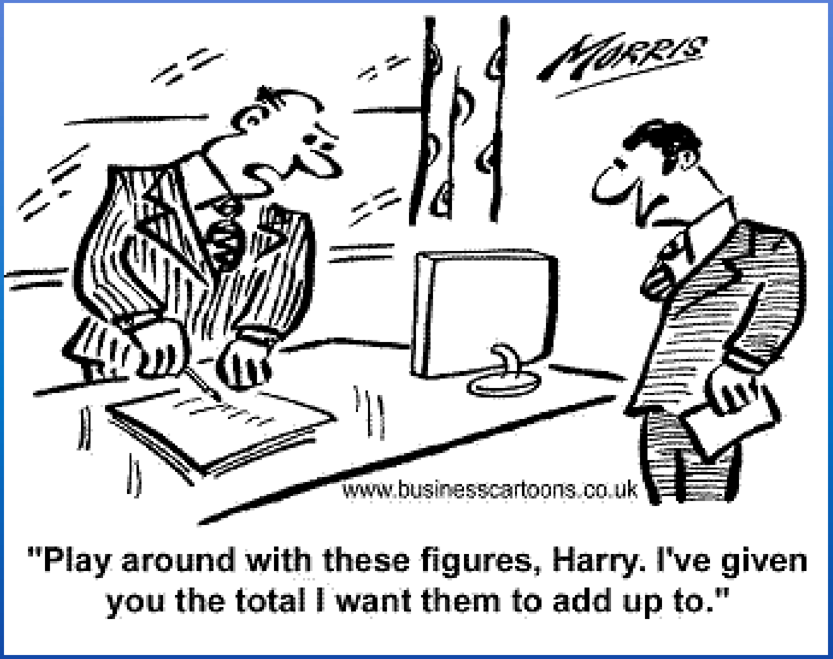
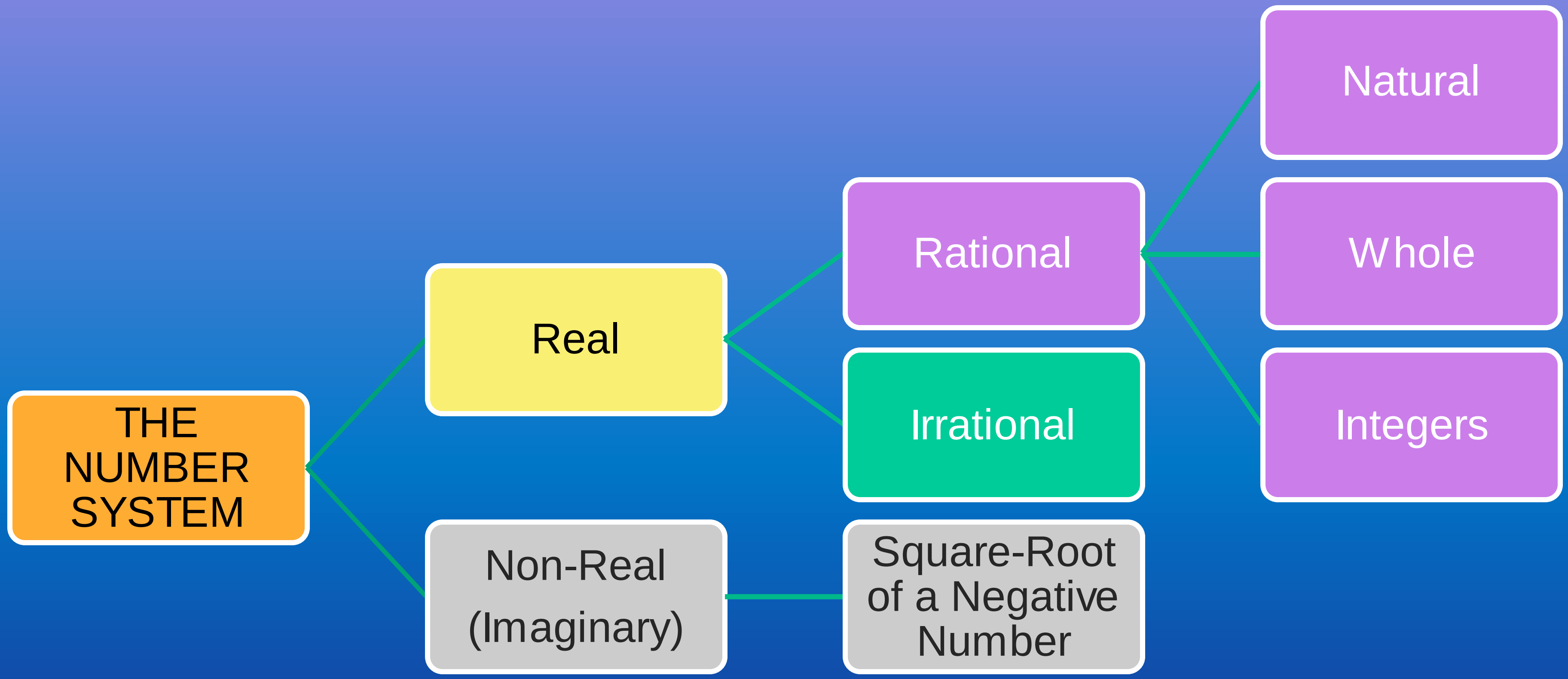
NUMBERS, OPERATIONS AND RELATIONSHIPS

