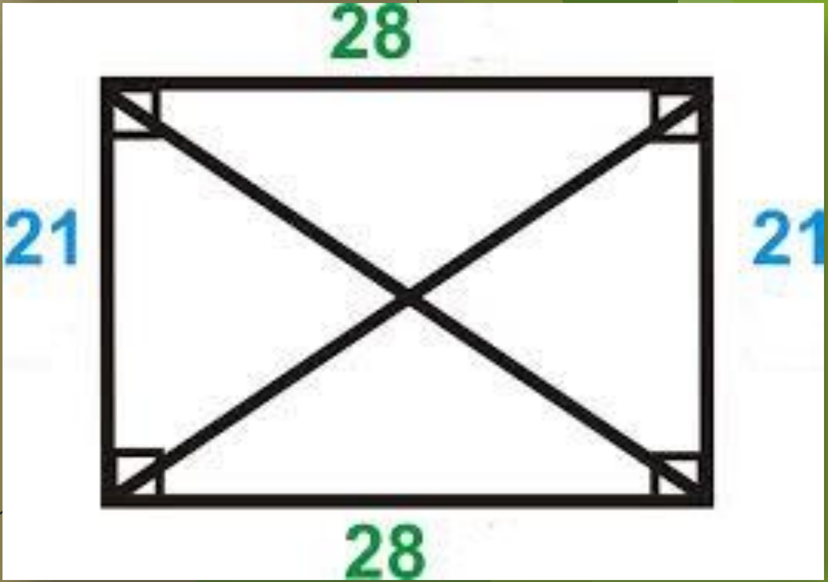


2.



3.

4.



2. Similarity(|||)

<u>Understanding Similarity based on Real-life Examples</u>

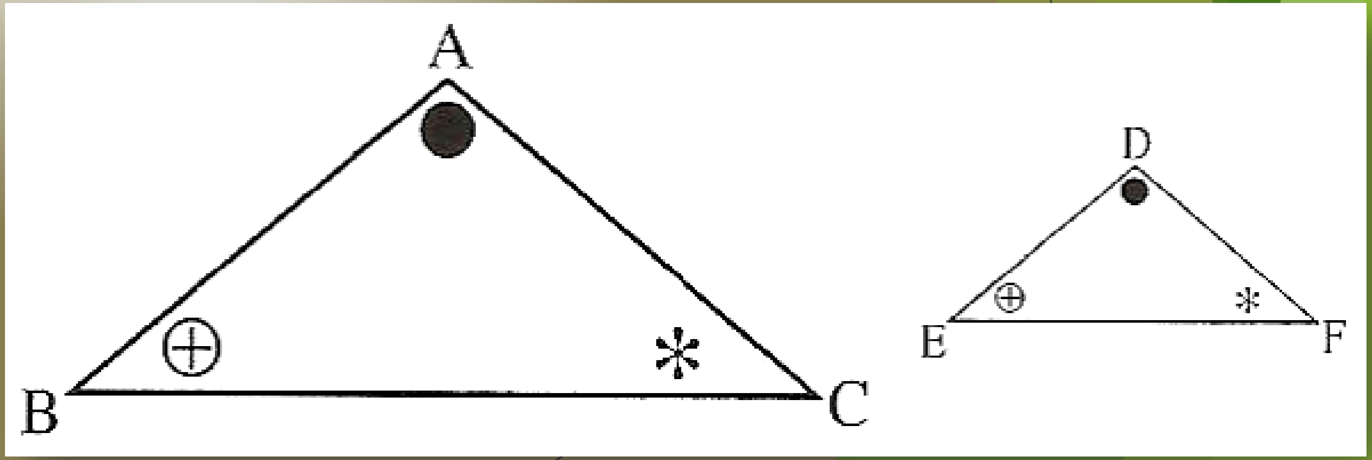
* Triangles are said to be similar if:

i. All pairs of corresponding **angles are equal**

i. All pairs of corresponding **sides are in the same proportion**

ie. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Δ ABC ||| DEF

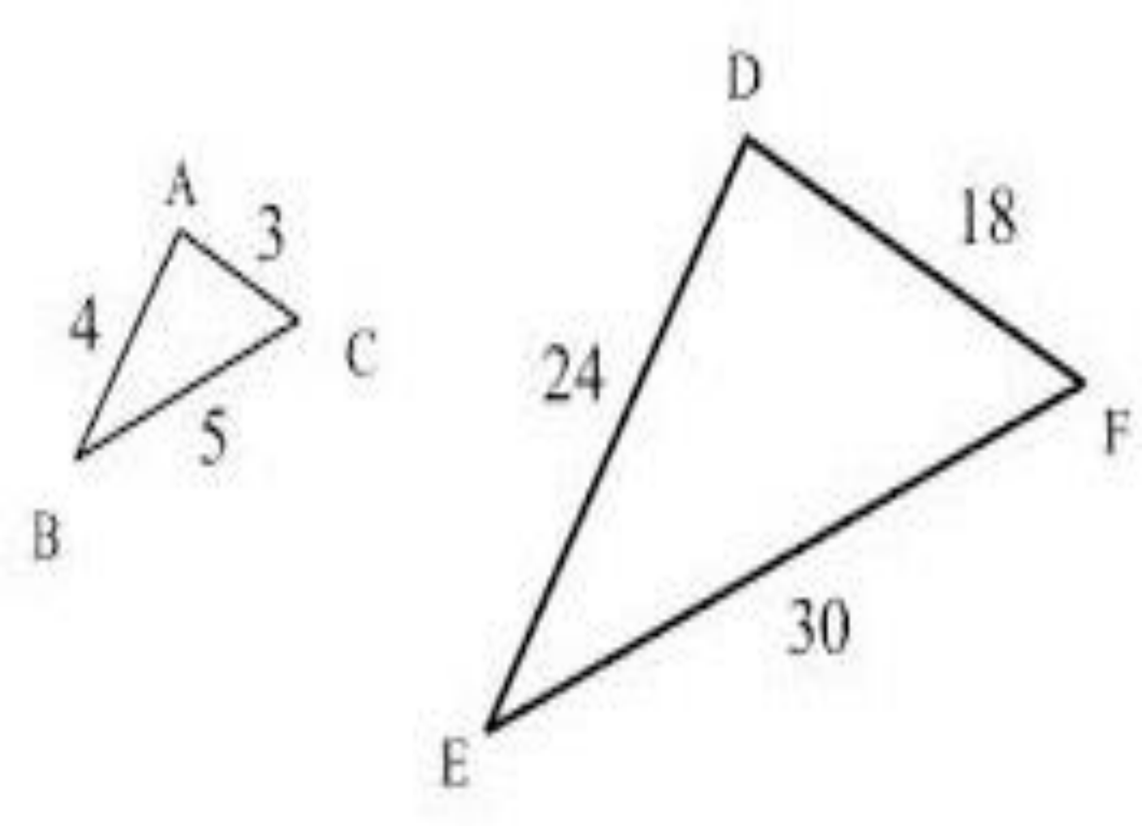


Examples

1. Prove that Δ ABC III Δ DEF:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

(sides in proportion)



∴ Δ ABC ||| Δ DEF

2.1 Prove that ∆ ADE III ∆ ABC:

In ∆ ADE and ∆ ABC

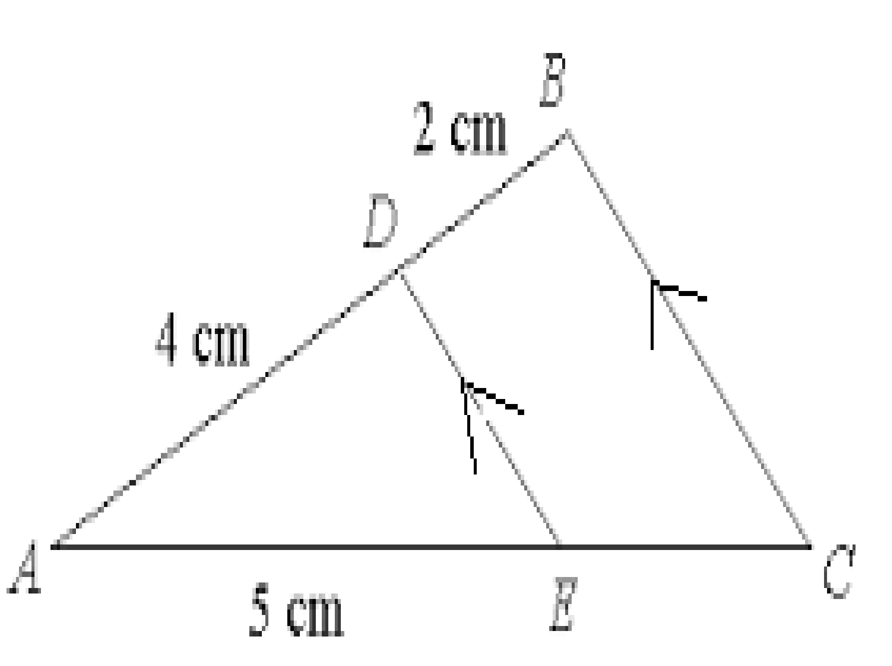
i. Â is common (given)

ii. D=B (corresponding $\angle's$; DE II BC)

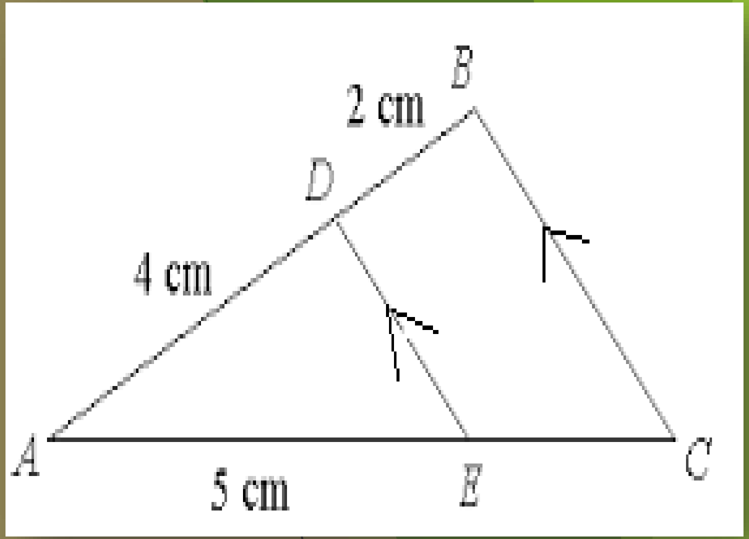
iii. Ê=Ĉ (corresponding $\angle's$;

DE II BC

∴ ∆ ADE III ∆ ABC



2.2 Determine the length of EC:



In Δ ADE III Δ ABC (given)

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$ (sides in proportion)

$\frac{4}{4+2} = \frac{5}{5+EC}$ ←Fill in lengths from diagram

$4(5 + EC) = 6 \times 5$ ←cross multiply

$20 + 4EC = 30$

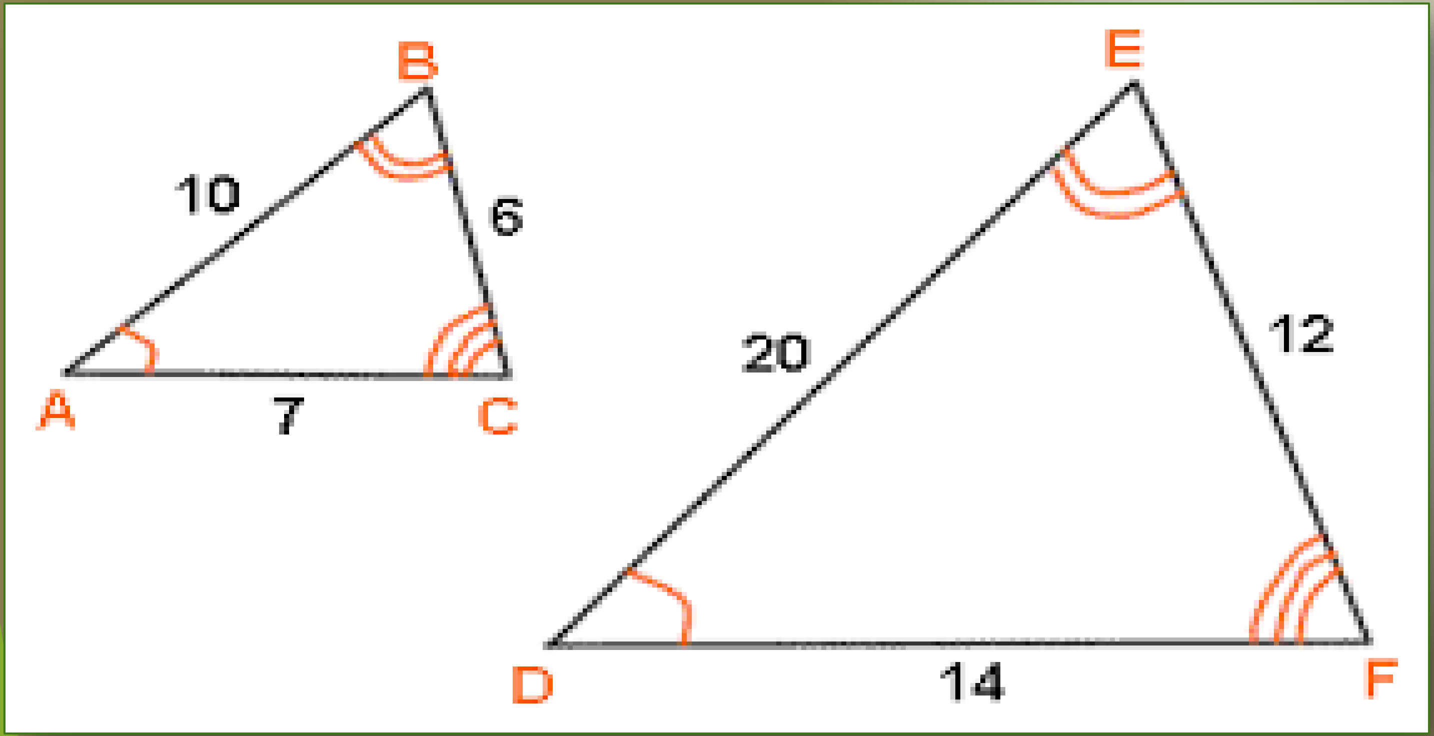
$4EC = 10$

$EC = 2.5\ cm$

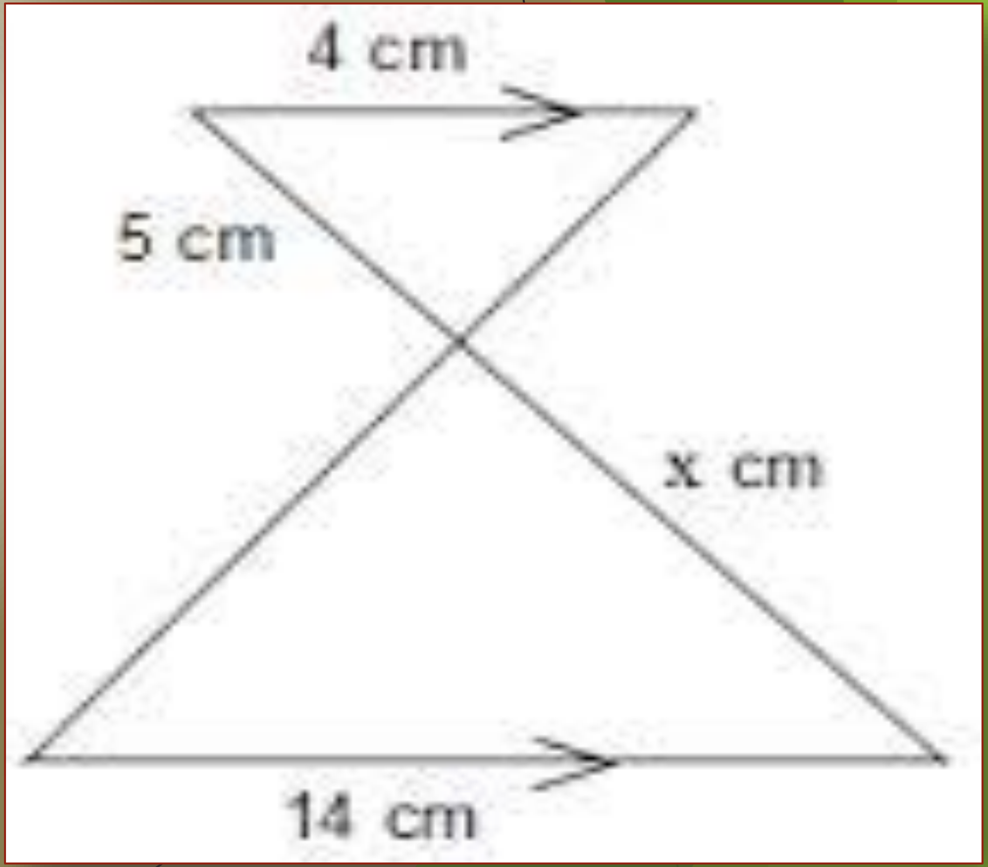
EXERCISE!

Similarity Example Problems

1. Prove that Δ ABC ||| Δ DEF



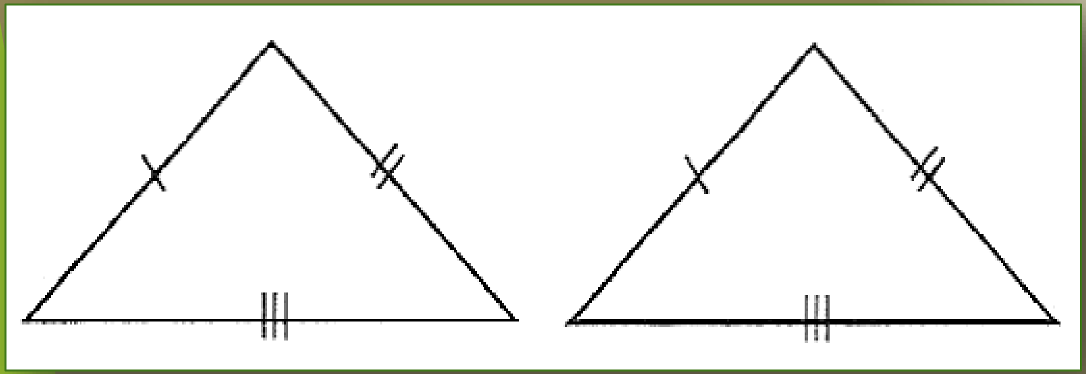
2. Solve for x



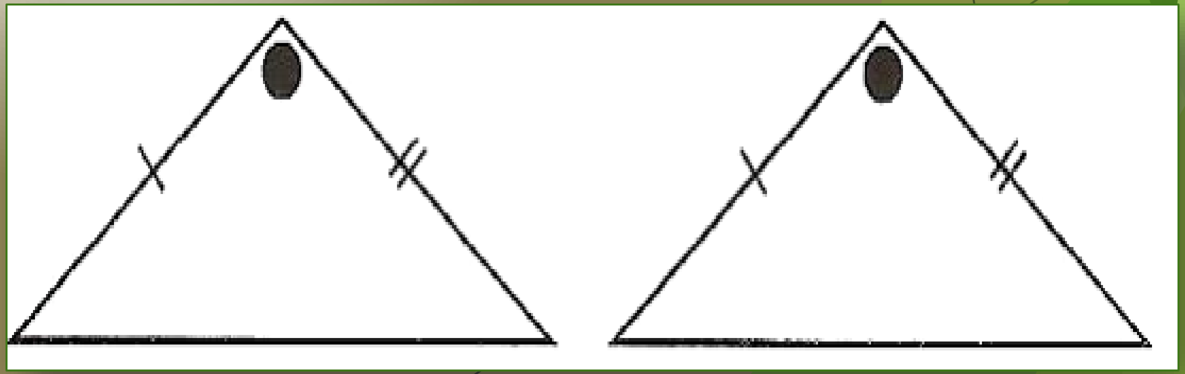
Congruency(≡)

* Triangles are said to be congruent if they have the **same shape and size:**
* There are 4 cases for congruency:

i. Side, Side, Side

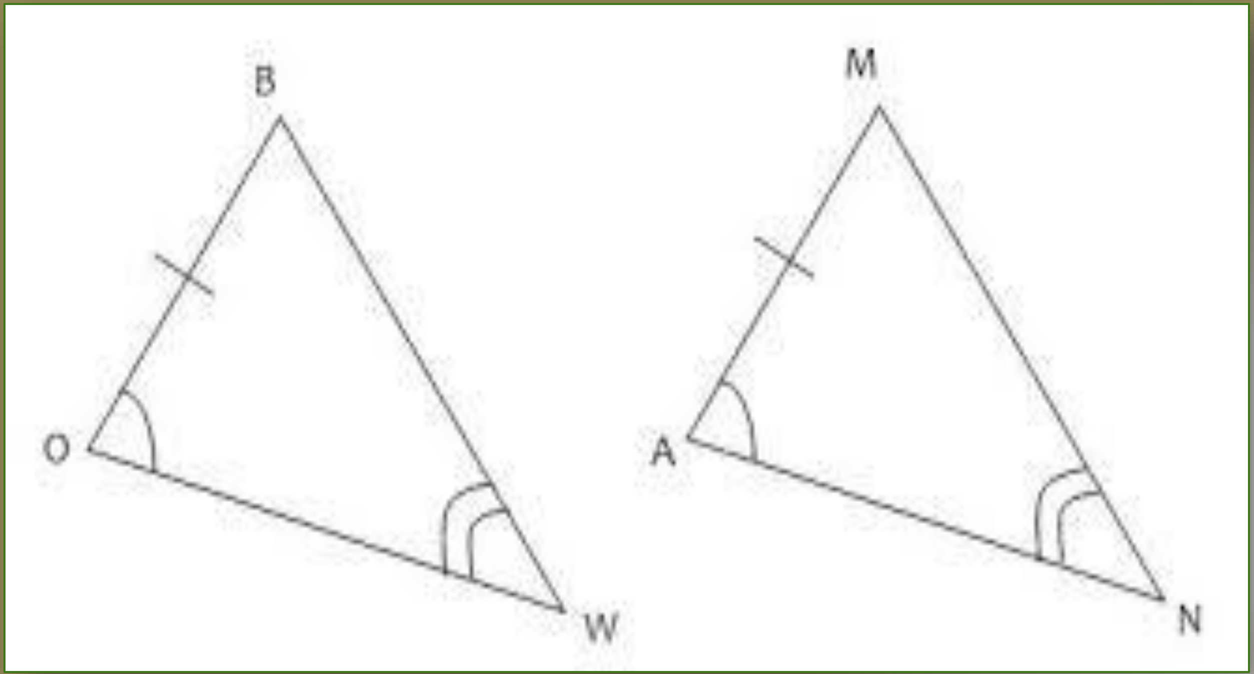


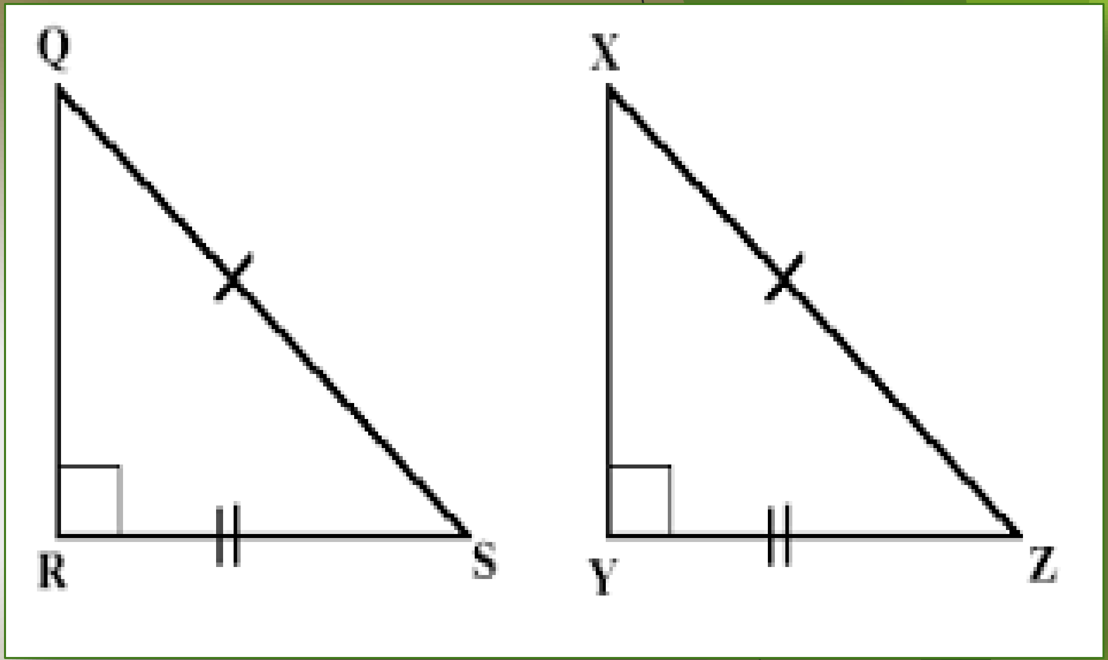
ii. Side, Angle, Side
[NOTE! Angle must be included!]



iii. Angle, angle, side

iv. 90° hypotenuse side





<u>Note! </u>the order of the vertices of a triangle is NB ie. Δ ABC III Δ DEF is not necessarily the same as Δ ABC III Δ EDF!

Cases for Congruency Song

Examples

1. Prove that ∆ ABC III ∆ ACD:

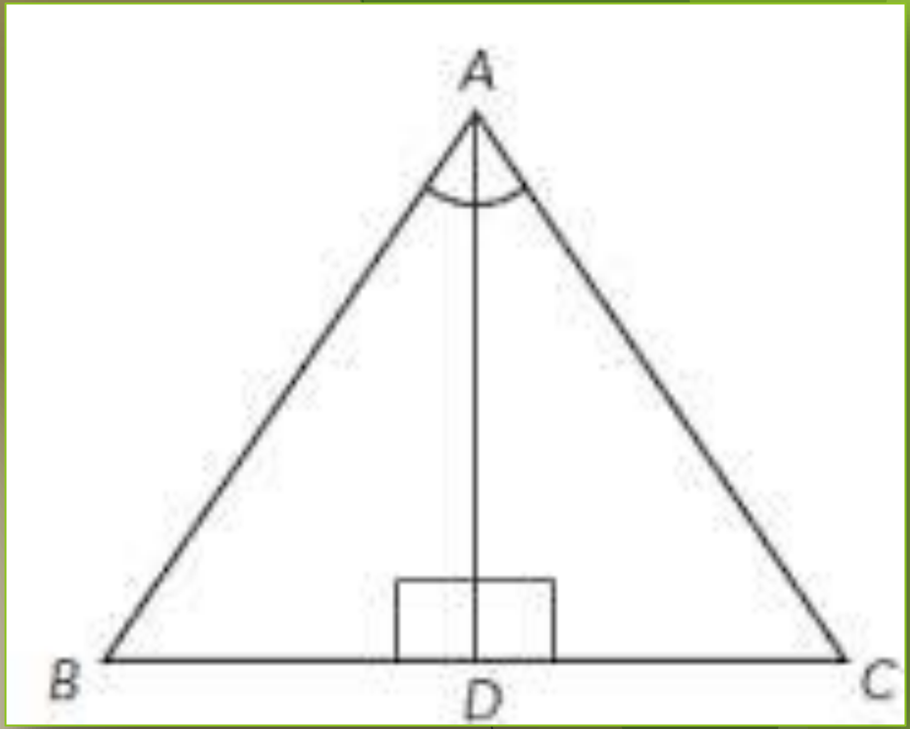
∆ ABC and ∆ ACD

i. BÂD=CÂD (given)

ii. BDA=CDA (given)

iii. AD is common

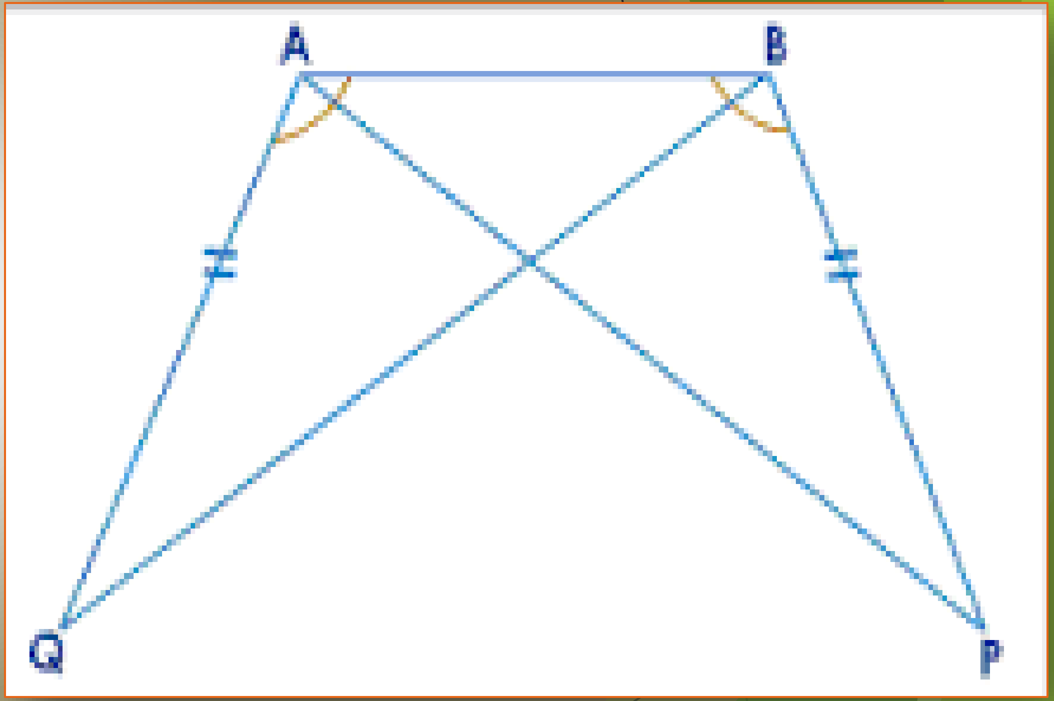
∴ ∆ ABC III ∆ ACD (a,a,s)



Note! Always remember to state the case for congruency!

2. Determine the length of QB, if PA = 7cm
Δ ABQ and Δ BAP

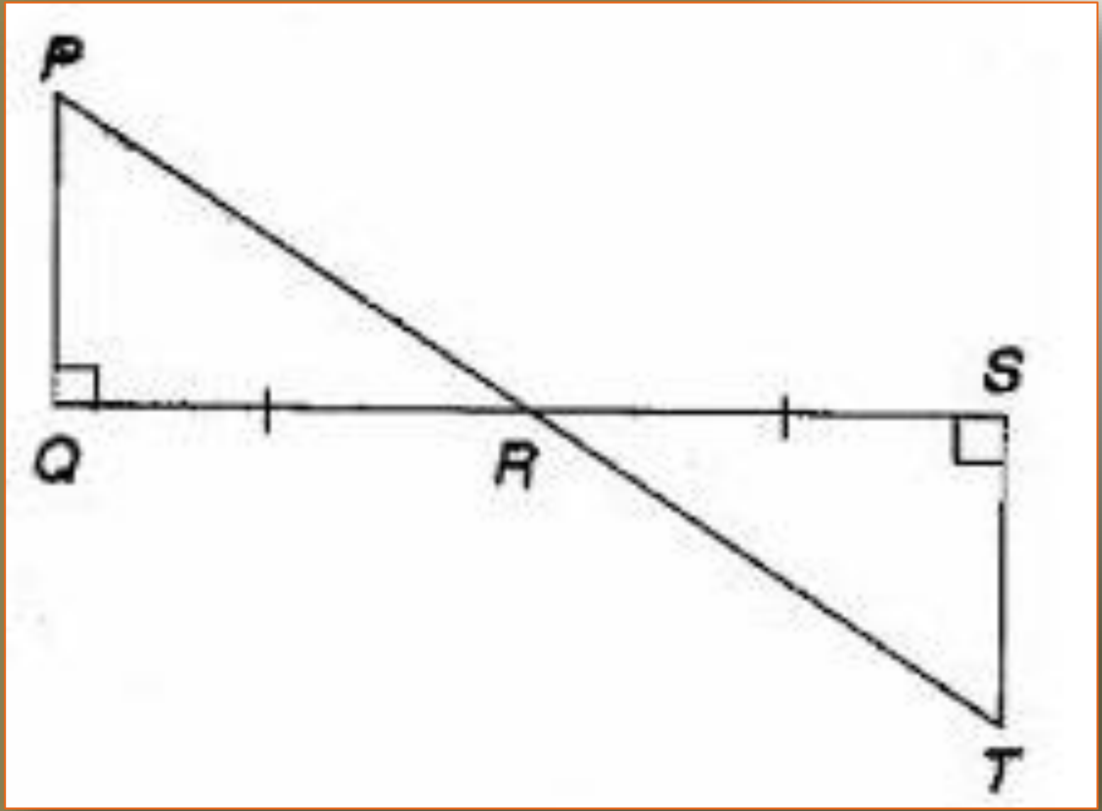
i. AQ=BP (given)
ii. A=B (given)
iii. AB is common
∴ Δ ABQ≡ Δ BAP (s,a,s)
∴ QB=PA (from congruency)
∴ QB=7cm (given PA=7cm)



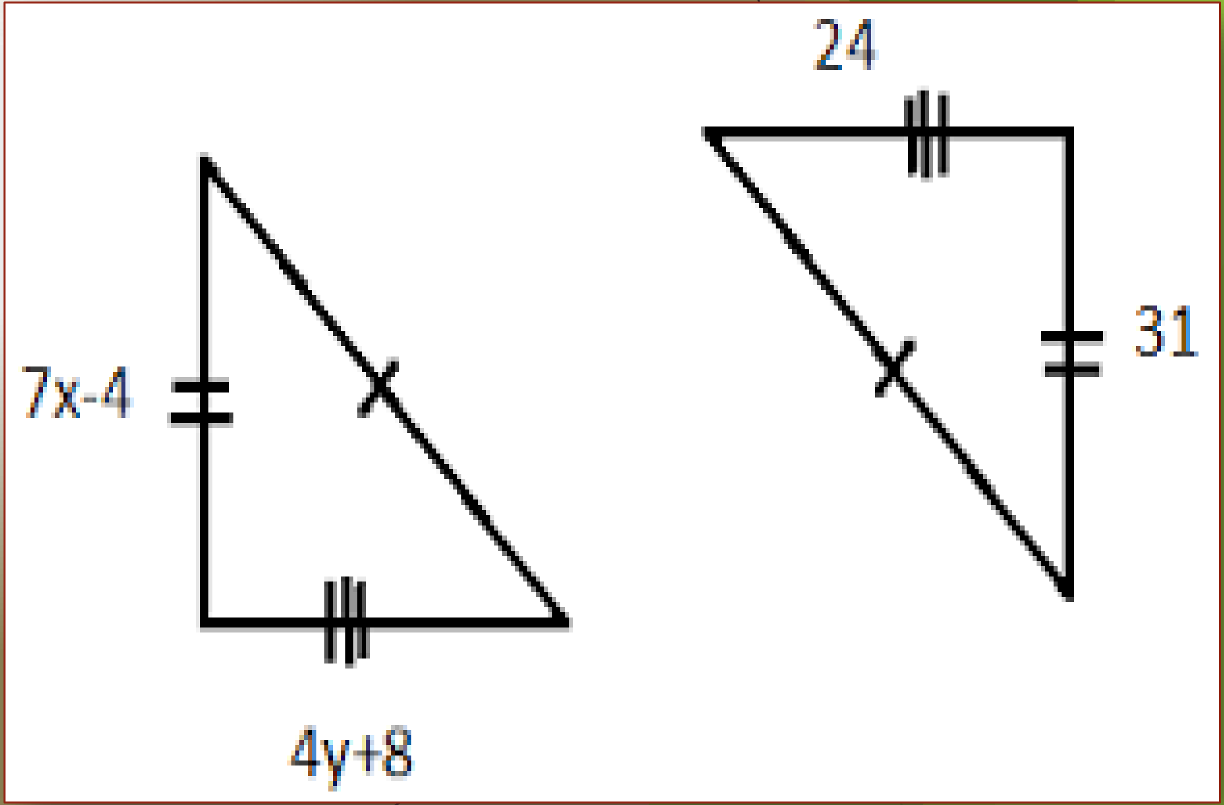
EXERCISE

Congruency Example Problems

1. Prove that Δ PQR $\equiv$ Δ TSR



2. Solve for x & y

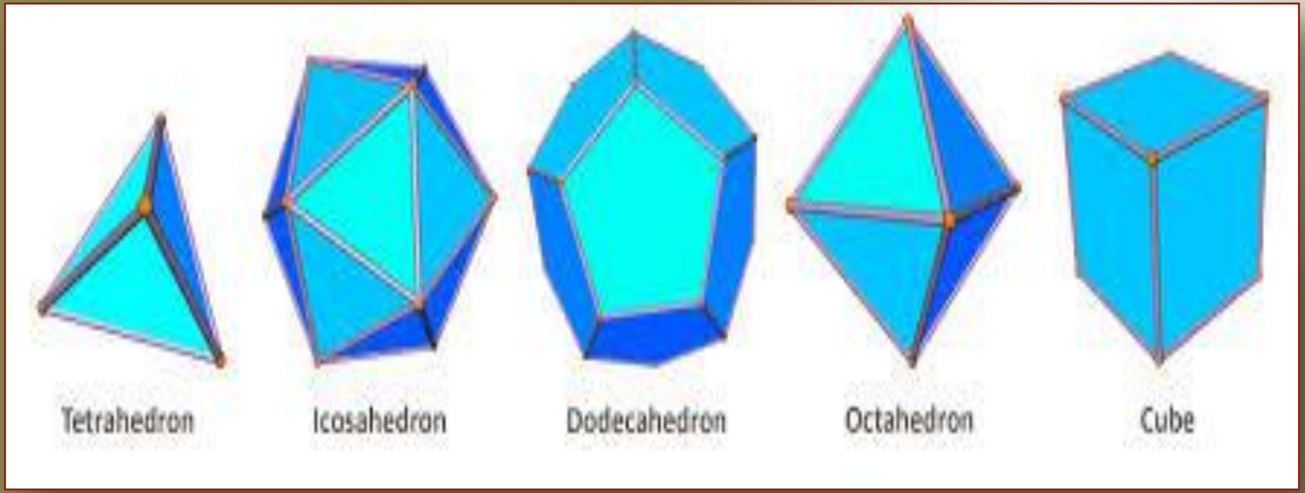


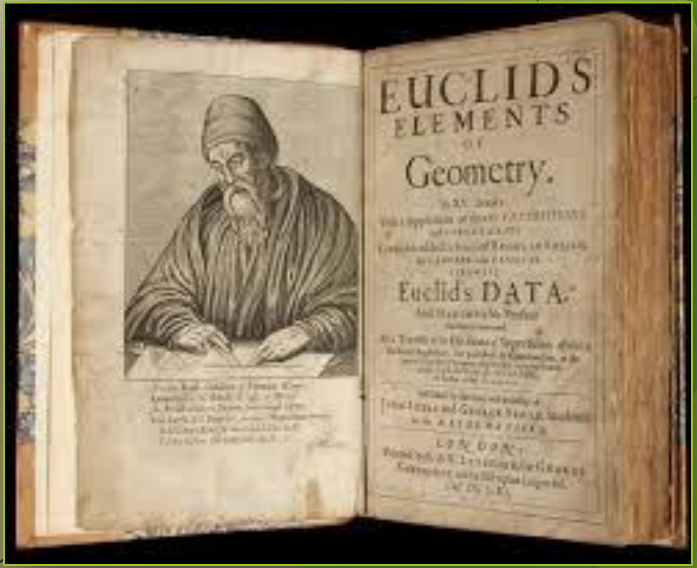
3.2 Geometry of 3D- objects

Classify 3D objects

- Platonic solids (aka regular polyhedral) have congruent faces (sides) made up of regular polygons

- There are 5 platonic solids
- Proved by Euclid in his book, “**Elements**”