- ❑ Whole numbers
- ❑ Exponents
- ❑ Integers
- ❑ Common Fractions
- ❑ Decimal Fractions



WHOLE NUMBERS

RECAP! The Number System



❑ REAL NUMBER SYSTEM

Natural Numbers (N)

Whole (i.e. no decimals), positive numbers that start at one.
(Also known as counting numbers)

Examples:

1; 2; 3; 4; 5; …. ; 47; …; 99 …

And what about 0?

✗, 0 is *not* a natural number!

Whole Numbers (N_0)

Whole (i.e. no decimals), positive numbers that start at zero.

Examples:

1; 2; 3; 4; 5; …. ; 47; …; 99 …

And what about 0?

✓ Yes, 0 is a whole number!

Integers ($\mathbb{Z}$)

Whole (i.e. no decimals), positive and negative numbers; including zero.

Examples:

…; -4; -3; -2; …; 2; 3; 4; …

And what about 0?

✓ Yes, 0 is an integer!

Rational Numbers ($\mathbb{Q}$)

Numbers that can be written as $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$.

- Includes terminating decimals (e.g. 0,4) and recurring decimals (e.g. $0,\dot{3}$; $0,\dot{7}\dot{5}$ etc.)

Examples:

5	$\frac{1}{2}$	0.3
$\sqrt{16}$	$-1\frac{6}{9}$	$\sqrt[3]{27}$

And what about 0? ✓ Yes, 0 is rational!

Irrational Numbers (Q')

All numbers that cannot be written as $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$.

i.e. numbers that recur *without* a pattern

Examples:

0,34791… -2,11816393…

$-\sqrt{6}$ $\sqrt[3]{17}$

And what about 0?

Summary of the Number System

✗, 0 is *not* an irrational number!

Exercise

Complete the following table by ticking the correct boxes:

Number	N_o	N	Z	Q	Q'
4,5					
$\frac{3}{2}$					
-591					
2,76					
$-3,\dot{6}$					
$\sqrt{121}$					
0					

Exercise

Continued …

Number	N_o	N	Z	Q	Q'
$\sqrt{6\frac{1}{4}}$					
$\frac{-13}{21}$					
-0,343434..					
$\sqrt[3]{-8}$					
$\sqrt[3]{729}$					
π					

❑ MULTIPLES

Natural numbers that are multiplied by another natural number
i.e. times – tables of the number!

Example: Multiples of 4…

4; 16; 64; 256; 1024 …4096

x 4 x 4 x 4 x 4

What is the next number?

Lowest Common Multiple (LCM)

Write out the multiples of each number and spot the lowest multiple that is common
- used when adding and subtracting fractions

LCM's

Example: Find the LCM of 4 and 7.

Multiples of 4: 4; 8; 12; 16; 20 24; 28; 32; 36; 40

Multiples of 5: 5; 10; 15; 20 25; 30; 35; 40 …

Common multiples …

LCM = 20

❑ FACTORS

Natural numbers that can divide into another number, without any remainders

Example: Factors of 48…

1; 2; 3; 4; 6; 8; 12; 16; 24; 48

Notice the pairs?
1 & 48
2 & 24 etc

Highest Common Factor (HCF)

Write out the factors of each number and spot the highest factor that is common
- used when simplifying fractions

HCF's (aka Greatest Common Factors – GCF's)

Example: Find the HCF of 30 and 40.