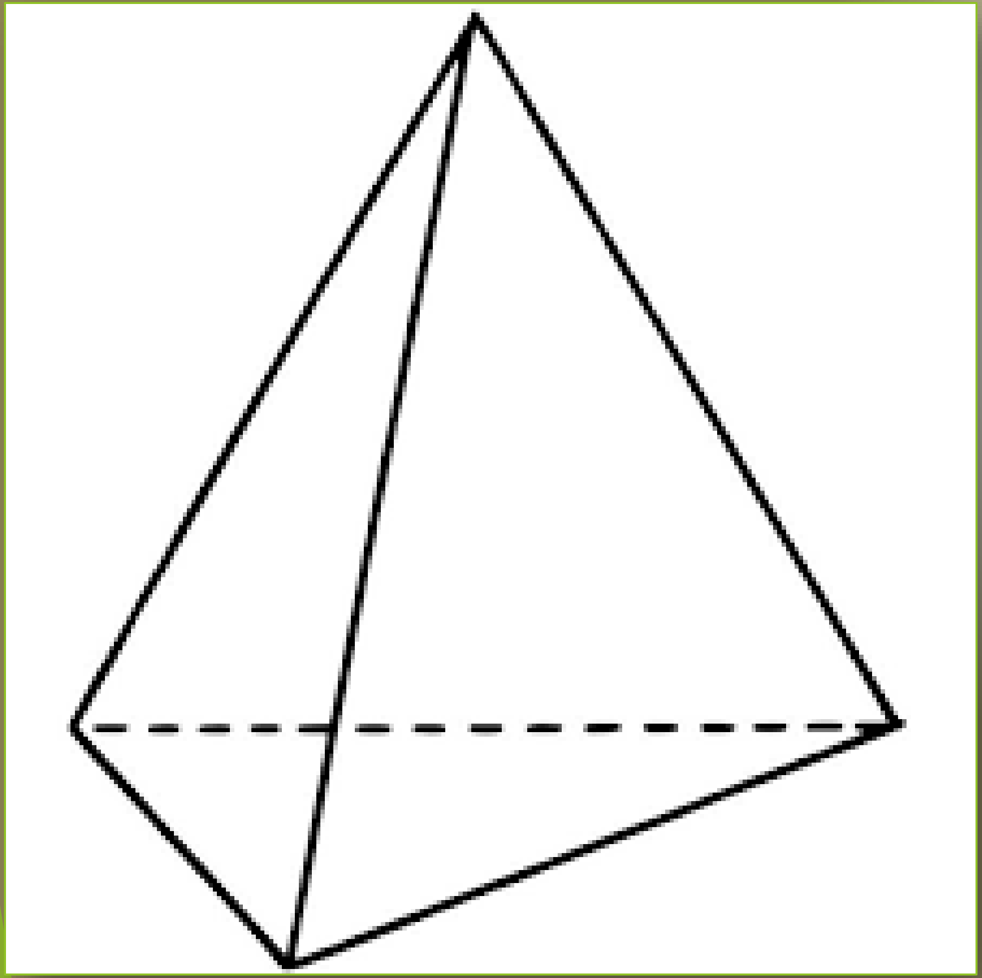


Platonic Solids – Part 1

Platonic Solids – Part 2

Properties of platonic Solids

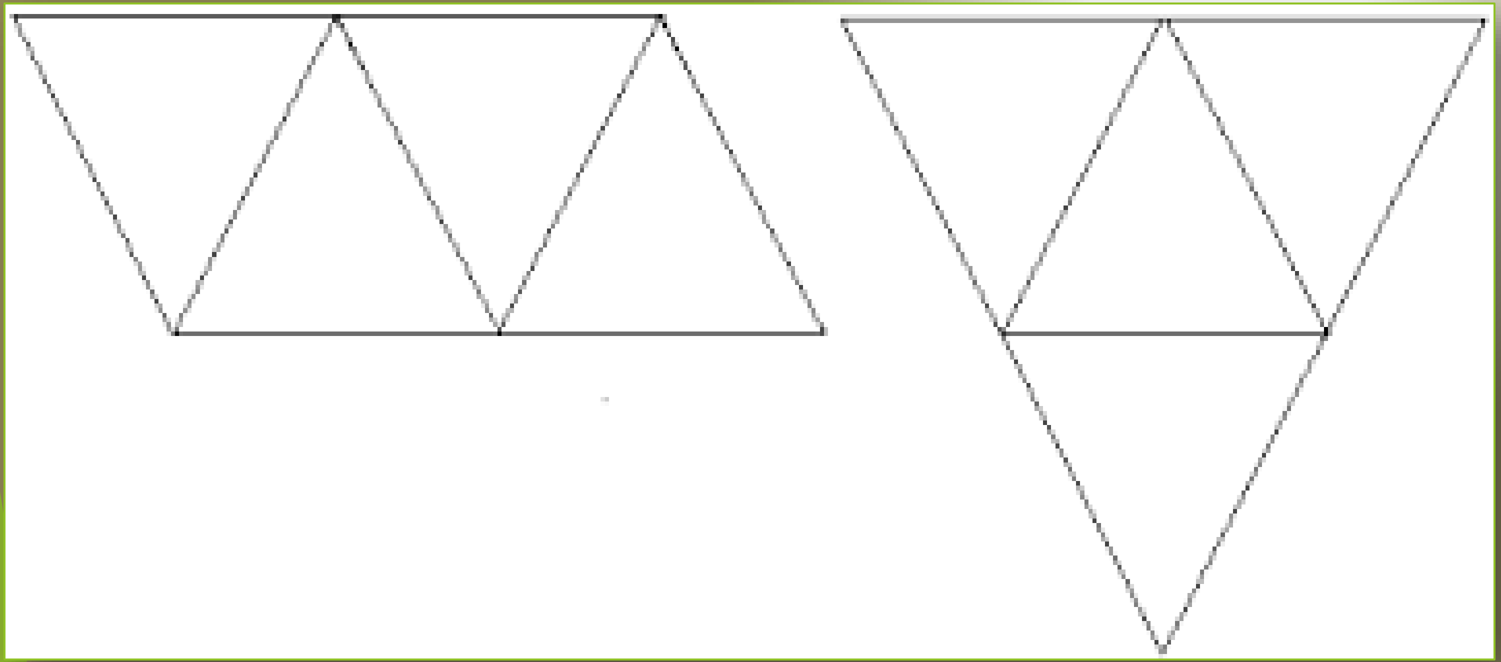
1. Tetrahedron



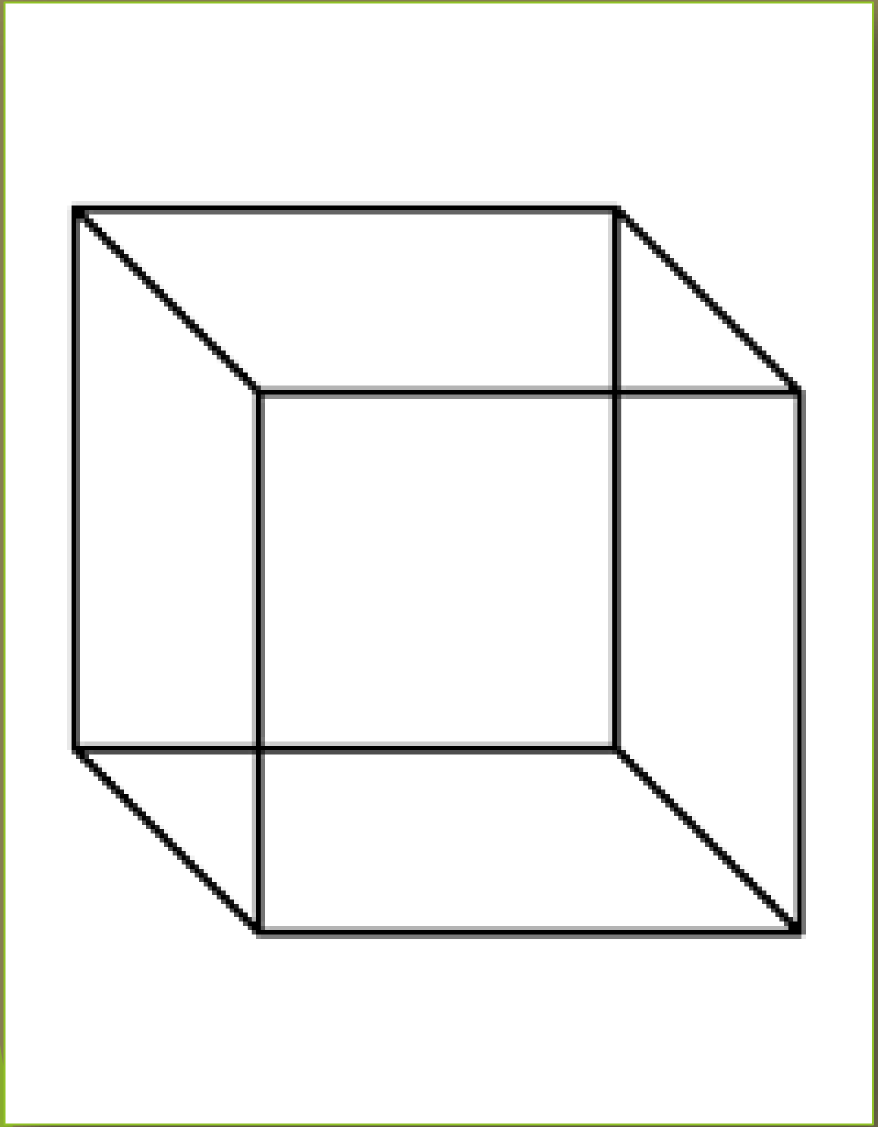
- No. of faces: 4 equilateral triangular faces
- No. of vertices: 4
- No. of edges: 6

- **A tetrahedron has 2 distinct nets!**

Can you draw them?



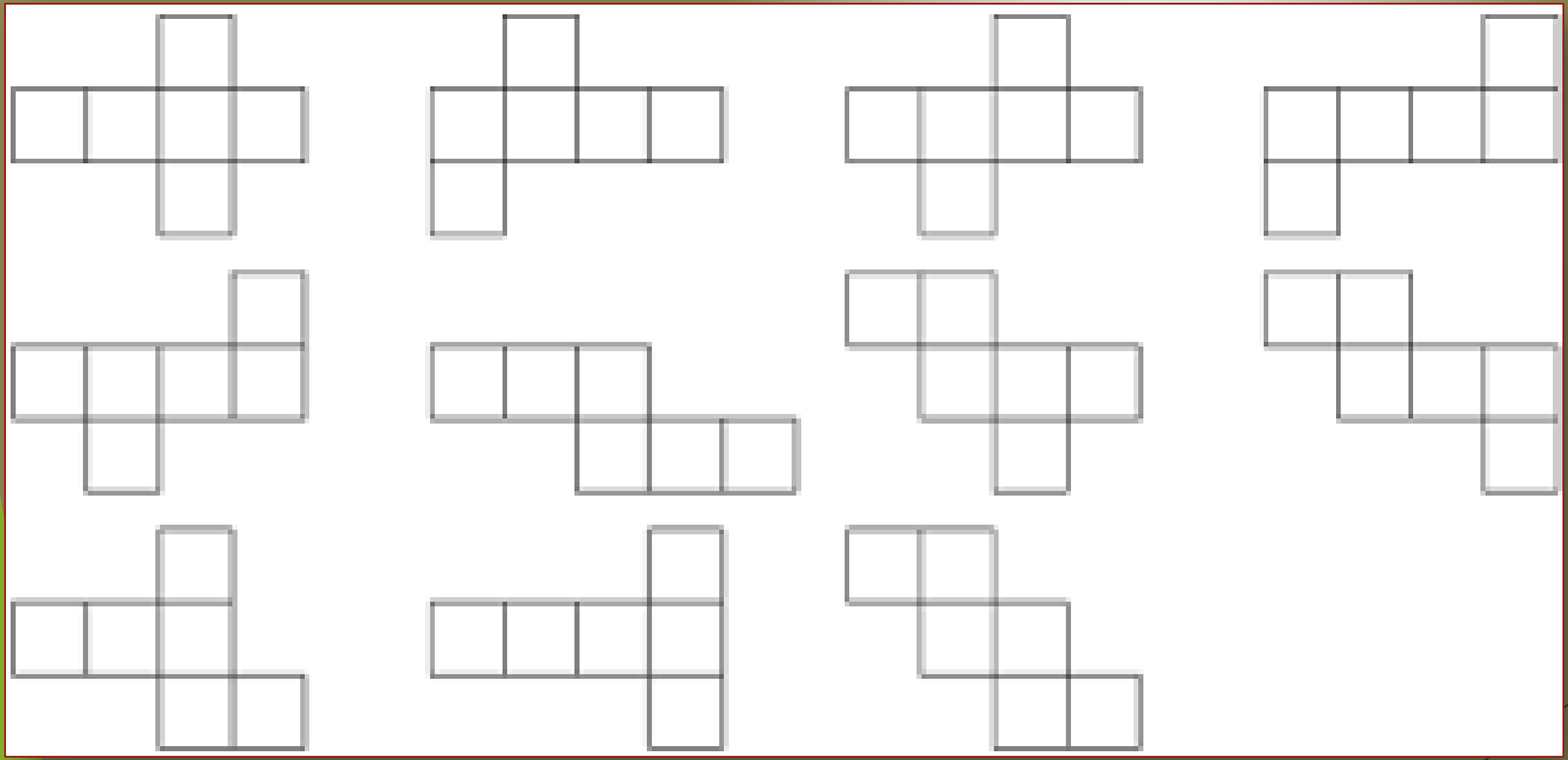
2. Cube



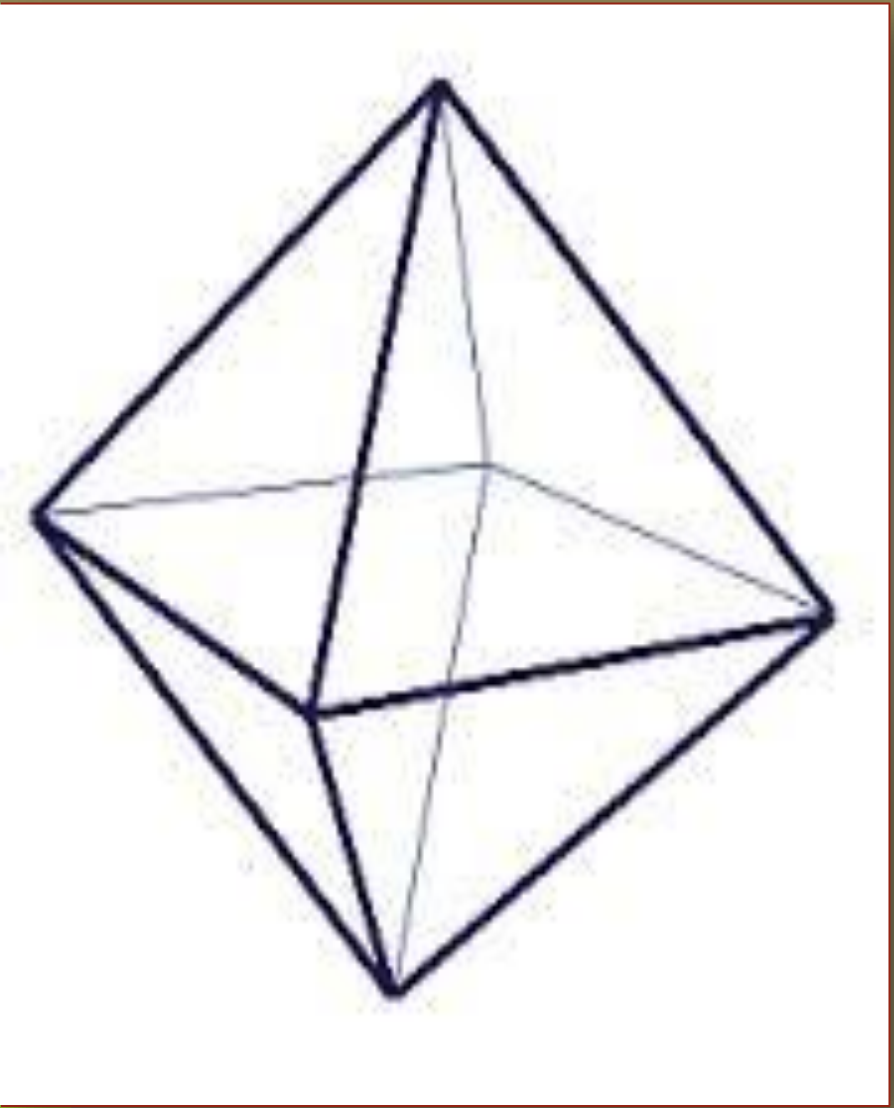
- **No. of faces:**
  6 square faces(faces meet at 90˚)
- **No. of vertices: 8**
- **No. of edges: 12**

- A cube has 11 distinct nets!

Can you draw all of them?



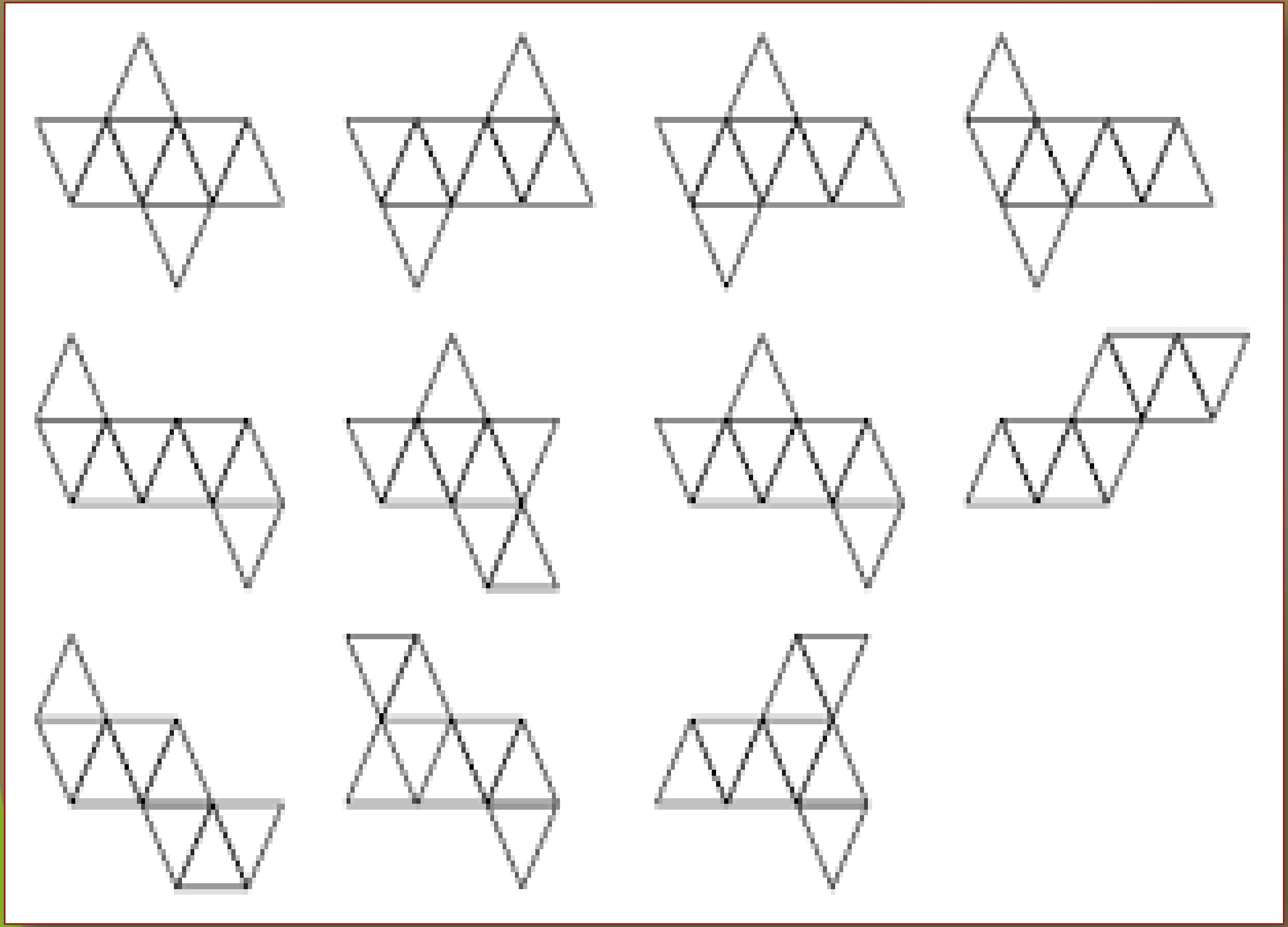
3. Octahedron



- No. of faces:

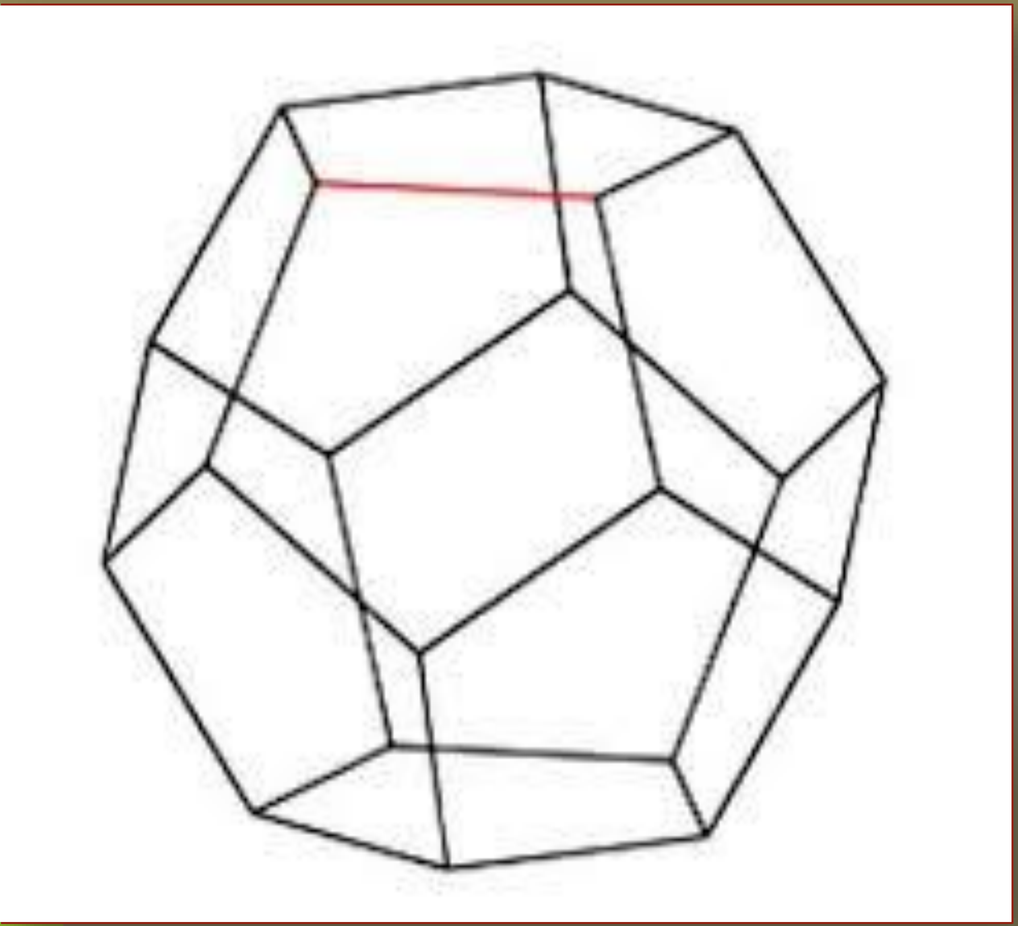
  8 equilateral triangular faces
- No. of vertices: 6
- No. of edges: 12

- A octahedron has 11 distinct nets!



Can you draw any of them?

4. Dodecahedron



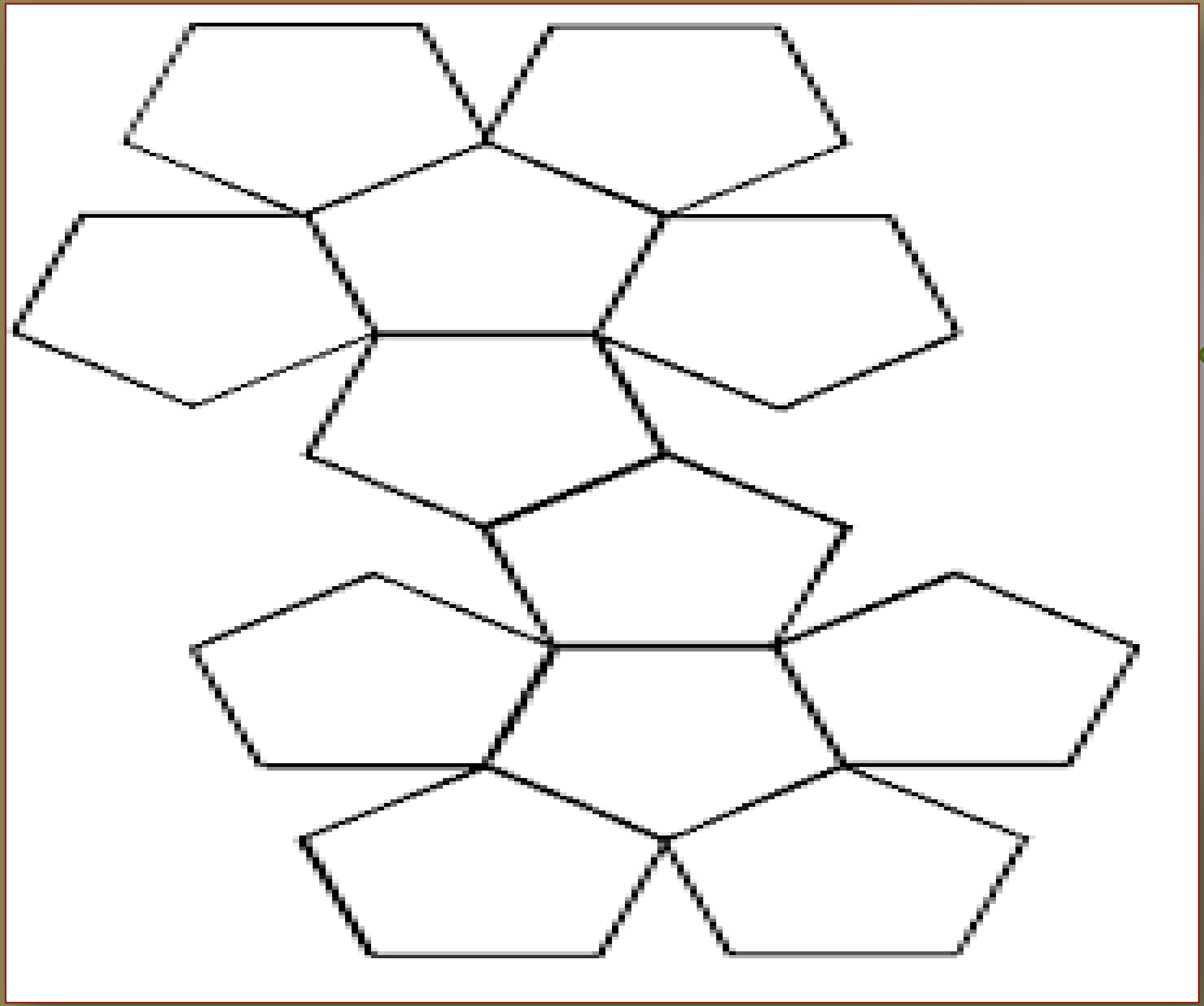
- No. of faces:

  12 pentagonal faces

- No. of vertices: 20

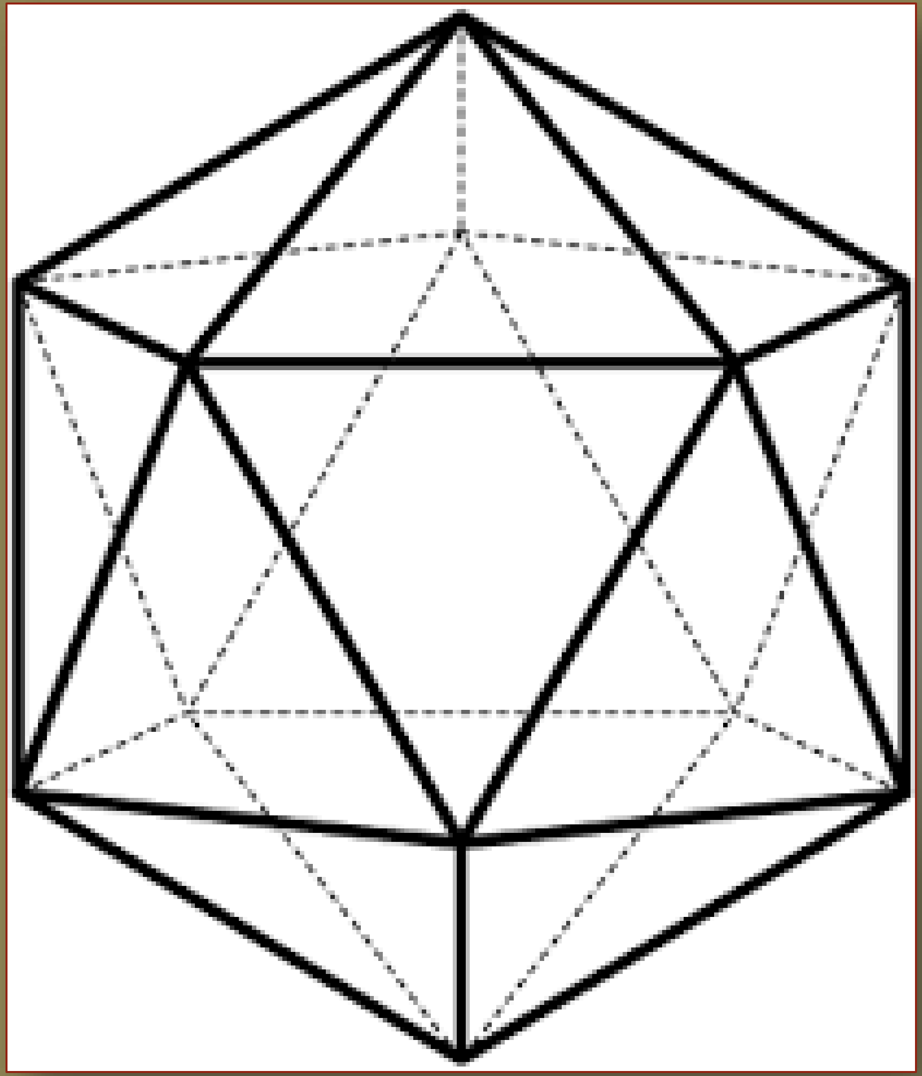
- No. of edges: 30

- A dodecahedron has 43 380 distinct nets!



Can you draw one?

5. Icosahedron



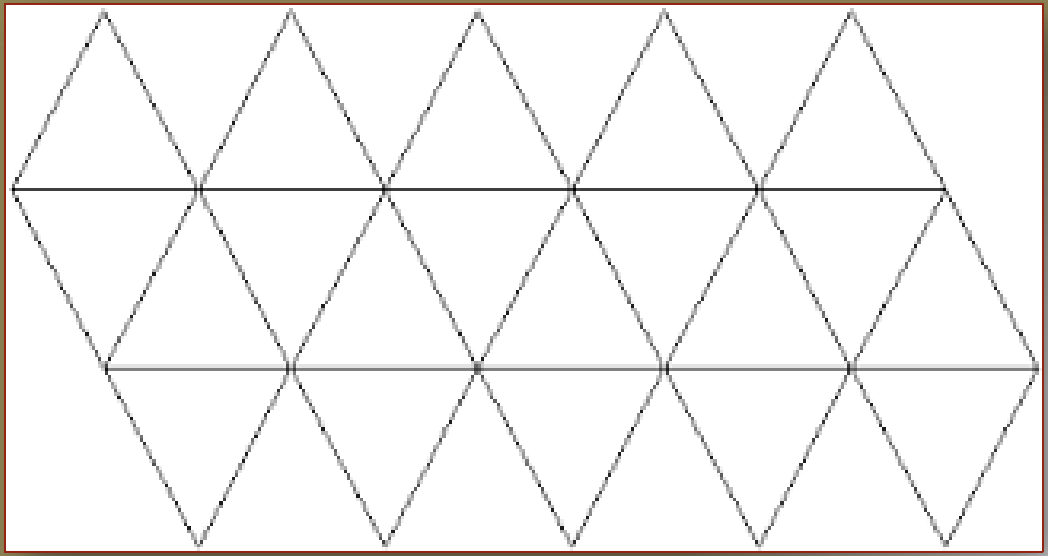
- No. of faces:

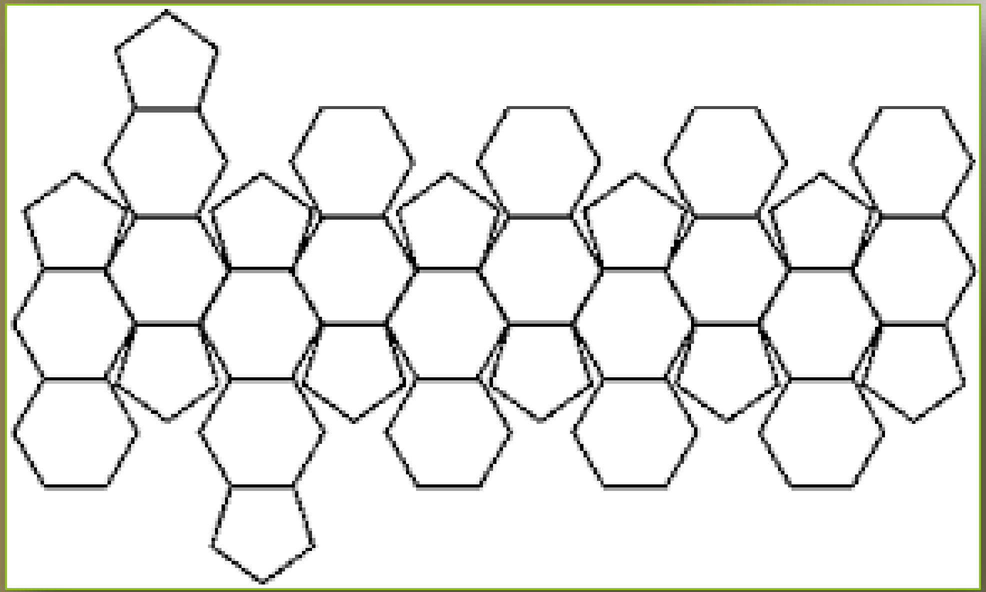
  20 equilateral triangular faces

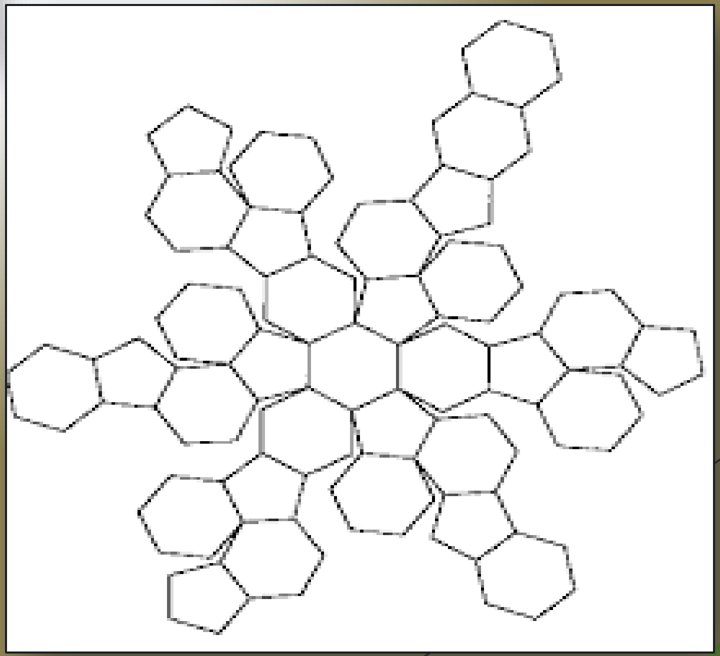
- No. of vertices: **12**

- No. of edges: **30**

- **An Icosahedron has 43 380 distinct nets!**



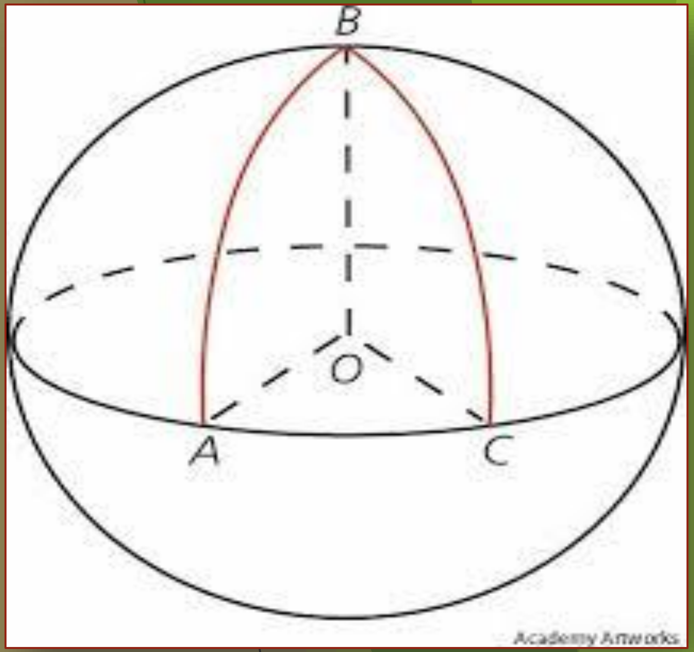




Can you draw one?

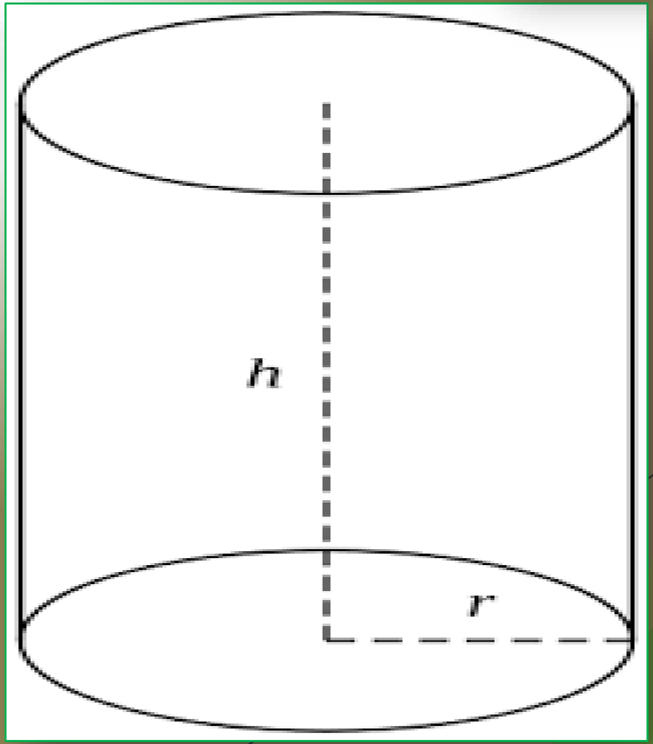
Spheres

- Are round solid figures, with every point on its surface equidistant from its centre



Cylinders

- Cylinders are closed solids, that have 2 parallel (circular or elliptical) base connected by a curved surface



[Platonic Solids Song]

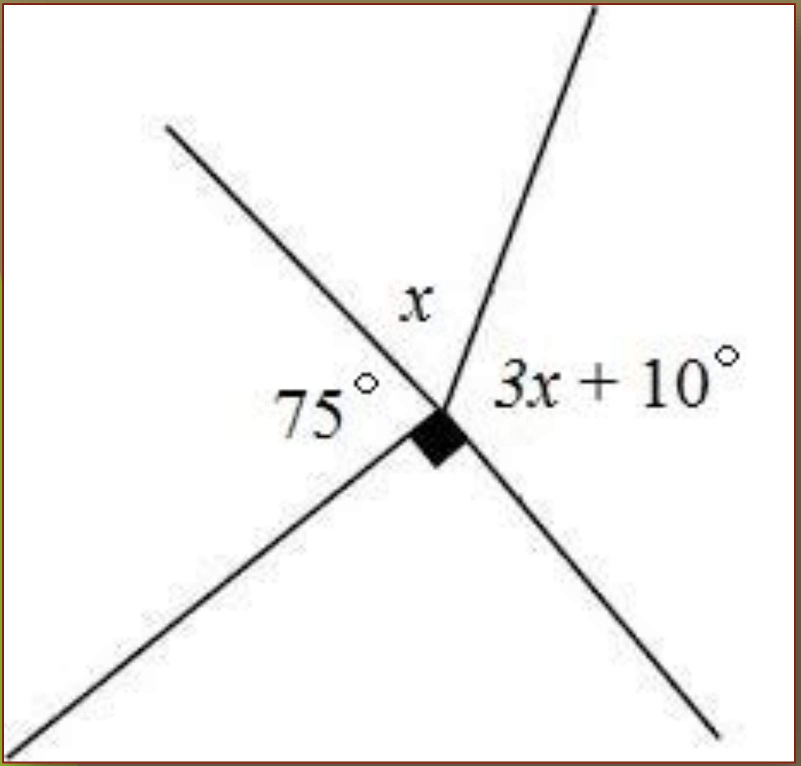
Your turn to be creative! Make your own platonic solid models by folding paper…

3.3 Geometry of Straight lines

Angle Relationships

1. Angles around a point add up to 360°

E.g. solve for



$$x + 75° + 3x + 10° + 90° = 360°$$

(angles around a point)

$$4x + 175° = 360°$$

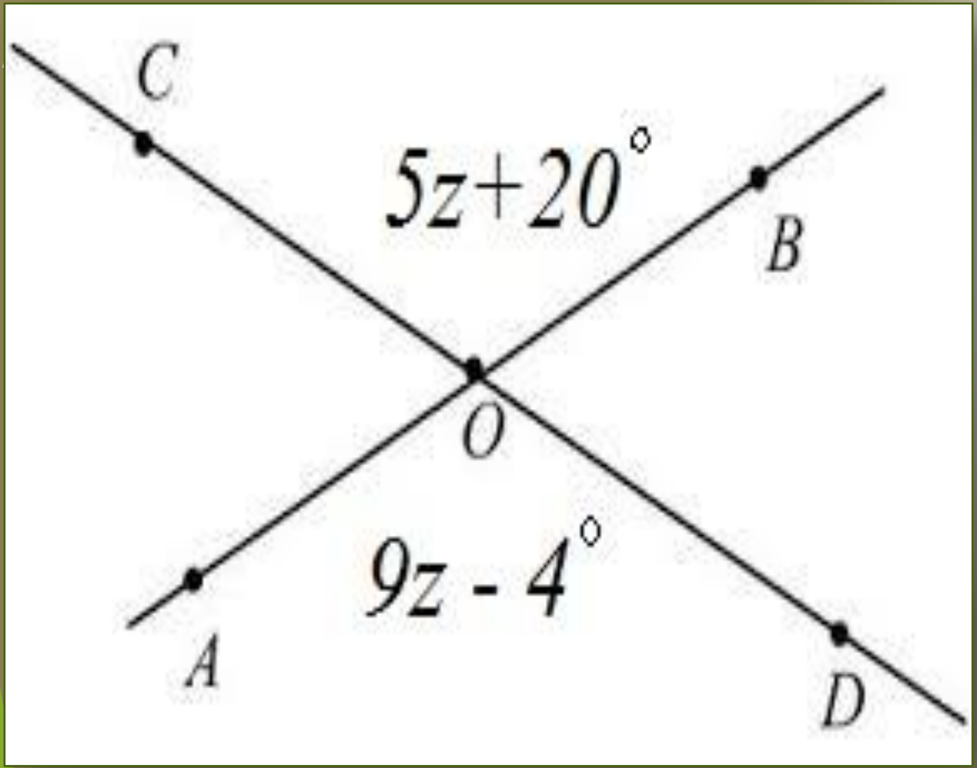
$$4x = 185°$$

$$x = 46.25°$$

Angles around a Point Examples

3. Vertically opposite angles are equal

E.g. solve for



$$5z + 20° = 9z - 4$$

(Vert.Opp $\angle's$)

$$5z - 9z = 4° - 20°$$

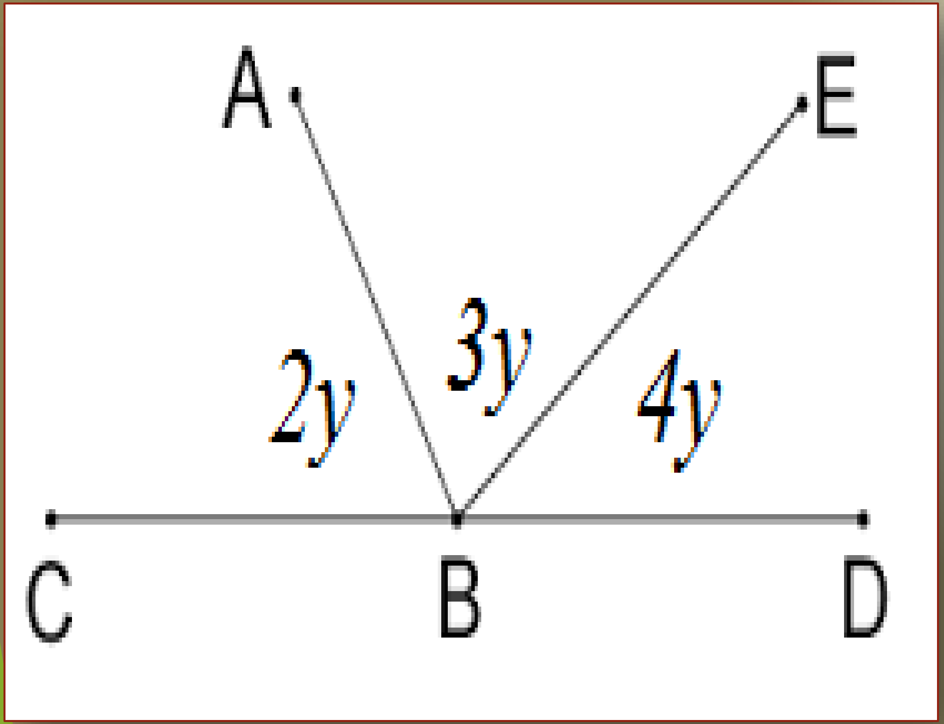
$$-4z = -24°$$

$$z = 6°$$

Vertically Opposite Angles Examples

2. Adjacent on a straight line add up to 180°

E.g. solve for y:



$$2y + 3y + 4y = 180°$$

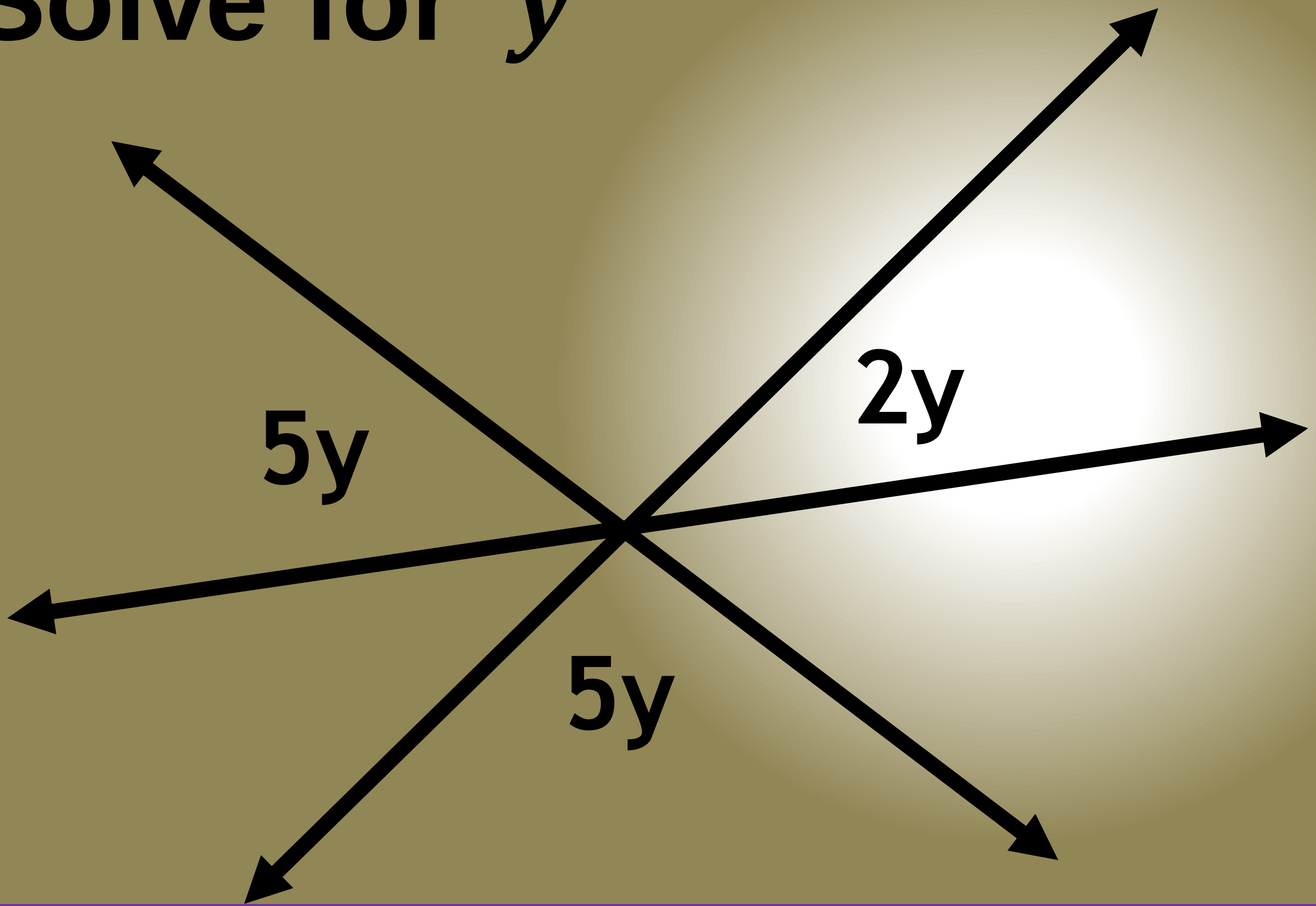
(Adj. angles on a str. line)

$$9y = 180°$$
$$y = 20°$$

Angles on a Straight Line Examples

EXERCISE

1. Solve for y



$*$ **Don′t forget to write reasons for statements!**

3. When parallel lines are cut by a transversal, then:

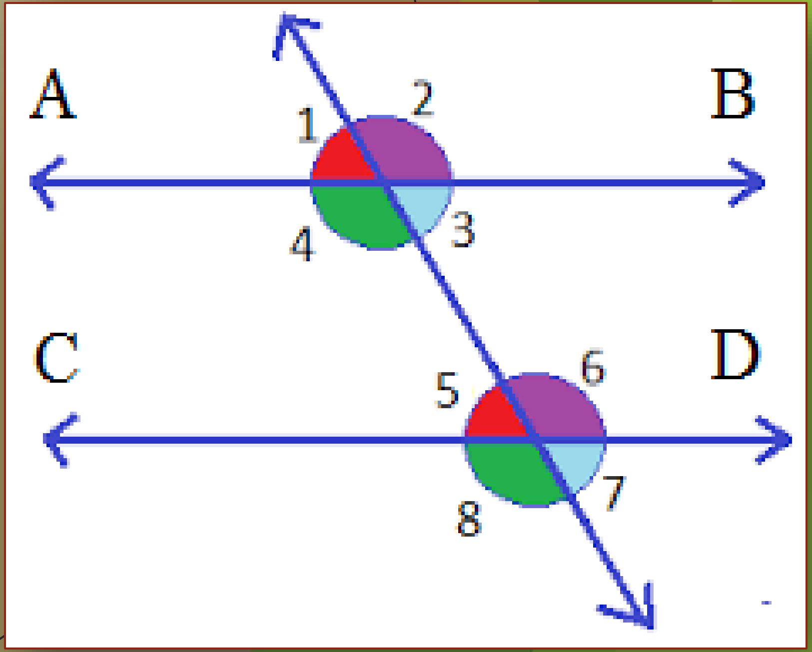
3.1. Corresponding angles are equal

F shape

E.g. if $4 = 120°, determine\ 8$

8=4 (corresp. $\angle' s$; $AB \parallel CD$)

$=120^{\circ}$



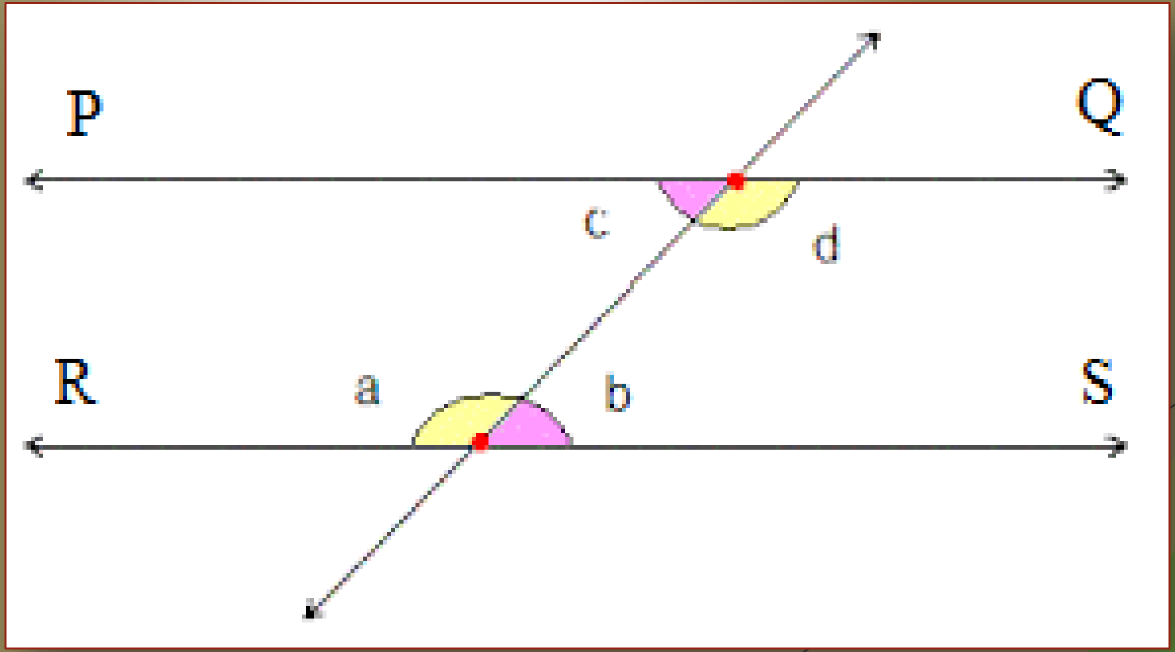
3.2. Alternate angles are equal

Z or N shape

E.g. if c =35˚, determine b.

b=c (alt.∟$'s\ PQ \parallel RS$)

= 35°



3.3. Co- interior angles add up to 180˚

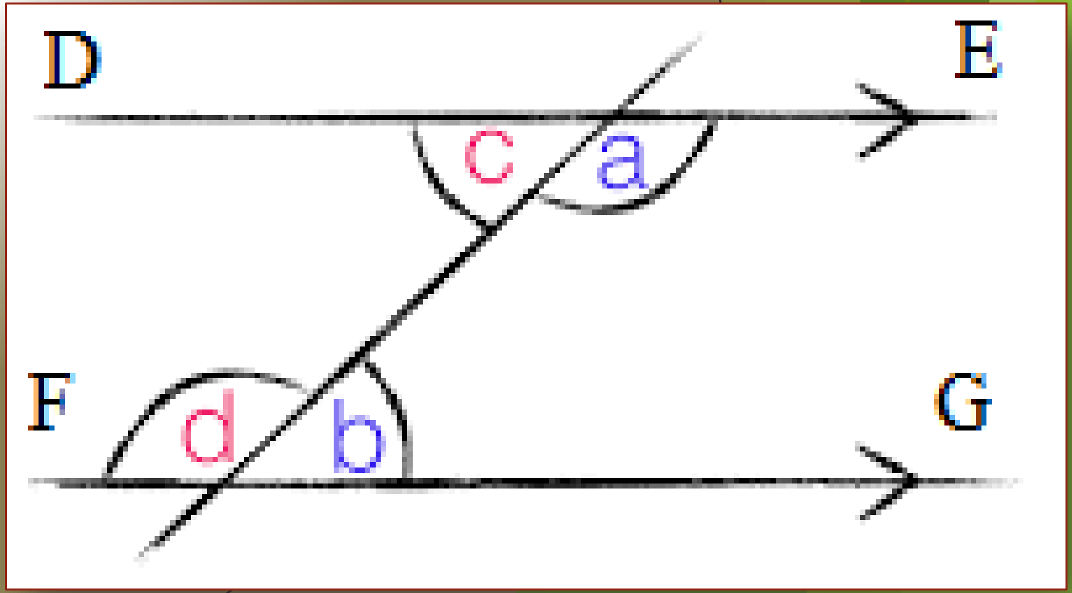
E.g. if a =130˚, determine b.

$$a + b = 180° \; (co - \text{int. } \angle's; \text{DE} \parallel \text{FG})$$

$$130° + b = 180°$$

$$b = 50°$$

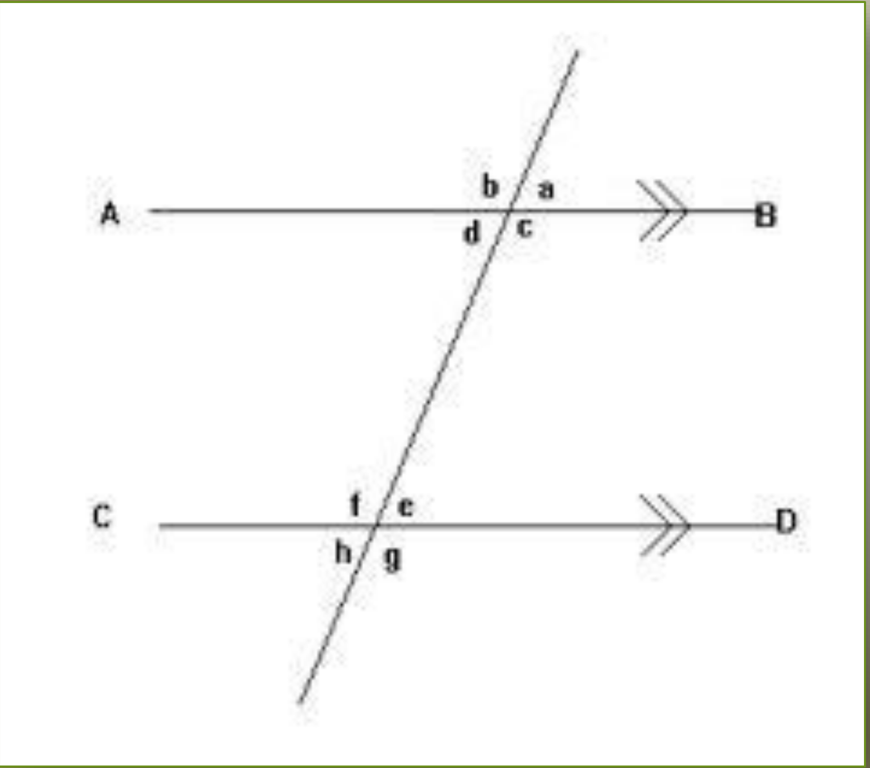
⊔ or ⊏ shape



EXERCISE

Angles formed by Parallel Lines & Transversals Example Problems

1. Solve for the unknown



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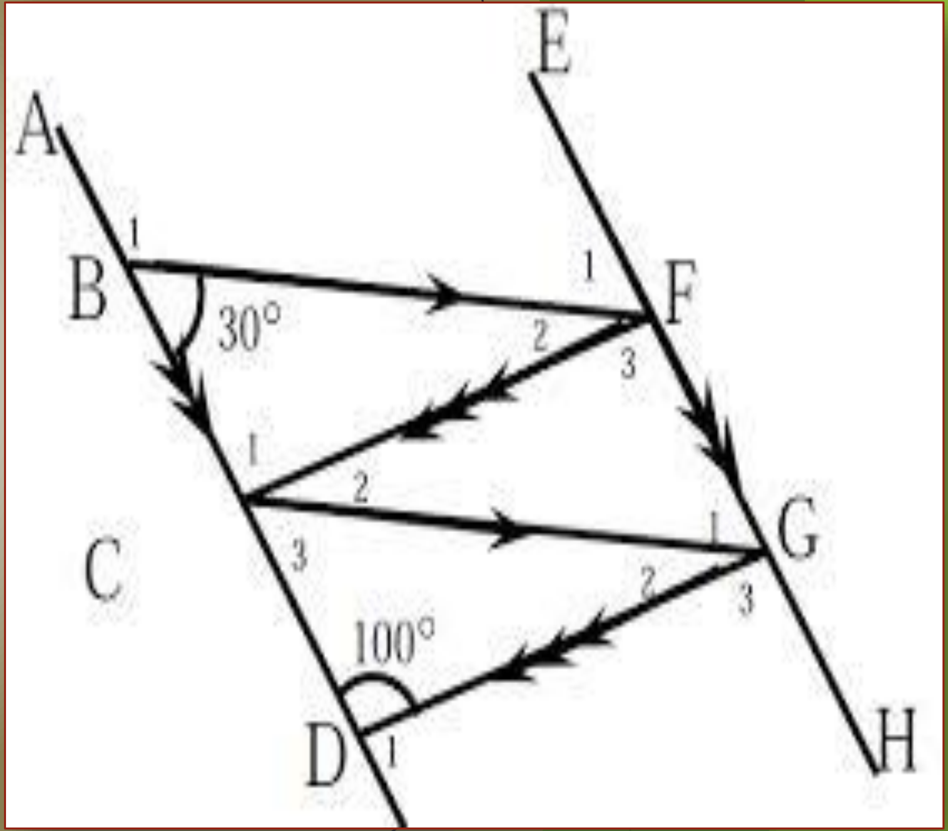


∗ Don't forget to write reasons for statements!

3.4 Transformation Geometry

Transformations

⇒ Transformations occur when a **point or object is moved**

⇒ If a figure's **shape & size remain the same**, the transformation is said to be **rigid**

⇒ Rigid transformations include**: translations, reflections & rotations**

⇒ The transformed point or object is **called the image**

⇒ **Notation:** $\boldsymbol{P(x, y)} \longrightarrow P'(\boldsymbol{x} \ldots \ldots ; \boldsymbol{y} \ldots .)$

transform point P' where
the general transformation
rule is applied to $(\boldsymbol{x}; \boldsymbol{y})$

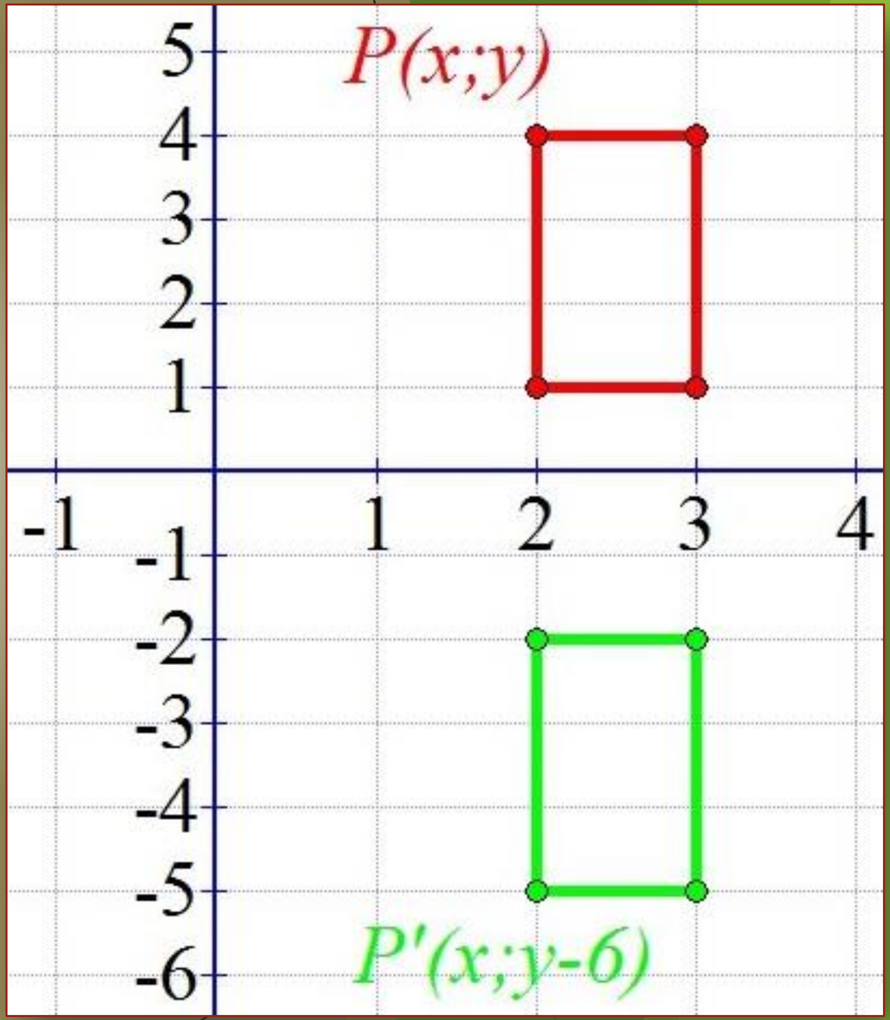
Translations

⇒ These transformations include:

i. Vertical translations
up & down movements

- Only affect the ***y*-coordinates**

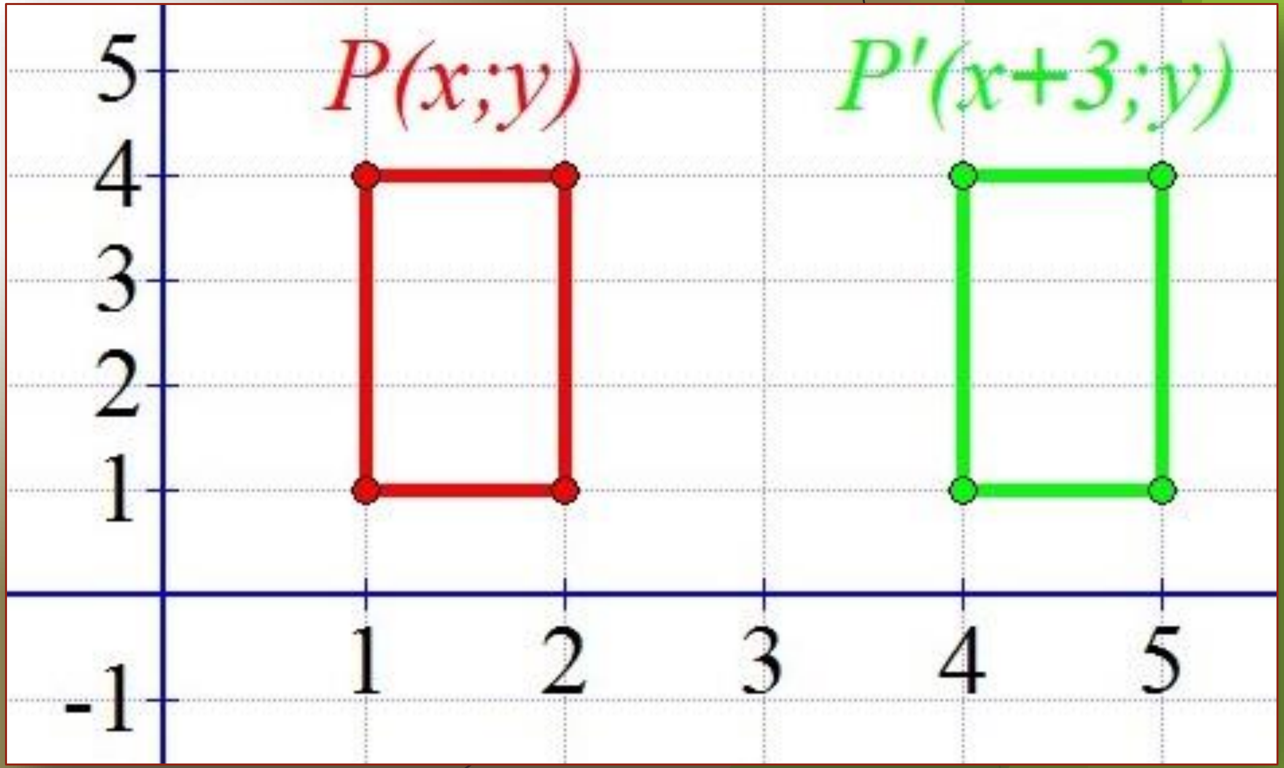
- $\mathrm{P}(x; y)] \rightarrow \mathrm{P}'(x; y \pm a)$
[where a is a constant]



ii. Horizontal translations

- **Left & right** movements
- Only affect the ***x*-coordinates**
- P$(x; y)\rightarrow$ P$'(x \pm a; y)$

[where a is a constant]

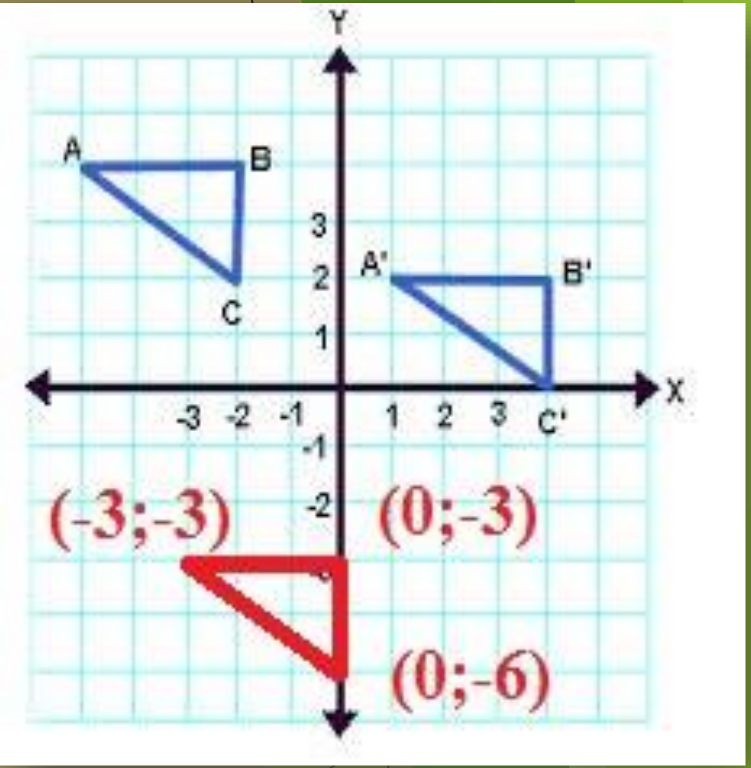


[Translating Shapes]