Factors of 30: 1; 2 3; 4 6; 10 15; 30

Factors of 40: 1; 2 4 5; 8; 10 20; 40

Common factors

HCF = 10

Prime Factors

Numbers which only have 2 factors
i.e. 1 and itself

Prime vs Composite Numbers using Divisibility Rules

Examples:

2; 3; 5; 7; 11; 13; 17; 19; 23; …

What about 1?

1 is not prime, as it only has 1 factor!

Prime Factorization

We use the ladder method to write a number in terms of its prime numbers

Eg: Write 80 as a product of its prime factors

$$\therefore 300 = 2^2 \times 3 \times 5^2$$

2	300
2	150
3	75
5	25
5	5
	1

Keep dividing by the prime number until you are left with a remainder – then use the next prime number

Using prime factorization to find the LCM

1. Write each number as a product of its prime factors
2. Multiply the *highest power of each prime factor of each number* together, to find the LCM

Example: Find the LCM of 6 and 8 by means of prime factorization.

Prime factors of 6:

$\therefore \mathbf{6 = 2 \times 3}$

$\boxed{3}$

2	6
3	3
	1

2	8
2	4
2	2
	1

First prime number … highest power

Prime factors of 8:

$\therefore \mathbf{8 = 2^3}$

2^3

LCM $= 2^3 \times 3 = 24$

Second prime number … highest power

Using prime factorization to find the HCF

1. Write each number as a product of its prime factors
2. Multiply the *highest power of each prime factor that is common* to find the HCF

Example: Find the HCF of 30 and 40 by means of prime factorization.

Prime factors of 30:

$\therefore \mathbf{30 = 2 \times 3 \times 5}$

2 5

2^1 5

2	30
3	15
5	5
	1

2	40
2	20
2	10
5	5
	1

Prime factors of 40:

$\therefore \mathbf{40 = 2^3 \times 5}$

2 5

2^1 5

Common factors

Highest power of each common factor

$HCF = 2^1 \times 5 = 10$

Exercise

1. Find the factors of:
   1.1. 49
   1.2 64
   1.3 108

2. Find the multiples of:
   2.1. 7
   2.2 12
   2.3 24

Exercise

3. Find the HCF of:
   3.1. 12 and 24
   3.2 32 and 56
   3.3 45 and 90

4. Find the LCM of:
   4.1. 4 and 14
   4.2 18 and 22
   4.3 35 and 48

Exercise

5. Write the following numbers as a product of its prime factors:
   5.1. 66
   5.2 75
   5.3 210

6. Find the LCM of 16 and 26 by means of prime factorization.

7. Find the HCF of 84 and 152 by means of prime factorization.

❑ RATIOS

Used to compare quantities of the same type (e.g. length, age, sweet, money etc)

- Can be written in 3 different forms …
  i) 1 : 2
  ii) $\frac{1}{2}$
  iii) 1 to 2
- Order of the numbers is very important
- Must be simplified as far as possible

Writing Ratios

Examples:

1. Write 42 : 54 in its simplest form.

   42 : 54

   7 : 9

   Divide both sides by the HCF i.e. 6

2. Andrew is 8 years old, while his father is 40 years old. Express their age in the simplest ratio.

   8 : 40

   1 : 5

   Divide both sides by the HCF i.e. 8

3. Manny and Pip divide a box of Smarties in the ratio of 2 : 3. If there are 25 Smarties in a box, how many Smarties does Pip get?

Total no. of parts = 2 + 3 = 5

Remember Order!

Manny : Pip

2 : 3

Pip = $\frac{3}{5} \times 25 = 15\ Smarties$

4. Jim and Brian share their profits in the ratio of 4 : 5. If Brian gets R15 000, then how much does Jim get in profit?

Remember Order!

Jim : Brian

4 : 5

? : R15 000

R15 000 represents 5 parts

1 Part = $R15\ 000 \div 5 = R3\ 000$

Jim = 4 parts $\times\ R3\ 000 = R12\ 000$

Working with Ratios

Exercise

1. Simplify the following ratios:
   1.1. 24 : 60
   1.2 100 : 44
   1.3 240 : 300

2. A recipe calls for 450g flour, 200g sugar and 120g cocoa. Write ratio of flour : cocoa : sugar in its simplest form.

Exercise

3. A balloon company manufactures red and yellow balloons in the ratio of 5 : 3. If the company manufactures 12 000 balloons in total, then how many red balloons were made?

4. A detergent bottle says that the cleaning liquid to water should be mixed in the ratio of 2 : 3. If Sally used 120ml of cleaning liquid, then how much water must she add?

❑ RATES

Used to compare two different quantities

e.g. distance & time	… km / hr	
volume & time	… kl / min	
money & mass	… R / g	etc

- Can be written in ratio form
- Order of the numbers is very important
- Must write down the units

Examples:

Speed, Distance & Time

1. Pam travels at 70 km / h. If she travels for 3 hours, how far did she travel?

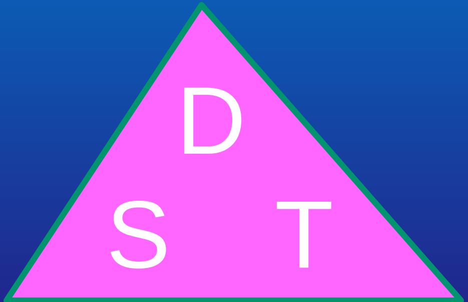
   70 km : 1 hr

   $3\ hrs \times 70\ km = 210\ km$

Rate: 70 km per 1 hr

2. Determine how long it took Sandy to reach the beach, if she travelled at 60 km/h and the beach is 20 km away?

$$\begin{aligned} \text{Time} &= \text{Distance} \div Speed \\ &= 20\ km \div 60\ km/h \\ &= 0.33\ hr \end{aligned}$$



3. The rate of water flowing out of a hose pipe is 0,8 litre per minute. How many liters of water would be used if James waters the garden for 1,5 hours?

No. of litres = $0{,}8\ l \times 90\ mins = 72\ l$

4. Which is cheaper? A 500ml colddrink for R6,50 or a 350ml colddrink for R5,80?

* 500ml → $R6{,}50 \div 500ml = R0{,}013/ml$
* 350ml → $R5{,}80 \div 350ml = R0{,}016/ml$

The 500ml colddrink is cheaper.

Rate per hour

Exercise

1. Which is a better buy?
   * 5kg bag of flour at R23,99
   * 2kg bag of flour at R14,80
   * 500g bag of flour at R6,20

2. Determine the speed at which a truck was travelling if t covered a distance of 2 400 km in 25 hours?

Exercise

3. Liam calculates that it costs him R1,80 per kilometer in fuel. Determine his total cost of travelling to and from work each day if he stays 23km from work.

4. How long would it take a tortoise to cover a a distance of 1km, if the tortoise moves at a speed of 2 meters per minute?

❑ DIRECT PROPORTION

- Quantities are said to be in direct proportion when both quantities increase / decrease in the same ratio

  i.e. as x gets bigger, so y gets *bigger too* by the same ratio (same applies for smaller)

  $$\therefore \frac{y}{x} = constant\ ratio$$

- The graph of a direct proportion relationship is a straight line

Examples:

1. Find the missing values in the table:

x	2	3	9	B
y	5	7,5	A	120

$$\text{A} = 9 \times 2{,}5 = 22{,}5$$

$$\text{B} = 120 \div 2{,}5 = 48$$

$$Constant\ ratio = \frac{y}{x} = \frac{5}{2} = \frac{7{,}5}{3} = 2{,}5$$

2. A school works according to the fact that each learner needs 250ml of juice in a day. How many litres of juice are needed at the athletics day, if the school has 1 241 learners?

Typical Direct Proportion Problems

As the number of learners increases, so does the volume of juice needed by the same ratio $\therefore Direct\ Proportion$!

$Constant\ ratio$
$= 250\ ml$

No. of liters of juice

$= 250ml\ \times 1\,241$

$= 310\,250ml$

$= 310{,}25\ liters$

❑ INDIRECT / INVERSE PROPORTION

- Quantities are said to be in indirect / inverse proportion when their products remain constant

  i.e. as x gets bigger, so y gets *smaller* by the same ratio (and vice versa)

  $$\therefore x \times y = constant\ product$$

- The graph of an indirect / inverse proportion relationship is a hyperbola

Examples:

1. Find the missing values in the table:

x	2	3	8	B
y	60	40	A	5

$$\text{A} = 120 \div 8 = 15$$

$$\text{B} = 120 \div 5 = 24$$

$Constant$
$product$
$= x \times y$
$= 2 \times 60$
$= 3 \times 40$
$= 120$

2. It takes twelve men four days to paint a school building. How long would it take to paint the school if only eight men were available for the job?