Examples

1. Describe the transformation that has occurred

   $\Delta ABC\ has\ moved\ 2\ units\ down$
   $and\ 6\ units\ to\ the\ right$



2. Write down the transformation rule

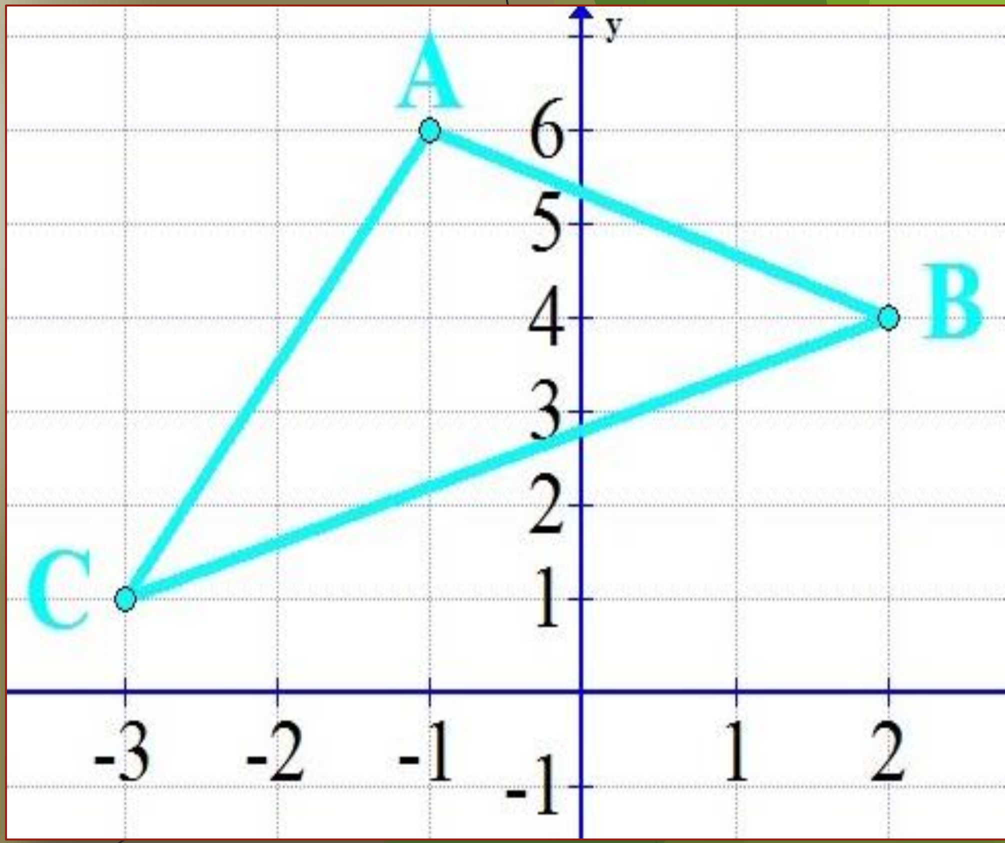
   P$(x; y)\rightarrow$ $\mathrm{P}'(x+6; y-2)$

3. Draw the transformed triangle if it is translated 2 units to the right and 7 units down

EXERCISE!

1. Draw the image of ABCD, if it is translated 4 units to the left and 5 units up

2. Write the transformation rule described in Question 1



2. Reflections

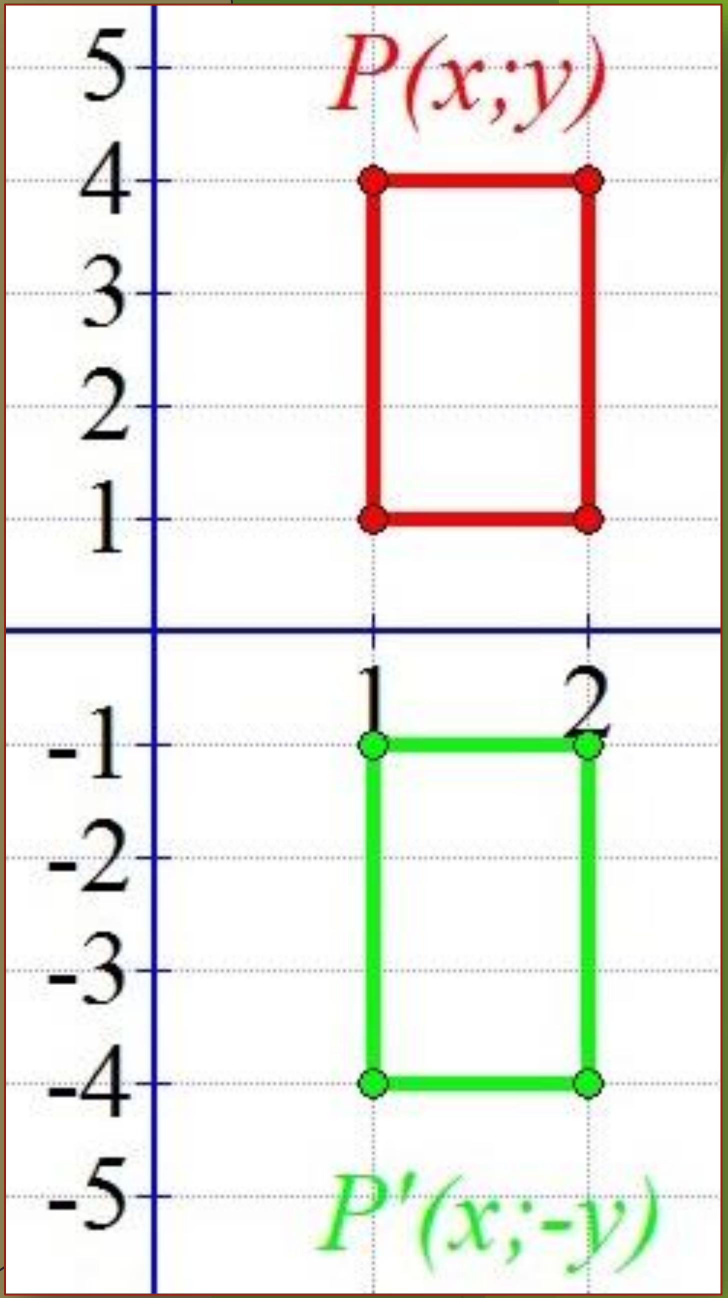
⇒ These transformations include:

i. Reflections about the x $axis$

The transformed point or object is a **mirror image** across a **horizontal line of** $\mathbf{y = 0}$

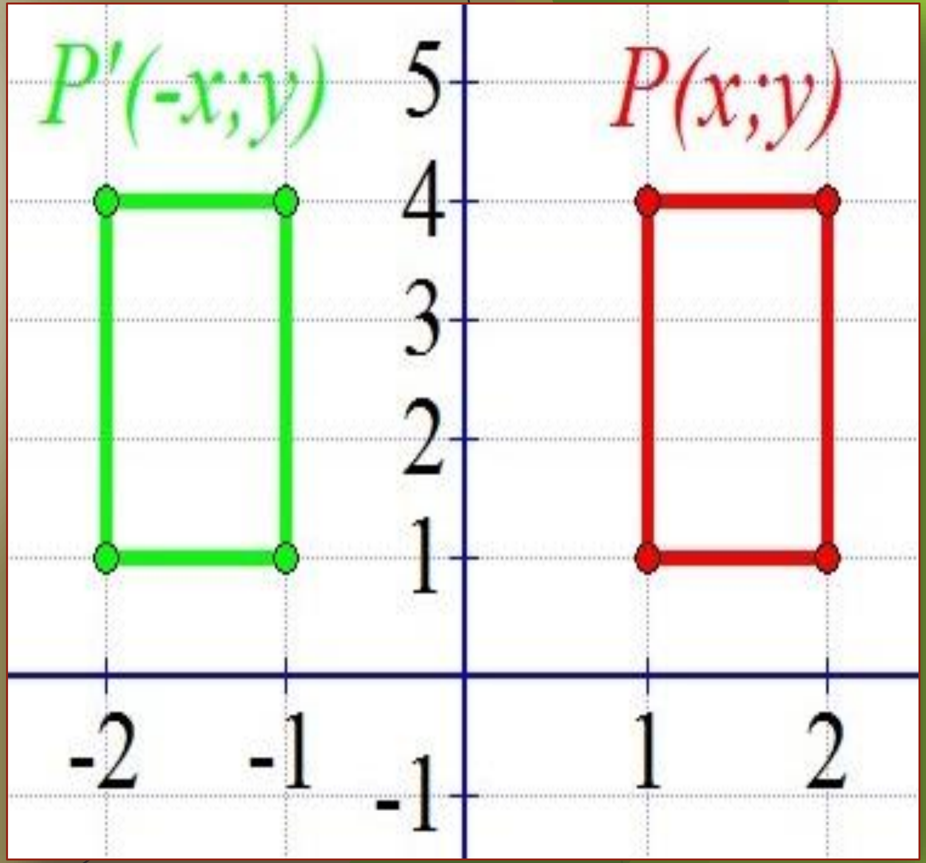
i.e the $\boldsymbol{x}$ $\boldsymbol{axis}$

- $\boldsymbol{x}$ co ordinates stays the same, $\boldsymbol{y}$ **co ordinates changes sign**
- $\mathrm{P}(x; y) \rightarrow \mathrm{P}'(x; y)$



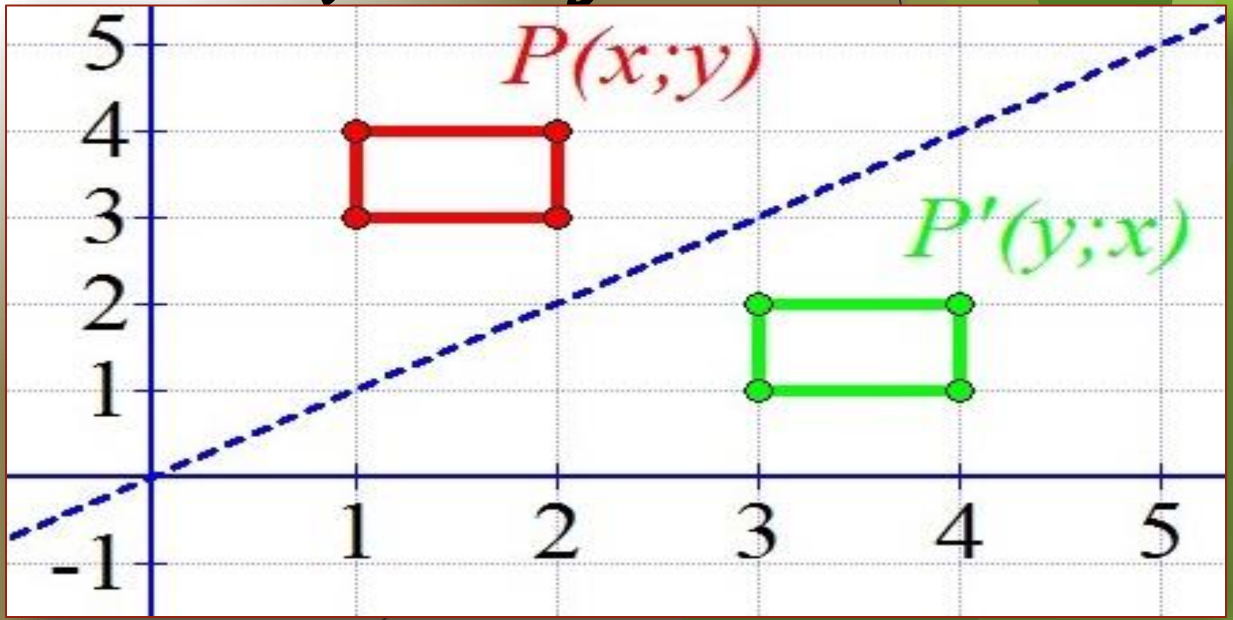
ii. Reflections about the y $axis$

- The transformed point or object is a **mirror image** across a **vertical line of** $\boldsymbol{x = 0}$ **i.e** the $\boldsymbol{y - axis}$
- $\boldsymbol{x}$ **co ordinates change sign,** $\boldsymbol{y}$ co ordinates stays the same
- P$(x; y)$→ P$'(-x; y)$



iii. Reflections about the line $x = y$

- The transformed point or image is a **mirror image** across a **diagonal line** that **intersects** the **origin** $(0 = 0)$ - ie. The line $x = y$
 i.e the $y -$ $axis$
- **Swop the co ordinates of x,and y around**
- $P(x; y) \rightarrow P'(-x; y)$



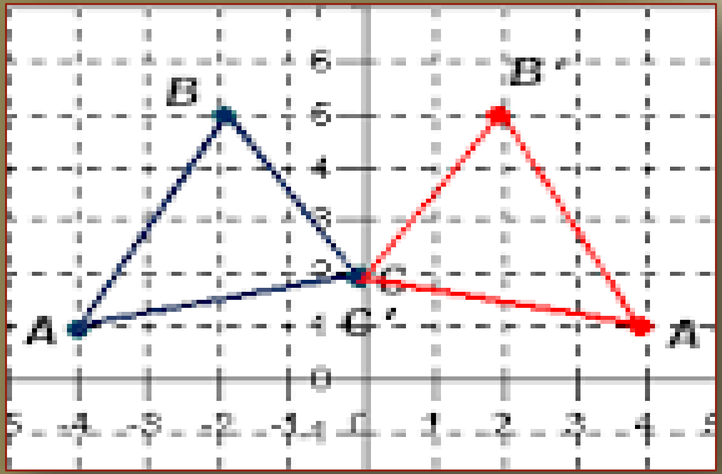
Reflecting Shapes

Examples

For each of the following:

i. Describe the transformation
ii. Write down the line of symmetry rule
iii. Write down the general transformation rule

ie. The line of reflection



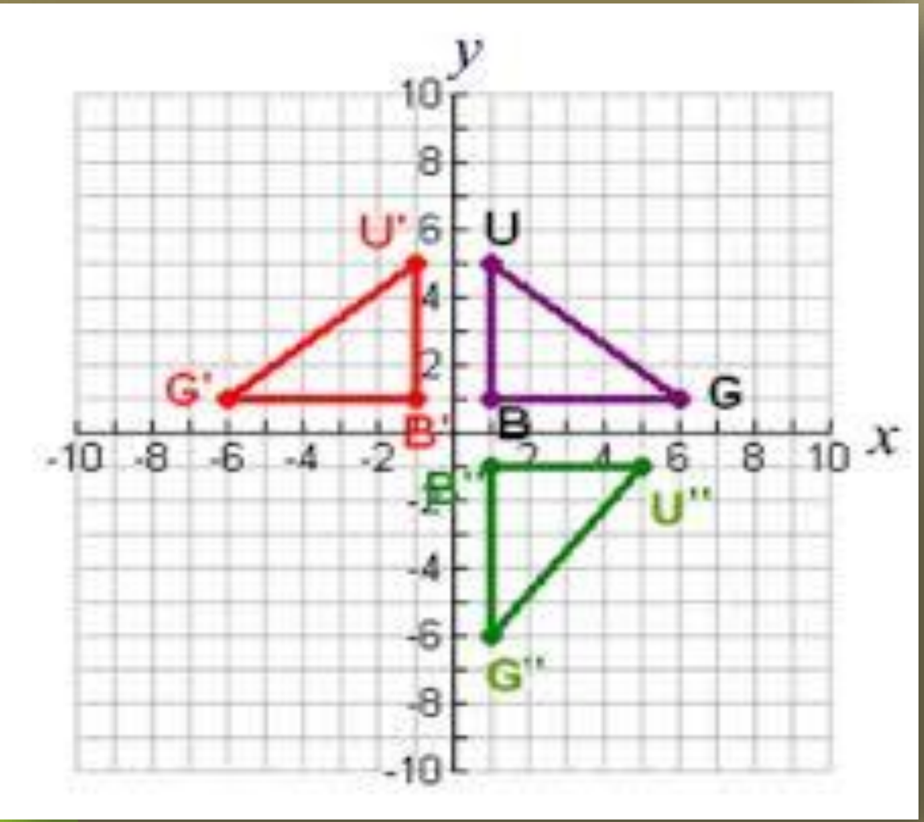
i. reflect about the y $axis$

ii. $x = 0$

iii. $\mathbf{P}(x; y) \rightarrow \mathbf{P}(-x; y)$

2. For U’G’B’→ U”G”B”

(ie.red to green transformation)



For U’G’B’→ U”G”B”:

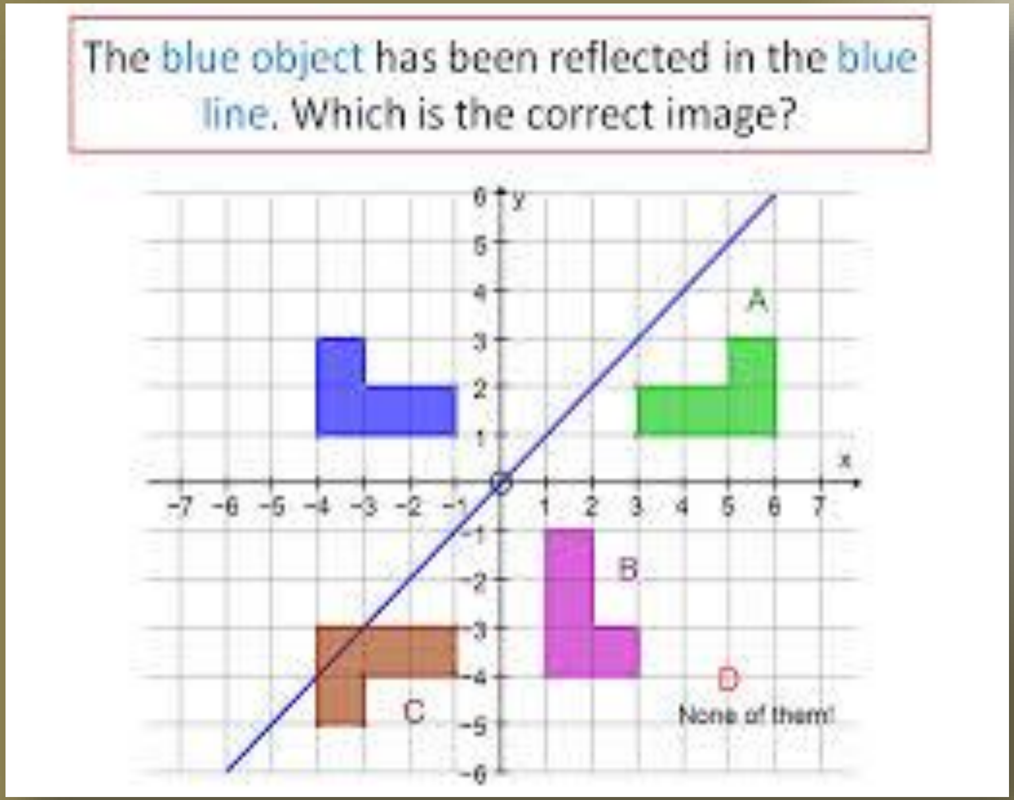
i. **reflect about the** $y = x\ line$

ii. $y = x$

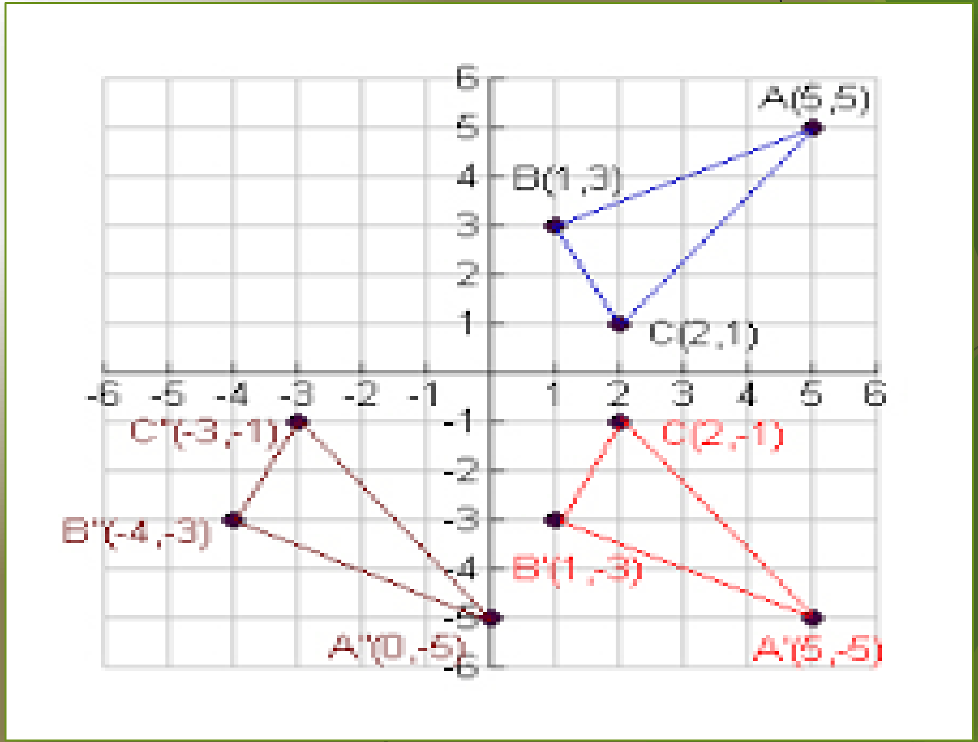
iii. P$(x; y)$→ $\mathrm{P}'(-x; y)$

EXERCISE!