As the number of men decreases, so the time needed to complete the painting must increase $\therefore$ *Indirect Proportion*!

$Constant\ product$
$= 12 \times 4 = 48$

No. of days
$= 48 \div 8$
$= 6\ days$

Direct vs Indirect Proportion:

Working with Direct & Indirect Proportion Examples

Proportionality Constants in Direct & Indirect Proportion

Summary of Direct & Indirect Proportion

Exercise

1. A recipe that makes 12 cupcakes calls for 500g of flour. How many grams of flour are needed to make 10 cupcakes?

2. If it takes two people four hours to clean a house, then how long will it take four people to clean the same house?

Exercise

3. Find the missing values in the table below:

x	2	3	A	6
y	18	12	8	B

4. If it takes ten people four hours to make 220 Christmas decorations, then how many Christmas decorations can be made in four hours, if there are 15 people?

Exercise

5. Find the missing values in the table below:

x	30	20	10	B
y	6	4	A	1,5

6. Warren travels for 2,5 hours at a speed of 88 km/hr. How long will it take him to travel the same distance, if he travels at 120km/hr?

❑ FINANCE

- VAT (Value Added Tax) is a government tax of 14%, that is added onto the price of most goods and services

VAT Calculator

Example:

Determine the VAT-incl. price of a solar panel if it is quoted as R8 700 excl. VAT.

$$VAT = \frac{14}{100} \times R8\,700 = R1\,218$$

$$Price\ incl.VAT = R8\,700 + R1\,218 = R9\,918$$

- **<u>Discounts</u>** (usually given as a %):
  To find the sale price, determine the Rand value of the discount and subtract it from the original price of the item

Discounts & Mark-Ups

Example:

An engagement ring of R18 999 is advertised as being 20% less on sale. How much can the groom expect to pay for the engagement ring?

$$\text{Discount} = \frac{20}{100} \times R18\,999 = R3\,799{,}80$$

$$\text{Sale price} = R18\,999 - R3\,799{,}80 = R15\,199{,}20$$

- **Profit or Loss** = Selling Price – Cost Price
  = Income – Expenses

  * If the number is positive $\rightarrow Profit$
  * If the number is negative $\rightarrow Loss$

Example:
Calculate the profit or loss that a T-shirt manufacturer makes if they buy T-shirts for R22 100 and receive an income of R26 900.

$\text{Profit} = \text{R}26\,900 - \text{R}22\,100 = \text{R}4\,800$

- **Simple Interest:**
    * Interest is calculated on the original amount (called the principle)
    * Interest remains constant for the entire life of the loan or investment

$$A = P(1 + ni)$$

A = total amount
P = principle amount
n = no. of years
i = interest rate

Understanding Simple Interest

Example:

Ann invests R45 000 of her inheritance in a bank account that accrues simple interest of 12,6% p. a. Determine the value of Ann's investment after seven years.

$$A = P(1 + ni)$$
$$= 45\,000\left(1 + 7 \times \frac{12,6}{100}\right)$$
$$= R84\,690$$

Note! Interest rate is given as a %!

- **Compound Interest:**

* Interest is calculated on the original amount (principle), plus on the interest that has accumulated
* Interest increases exponentially

$$A = P(1 + i)^n$$

A = total amount

P = principle amount

n = no. of years

i = interest rate

Understanding Compound Interest

Example:

Nellis takes out a loan of R12 700, in order to buy himself a quad bike. The bank charges him interest of 14,5% compounded annually. Determine how much Nellis will pay back in total, if he pays off his loan in 24 months.

$$A = P(1+i)^n$$
$$= 12\ 000\ (1 + \frac{14,5}{100})^2$$
$$= R15\ 732,30$$

Note! The formula works in years … 24 months = 2 yrs

- **Hire Purchase:**
  * The buyer puts down a deposit on a product and pays off the outstanding balance (plus interest) in monthly instalments over a period of time
  * Usually used for furniture & appliances
  * Simple interest is calculated on the outstanding balance

Remember:

$$A = P(1 + ni)$$

Example:

Joshua Door advertises a leather lounge-suite for R26 999 cash; or on HP with a 10% deposit, interest at 11,5% p.a. over a period of 36 months. Calculate the monthly instalments and how much you would save if you bought it cash.

$$Deposit = \frac{10}{100} \times R26\,999 = R2\,699{,}90$$

$$Balance = R26\,999 - R2\,699{,}90 = R24\,299{,}10$$

$$A = P(1 + ni)$$

$$= 24\,299{,}10\left(1 + 3 \times \frac{11{,}5}{100}\right)$$

$$= R32\,682{,}29$$

Interest is only calculated on the outstanding balance

$$Monthly\ instalments = \frac{R32682{,}29}{36}$$
$$= R907{,}84$$

$$Total\ paid\ \ on\ HP$$
$$= Deposit + Monthly\ instalments$$
$$= \ R2\ 699{,}90\ + \ (R907{,}84 \times 36)$$
$$= \ R2\ 699{,}90\ + R32\ 682{,}24$$
$$= \ R35\ 382{,}14$$

Saving = Total paid on HP – Cash Price
= R35 382,14 – R26 999
= R8 383,14 i

Hire Purchase Calculations

Exercise

1. A car exhaust is quoted as R3 780 excl. VAT. Determine the cost of the exhaust incl. VAT.

2. A tent is on special for 25% less. If the tent costs R1 299, determine the sale price.

3. A florist spends a total of R14 000 per month on expenses. If the florist receives an income of R16 500, determine the profit or loss of the business.

Exercise

4. Determine the total value of an investment, if the customer invests R60 000 for four years and receives a simple interest rate of 8,8% p.a.
5. Calculate the interest that a customer will pay if he takes out a loan of R44 000 over a period of six years, at a compounded interest rate of 12,4% p.a.
6. Determine the monthly instalment if a customer buys a TV on HP, with a 12,5% deposit on the cash price of R10 999; and pays it off over 24 months, at an interest rate of 7,6% p.a.

EXPONENTS

❑ EXPONENTIAL FORM

Exponent Terminology

Instead of writing $2 \times 2 \times 2 \times 2 \times 2$,

Factor form

we can write 2^5

Exponential form

Terminology:

Power

Coefficient

$-6x^4$

Exponent

Base

❑ ZERO EXPONENT

Anything to the power of 0 = 1

$$a^0 = 1$$

(where a $\neq$ 0)

Examples:

- $4^0 = 1$
- $4x^0 = 4 \times 1 = 4$
- $(4x)0 = 1$

Only x is being raised to the power of 0

Everything is being raised to the power of 0

❑ MULTIPLYING EXPONENTS

When you are multiplying AND the bases are the same … add the exponents

$$a^m \times a^n = a^{m+n}$$

Multiplying Exponents

Examples:

- $x^2 . x^3 = x^{2+3} = x^5$
- $4x^5y^2 \times 3x^3y^4 = 12x^8y^6$
- $2^4 \times 2^6 = 2^{10}$

$\neq x^6$

Add the exponents of the SAME bases

❑ DIVIDING EXPONENTS

When you are dividing AND the bases are the same … subtract the exponents

$$\boldsymbol{a^{m} \times a^{n} = a^{m-n}}$$

Dividing Exponents

Examples:

- $x^8 \div x^2 = x^{8-2} = x^6$

$\neq x^4$

- $12x^6y^9 \div 3x^3y^3 = 4x^3y^6$
- $\frac{3^{10}}{3^5} = 3^5$

Subtract the exponents of the SAME bases

❑ RAISING A POWER TO A POWER

When you raise a power to a power … multiply the exponents

$$(\boldsymbol{a^m})^{\boldsymbol{n}} = \boldsymbol{a^{mn}}$$

Raising an Exponent to a Power

Examples:

- $(x^3)^2 = x^{3.2} = x^6$ $\neq x^5$
- $(4^5)^6 = 4^{30}$

❑ RAISING THE PRODUCT / QUOTIENT TO A POWER

When you raise the product / quotient to a power … multiply the exponents of each factor

$$(ab)^m = a^m b^m \quad or \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Examples:

- $(x^2y^3)^4 = x^{2.4}y^{3.4} = x^8y^{12}$
- $(-3x^5)^3 = (-3)^3x^{15} = -27x^{15}$
- $(-2x^6)^4 = (-2)^4x^{24} = 16x^{24}$

Watch out for negative signs!

Exercise

Simplify:

1. $3x^0 + (3x^4)^0$
2. $-6a^8b^5 \div 2a^4b^2$
3. $\dfrac{81x^6y^9z^4}{-3x^2y^5z^6}$
4. $\left(\dfrac{-2y^3}{x^7}\right)^3$
5. $x^4y^8z^0 \times x^3y^5z^8$
6. $(\dfrac{-3a^4}{b^6})^4$

❑ NEGATIVE EXPONENTS

If the exponent is NEGATIVE in the NUMERATOR … move the factor to the denominator and make the exponent POSITIVE

Negative Exponents

$$\boldsymbol{a^{-n} = \frac{1}{a^n}}$$

5 & b have positive exponents, so they stay at the top!