1.



2. Describe the transformations and write down the general rule for transformations from **ABC**→ $A'B'C$ →**A''B''C''**



Enlargement

⇒ Enlargement involve the enlarging of an object, in the **same proportion** by a factor – called the **scale factor**

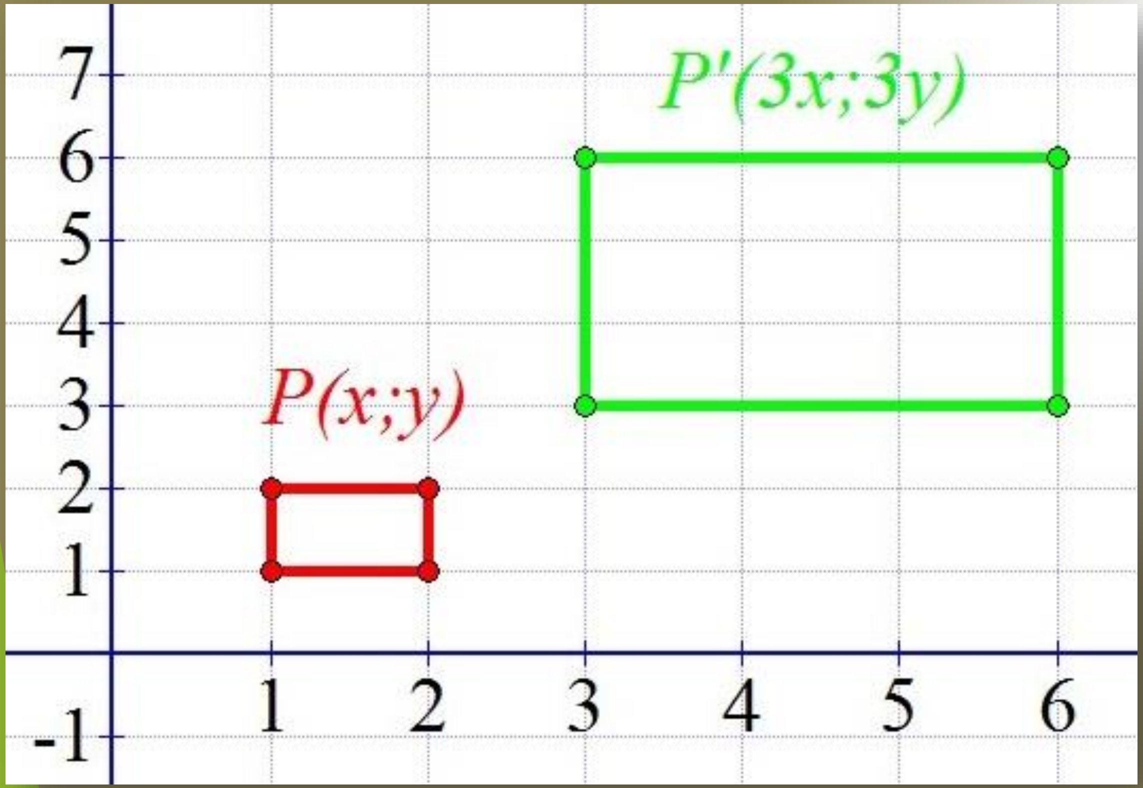
- The size of the angles stay the same; while **object get bigger**
- P($\boldsymbol{x}$; $\boldsymbol{y}$)→ $\mathrm{P}'(\mathrm{a}x; ay)$

[where a is a constant and $a > 1$]

Enlargements

Example

$\Delta ABC \to A'B'C$ has been enlarged by a factor of 3

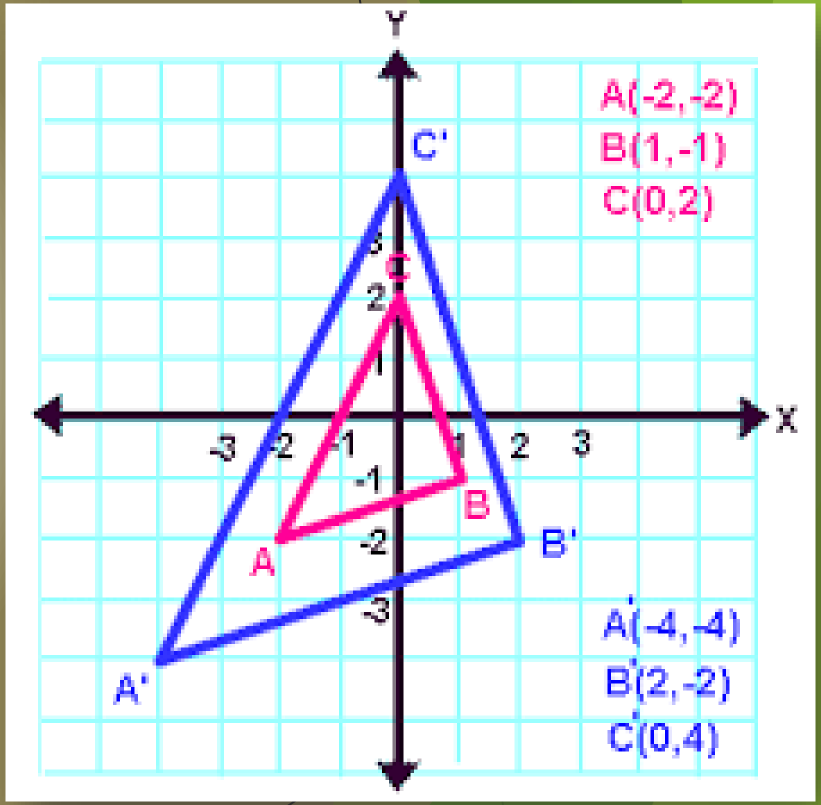


- Each co-ordinate is multiplied by 3
  e.g. A(1;1)→ $\boldsymbol{A'(3;3)}$
  $\therefore \boldsymbol{P(x;y) \to P'(x;3y)}$
- The area will be 3 times longer
- The perpendicular will be **(3)**^{2}**=9** times bigger ie. $(\text{scale factor})^2$

EXERCISE!

Given the following transformation:

1. Describe the transformation
2. Write down the general rule for the transformations
3. Draw $A"B"C" \rightarrow A'B'C$ is enlarged by a factor of 4
4. By how many times larger will the area be from $A'B'C \rightarrow A"B"C"$



Reduction

⇒ Reductions involve reducing each length of an object, in the **same proportion** by a factor – called the **scale factor**

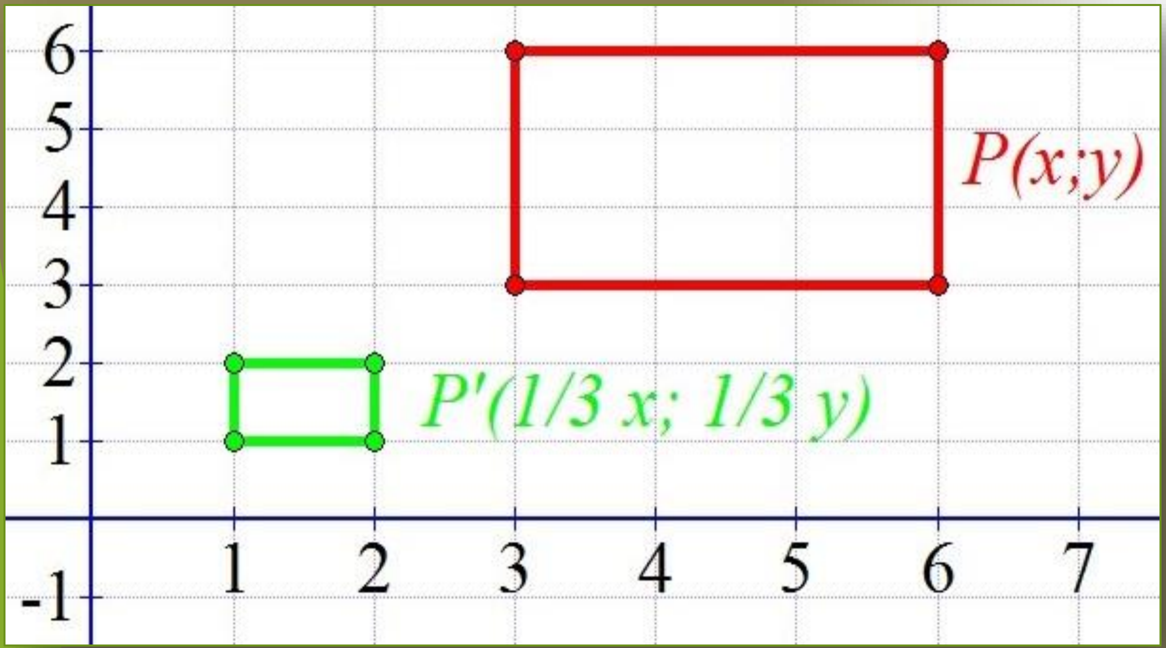
- The size of the angles stay the same; while **object get smaller**
- P$(x; y)$→ $\text{P}'(\text{a}x; ay)$

[where a is a constant and $0 < \text{a} < 1$]

Reductions

Example

$\Delta ABC \rightarrow A'B'C$ has been enlarged by a factor of 3



- Each co-ordinate is multiplied by 3

e.g. A(1;1)→ $A'(3;3)$

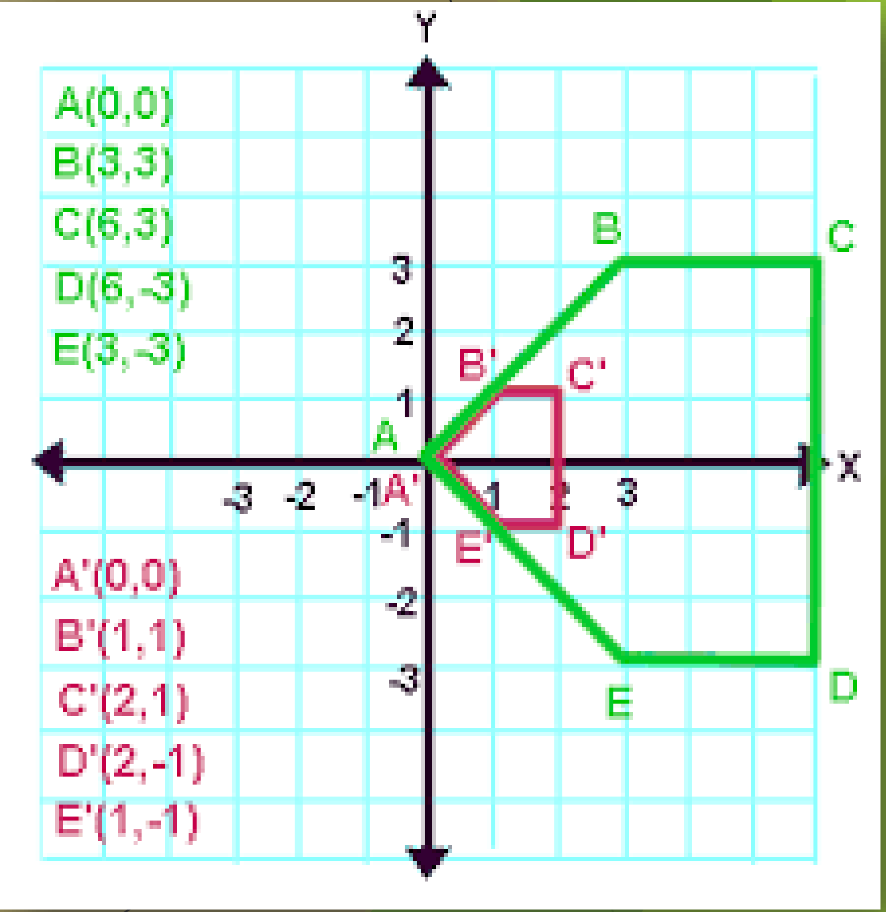
- The perimeter will be 3 times shorter

- The area be $(3)^2$=9 times smaller ie. $(\text{scale factor})^2$

EXERCISE!

Given the following transformation:

1. Describe the transformation
2. Write down the general rule for the transformation.
2. By what factor will original shape and images perimeter and area be reduced by?

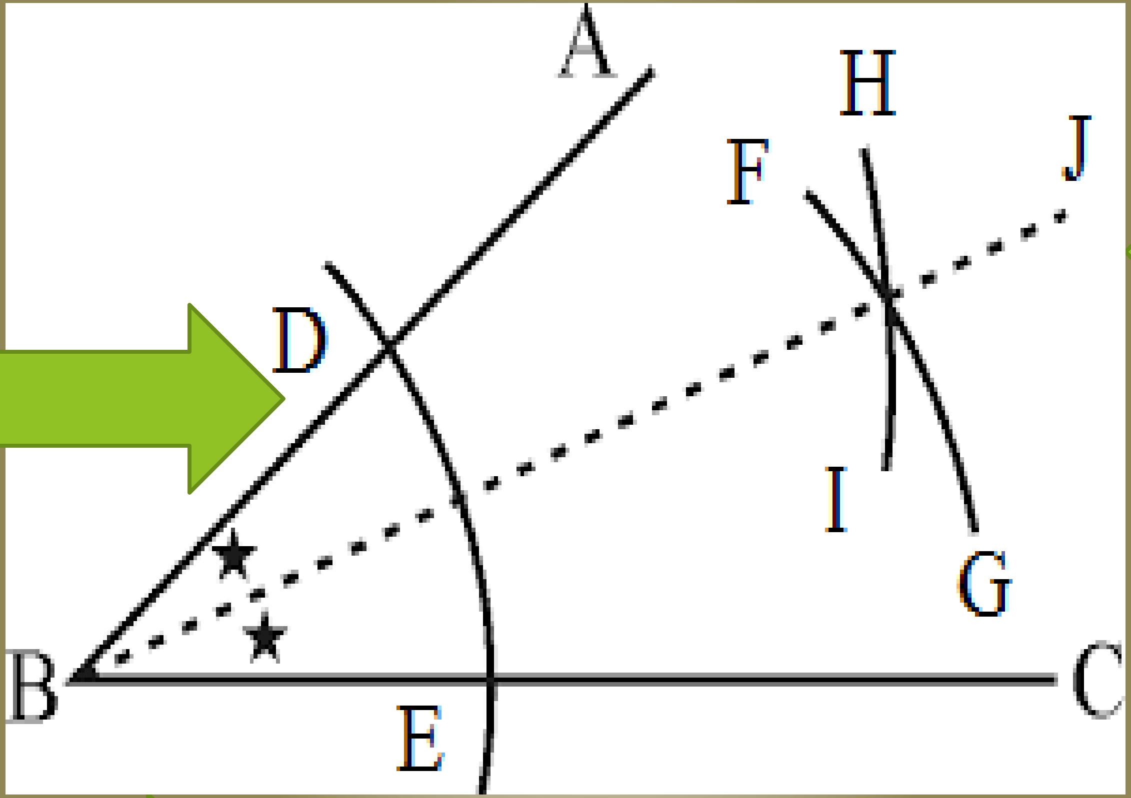


3.5 Construction of Geometric Figures

Constructions

1.Bisecting an angles

2. Place compass on *D & draw arc FG*



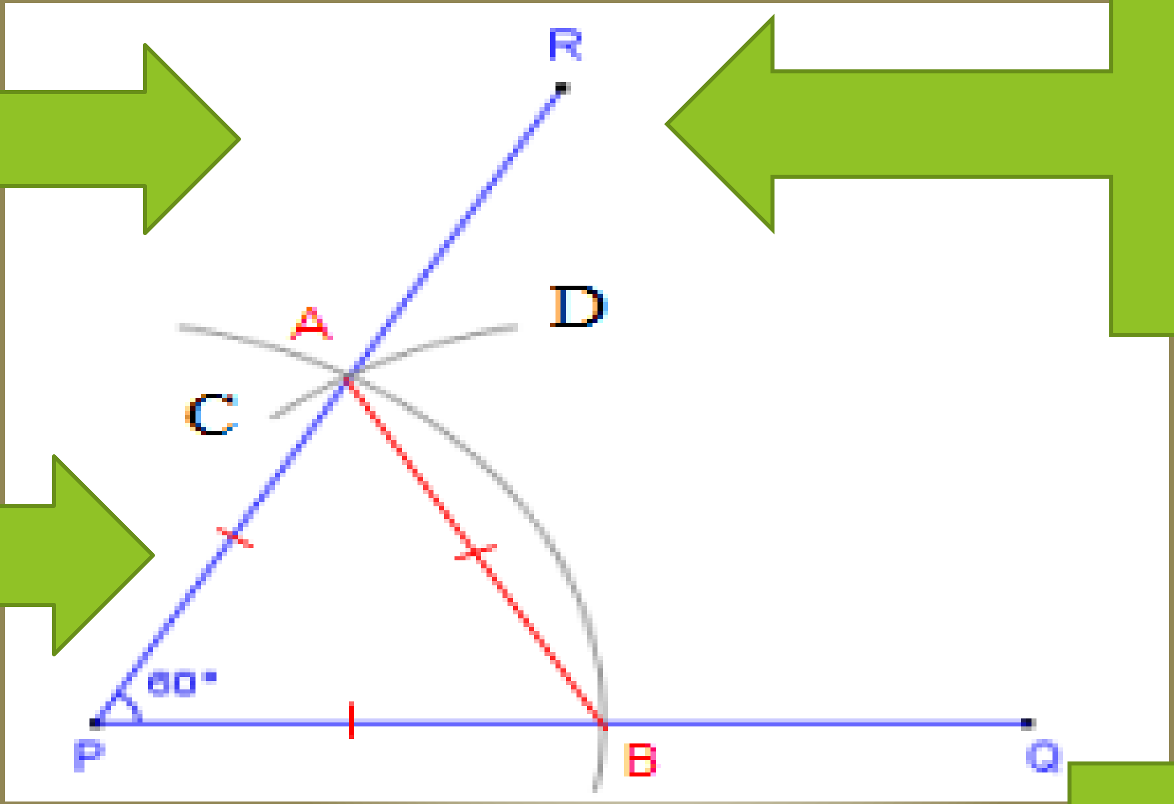
4. Draw line BJ to bisect angle ABC

1. Place compass on B & *draw arc DE*

3. Keep the compass the same & place on E . Draw arc HI

2. Constructing Angles: 60°;30°&15°

5. Bisect the angle of 30° to get an angle of 15°



3. Draw line PR to get angle RPB=60°

4. Bisect RPB to get an angle of 30°

[Constructing Different Triangles]

1. Place compass on P & *draw arc AB*

2. Keep the compass the same & place on B. Draw arc CD

3. Constructing a triangle given 3 sides

1. Draw line KJ

2. Measure the first given length on a compass (using a ruler)

3. Place the compass on K & draw arc M

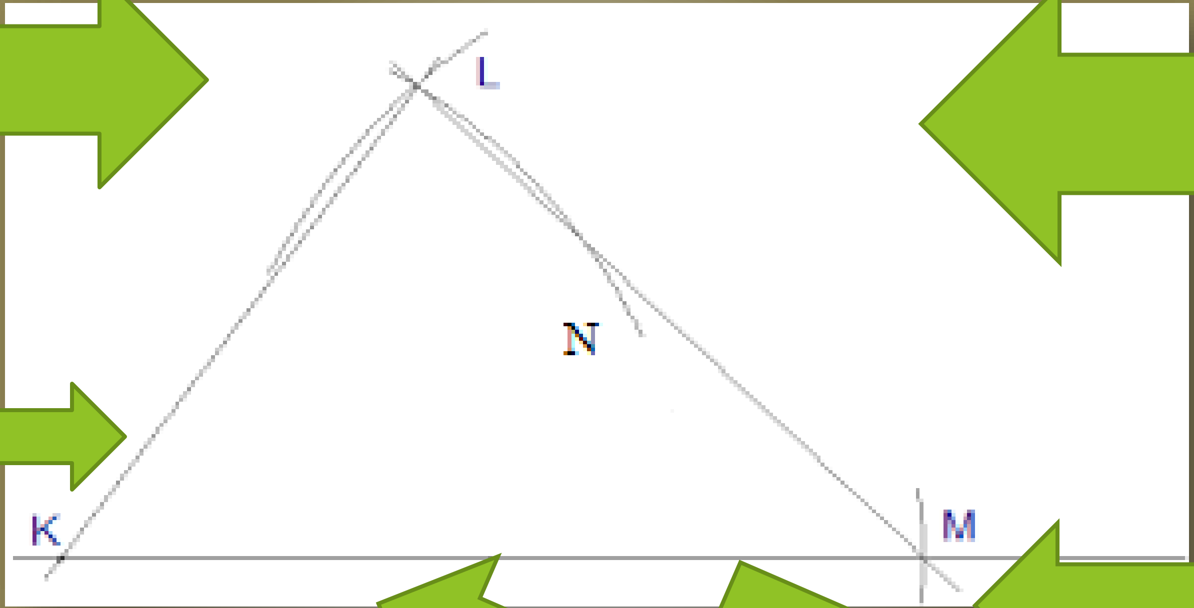
4. Measure the second given length on a compass (using a ruler)