Examples:

- $5a^{-2}b^3c^{-4} = \frac{5b^3}{a^2c^4}$
- $(2x^8y^3)^{-2} = \frac{1}{(2x^8y^3)^2} = \frac{1}{4x^{16}y^6}$

The whole term has a negative power

❑ NEGATIVE EXPONENTS

If the exponent is NEGATIVE in the DENOMINATOR … move the factor to the numerator and make the exponent POSITIVE

$$\frac{\mathbf{1}}{\boldsymbol{a^{-n}}} = \boldsymbol{a^{n}}$$

4 & y^6 have positive exponents, so they don't move!

Examples:

- $\frac{4}{x^{-5}y^{6}z^{-7}} = \frac{4x^{5}z^{7}}{y^{6}}$
- $\frac{1}{(-2x^{9})^{-3}} = (-2x^{9})^{3} = -8x^{27}$

The whole term has a negative power

Exercise

Simplify:

1. $6x^{-3}y^{-4}$
2. $-4ab^{-3}$
3. $\frac{1}{x^{2}y^{-5}z^{-6}}$
4. $\frac{-6a^{3}}{b^{-9}}$
5. $x^{-4}y^{7}z^{-5} \times x^{6}y^{-2}z^{8}$
6. $(\frac{4a^{-7}}{b^{2}})^{-2}$
7. $\frac{3x^{7}}{(9x^{8})^{-2}}$

❑ EXPONENTIAL EQUATIONS

- **If the bases are the same, equate the exponents**
- **If the bases are not the same, prime factorize them and then equate the exponents**

Examples:

- $2^x = 2^9$

  $\therefore x = 9$

Bases the same … exponents equal!

Complex Exponential Equations

Examples:

- $5^{3x} = 5^{12}$

  $3x = 12$

  $\therefore x = 4$

- $3^{x} = 27$

  $3^{x} = 3^{3}$

  $\therefore x = 3$

- $4.2^{2x} = 32$

  $2^{2x} = 8$

  $2^{2x} = 2^{3}$

  $\therefore 2x = 3$

  $\therefore x = 1{,}5$

Make the bases the same by means of prime factorization

First divide by 4, so that 1 factor = 1 factor – thereafter make the bases the same and solve

Exercise

Solve for x:

1. $2^{6x} = 2^{12}$
2. $3^{4x+2} = 3^{10x}$
3. $7^{x} = 49$
4. $5^{2x} = 25$
5. $3.2^{x} = 24$
6. $9.3^{3x} = 81$
7. $8^{x} = 32^{x}$
8. $-4^{x} = 16$

❑ SCIENTIFIC NOTATION

* Used to write really big or really small numbers in a compact way
* In scientific notation, move the decimal comma until *after* the first non-zero digit and then multiply by ten to the power of ($\times 10^{\cdots}$) how ever many places you moved the decimal comma
* *Really big* numbers will have a positive exponent … i.e. $\times 10^{positive\ no}$
* *Really small* numbers will have a negative exponent … i.e. $\times 10^{negative\ no}$

Examples:

Scientific Notation

- $246 = 2{,}46 \times 10^2$

> - Decimal comma after first non-zero digit (4)
> - Big no … so $\times\ 10^{pos.\,no}$
> - Decimal moved 2 places

- $5\ 890\ 221 = 5{,}890221 \times 10^6$
- $4\ 020\ 000\ 000\ 000 = 4{,}02 \times 10^{12}$

Examples:

- $0{,}000357 = 3{,}57 \times 10^{-4}$

> - Decimal comma after first non-zero digit (3)
> - Small no … so $\times\, 10^{neg.\,no}$
> - Decimal moved 4 places

- $0{,}000\,000\,042 = 4{,}2 \times 10^{-8}$
- $0{,}005\,055\,000\,000 = 5{,}055 \times 10^{-3}$