5. Place the compass on K & draw arc L

6. Measure the third given length on a compass (using a ruler)

7. Place the compass on M & draw arc N

8. Join points to form Δ



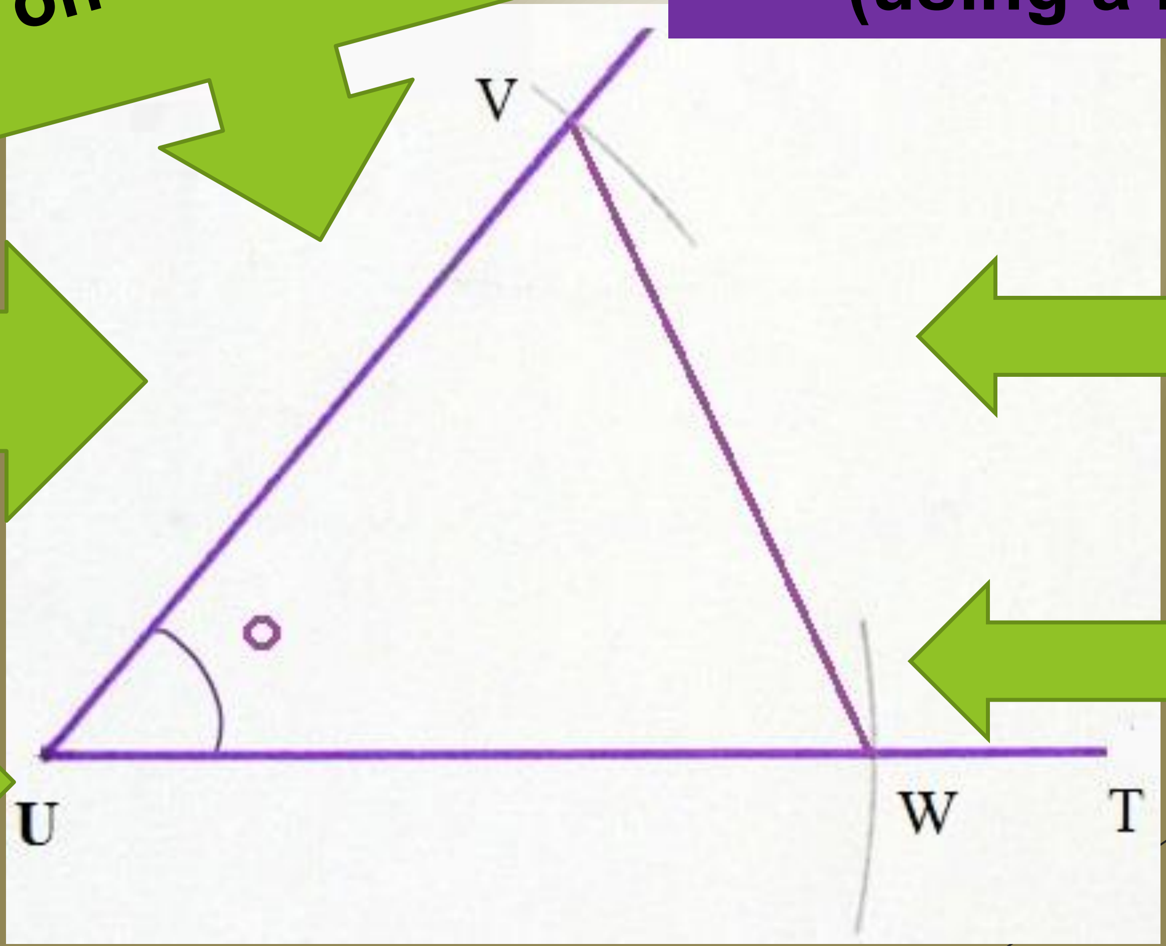
4. Constructing a triangle given 2 sides & a Non- Included Angle

6. Place compass on U and draw V

5. Measure the second length on a compass (using a ruler

2. Measure the First length on a compass (using a ruler)

4. Construct the given angle at the point B



7. Draw line VW

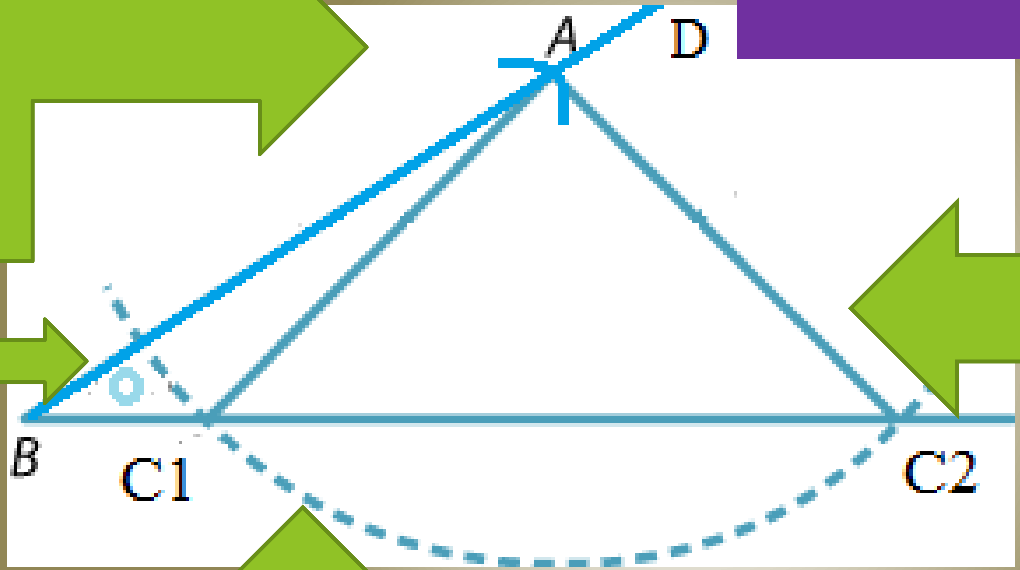
3. Place the compass on U & draw arc W

1. Draw line UT

6. Constructing a triangle given 2 sides & a non- included Angle

5. Measure the second given length on a compass (using a ruler

4. Construct the given angle at point B



6. Place compass on A and draw arc C1-C2

1. Draw line BD

2. Measure the First length on a compass (using a ruler)

8. Draw lines AC2 for triangle ABC2

3. Place the compass on D & draw arc C

7. Draw lines AC1 for triangle ABC1

Constructing Triangles & Quadrilaterals

6. Constructing a Quadrilateral given 3 sides & 2 Included Angles

1. Draw line AE

2. Measure the First length on a compass (using a ruler)

3. Place the compass on A & draw arc D

4. Construct the second given angle at point A

5. Measure the second given length on a compass (using a ruler

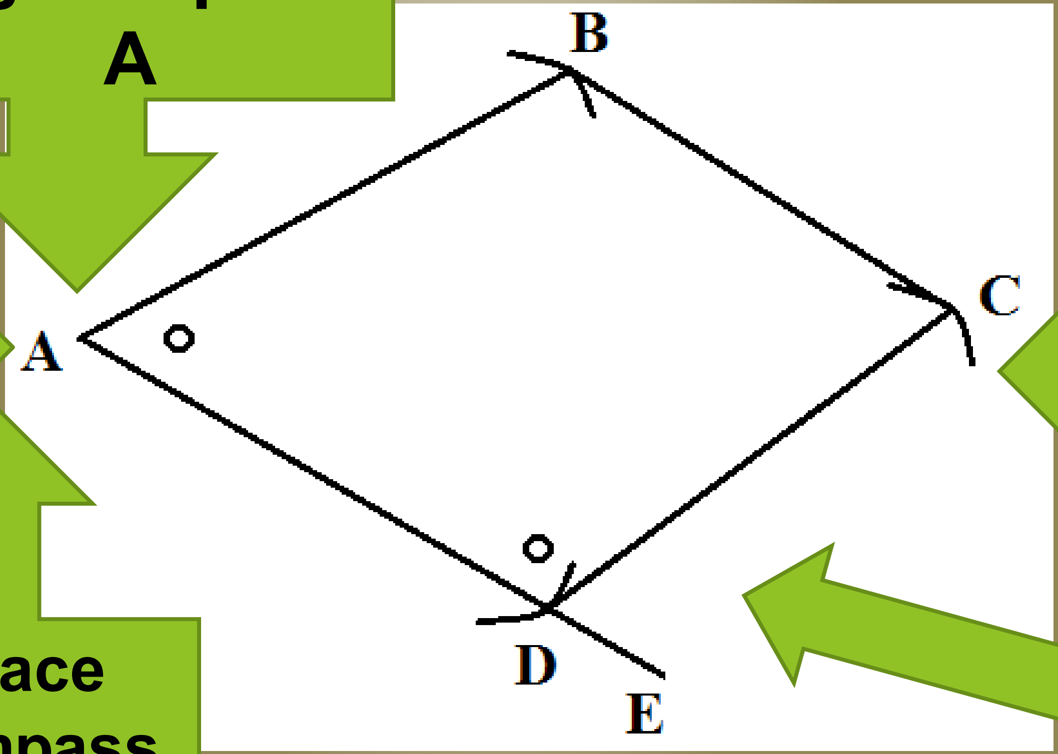
6. Place compass on A and draw arc B

7. Measure the Third length on a compass (using a ruler)

8. Place the compass on D & draw arc C

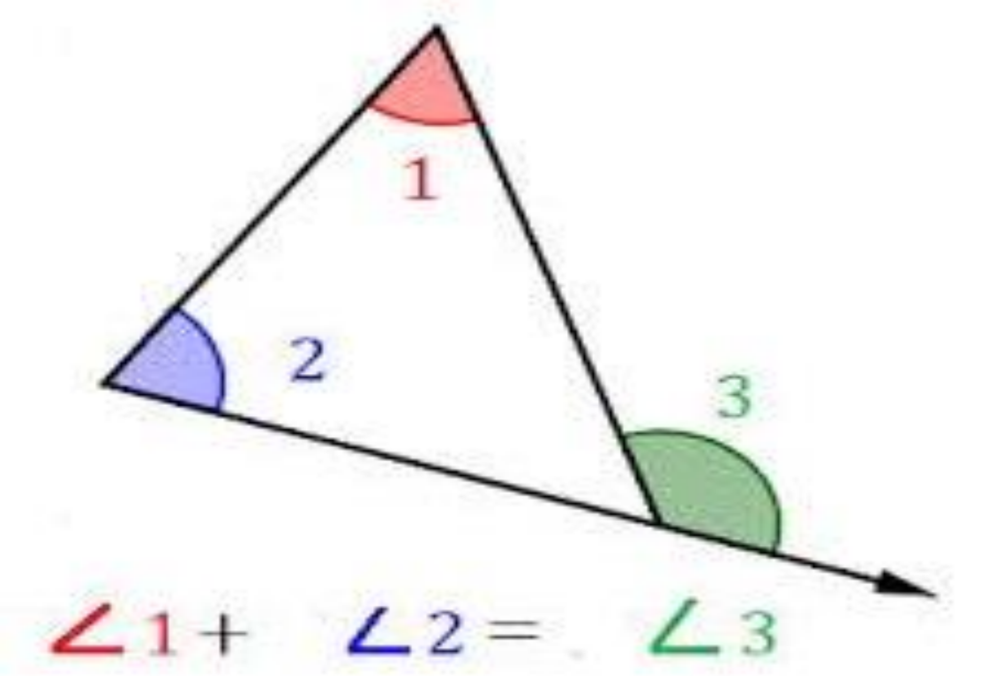
9. Construct the second given angle at point D

Draw line BC

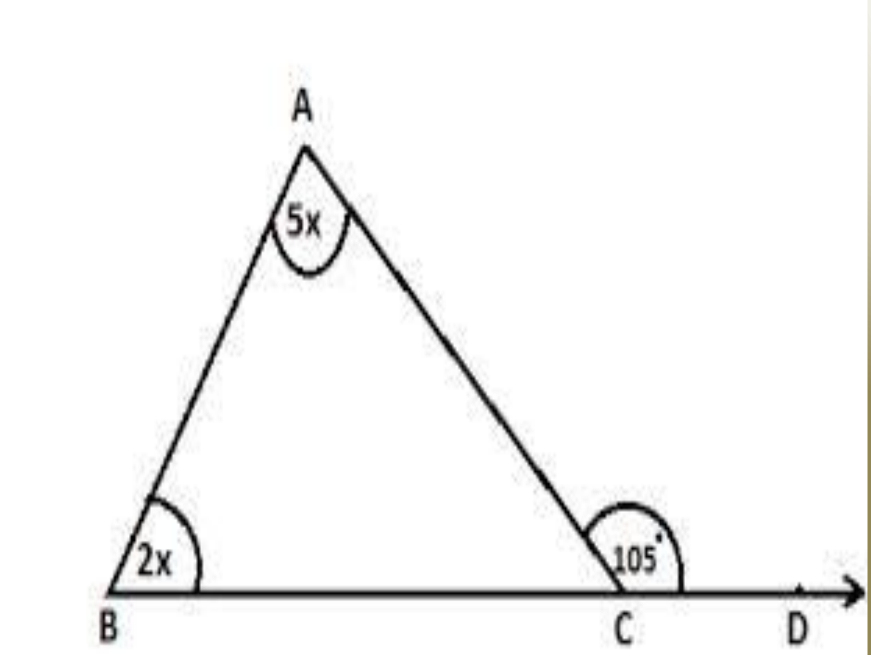


Exterior Angle of a triangle

The exterior angle of a triangle = the sum of the interior opposite angles



E.g.



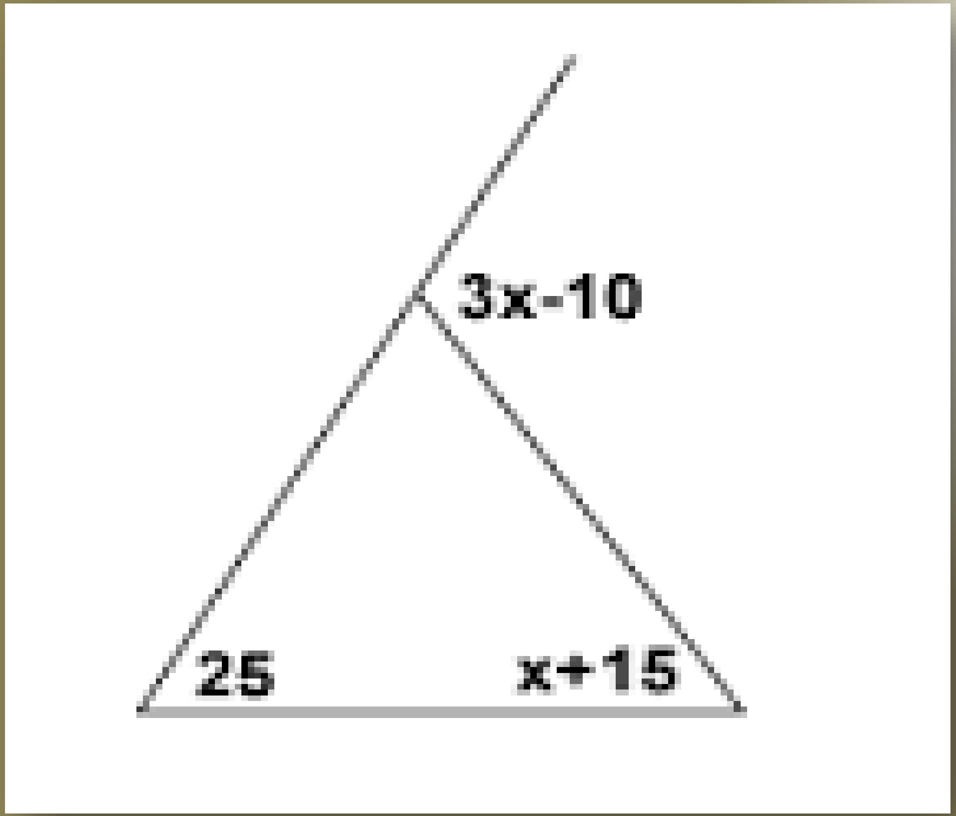
$2x + 5x = 105^\circ$ (ext.∠ *of* Δ*sum of* int. opp. ∠'s)

$7x = 105^\circ$

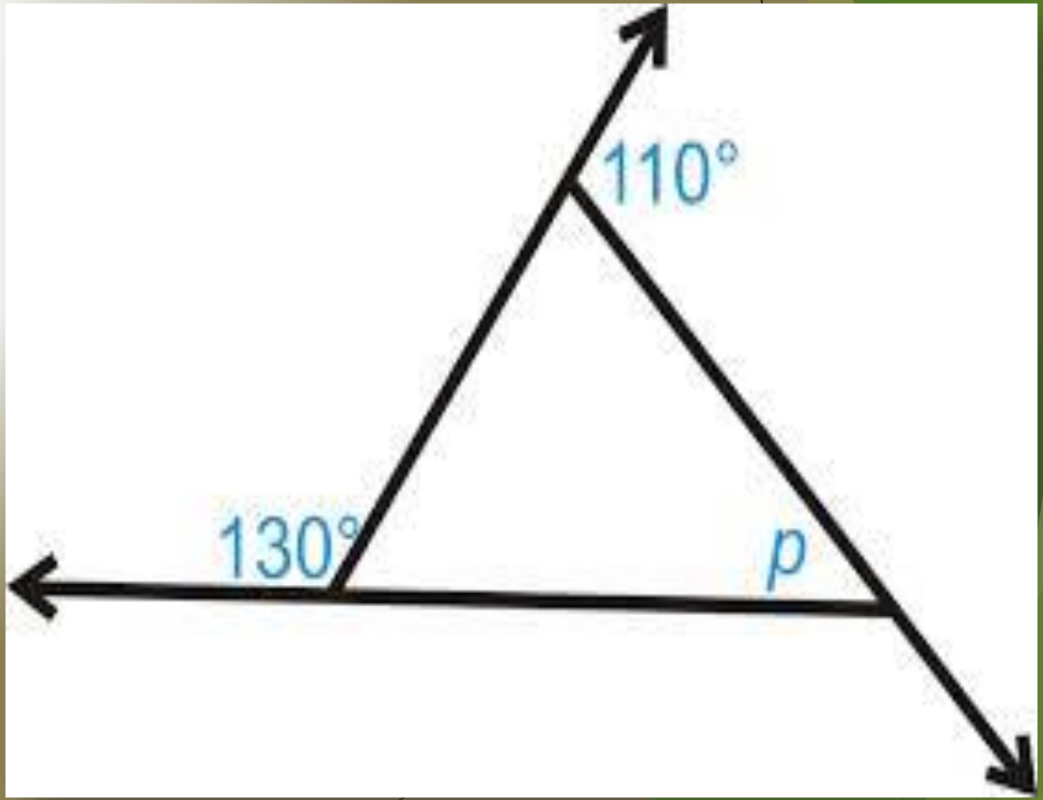
$x = 15^\circ$

EXERCISE!

1. Solve for x



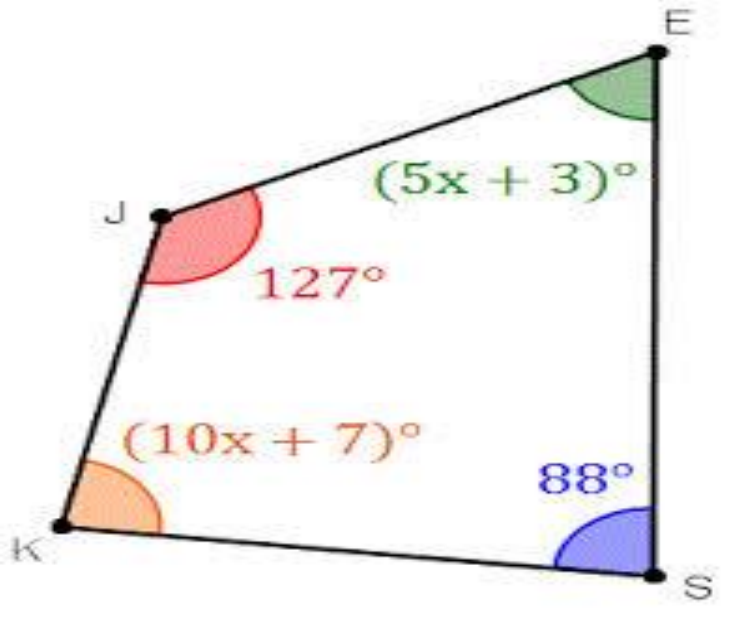
2. Solve for p



Interior Angles of a polygon

The sum of the interior angles of a polygon $=(n-2)\times 180^{\circ}$ sum of the (where $n =$ no. of sides of a polygon)

E.g.

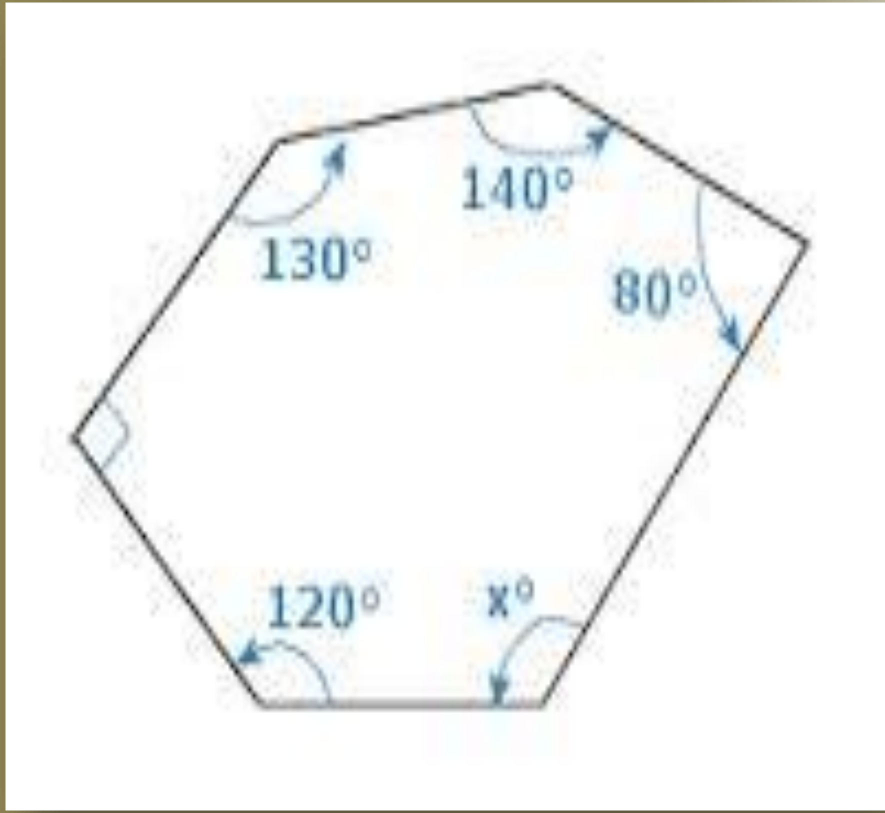


$$127^{\circ}+(5x+3)^{\circ}+88^{\circ}+(10x+7)^{\circ}$$
$$=(4-2)\times 180^{\circ}$$
$$225^{\circ}+15x=2\times 180^{\circ}$$
$$225^{\circ}+15x=360^{\circ}$$
$$15x=135^{\circ}$$
$$x=9^{\circ}$$

Calculating the interior & exterior angles of polygons

EXERCISE!

1. Solve for x



2. Solve for x