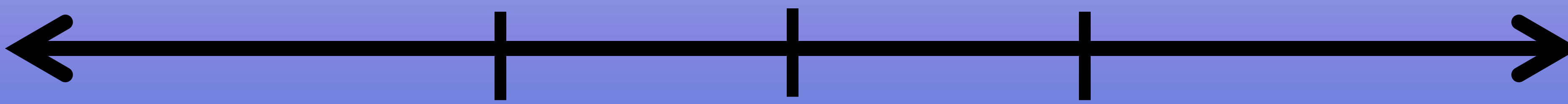
Exercise

Write the following numbers in scientific notation:

1. 66 789
2. 0,432
3. 1 083 201
4. 0,000 096
5. 12 121 000 000 000
6. 0,000 000 000 080

INTEGERS

❑ THE NUMBER LINE

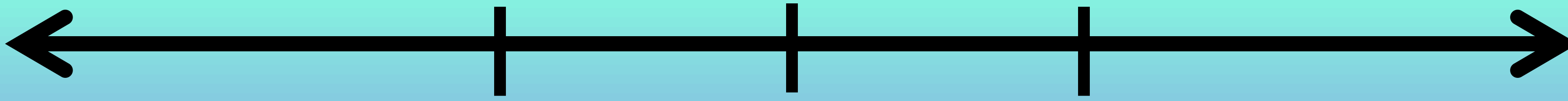


$$-\infty; \ldots -100; \ldots \quad 0; \ldots; 100; \ \ldots; \ +\infty$$

Negative no’s **0** **Positive no’s**

<u>The Number Line in Life</u>

- 100 is bigger than 50 … 100 > 50
- -100 is smaller than -50 … -100 < -500
- 0 is smaller than 2 … 0 < 2
- 0 is bigger than -2 … 0 > -2



$$-\infty;\ \ldots -\mathbf{100};\ \ldots\ \ \mathbf{0};\ \ldots;\ \mathbf{100};\ \ \ldots;\ +\infty$$

- Increase 100 by 50.

  100 + 50 = 150

- Increase -100 by 50.

  -100 + 50 = -50

Move 50 units to the RIGHT

- Decrease 100 by 50.

  100 – 50 = 50

- Decrease -100 by 50.

  -100 – 50 = -150

Move 50 units to the LEFT

Exercise

1. Fill in > or <:

   1.1 23 … -2

   1.2 -41 … 67

   1.3 83 … 32

   1.4 -49 … -88

   1.5 -101 … -100

   1.6 0 … 7

   1.7 -7 … 0

Exercise

2. Increase 30 by 40.
3. Decrease 30 by 40.
4. Increase 66 by 22.
5. Decrease 66 by 22.
6. Increase -10 by 25.
7. Decrease -10 by 25.

❑ SIGNS OF NUMBERS

A number can either be:

- Positive e.g. 5 or +5
- Negative e.g. -5

Multiplying (or dividing) signs:

- $+ \times + = +$
- $- \times - = +$

- $+ \times - = +$
- $- \times + = +$

Same sign … answer POSITIVE

Different sign … answer NEGATIVE

❑ MULTIPLYING INTEGERS

- 2.5
  =10

$$+2 \times +5 = +10$$

- (-2)(-5)
  = 10

$$-2 \times -5 = +10$$

- 2×-5
  $= -10$

$$+2 \times -5 = -10$$

- -2(5)
  = -10

$$-2 \times +5 = -10$$

Note the different ways of writing multiplication … Dot, × and Brackets!

DIVIDING INTEGERS

- $\frac{8}{4} = 2$

$+8 \div +4 = +2$

- $\frac{-8}{-4} = 2$

$-8 \div -4 = +2$

- $-8 \div 4 = -2$

$-8 \div +4 = -2$

- $8 \div -4 = -2$

$+8 \div -4 = -2$

Divisibility Rules

❑ ADDING & SUBTRACTING INTEGERS

- (3) + (+4)
  = 3 + 4
  = 7

$$-1 \times +4 = -4$$

- (3) – (-4)
  = 3 + 4
  = 7

$$-1 \times -4 = -4$$

- (3) + (-4)
  = 3 – 4
  = -1

$$-1 \times +4 = -4$$

- (3) – (+4)
  = 3 – 4
  = -1

$$-1 \times +4 = -4$$

❑ ADDITIVE INVERSE

The additive inverse is the number that you must add to another, to give a total of zero

Examples:

- -8 … +8 = 0
- 8 … -8 = 0

+8 is the additive inverse of -8

-8 is the additive inverse of 8

❑ COMMUNICATIVE PROPERTY

The order of adding integers does not matter!

$$x + y = y + x$$

Examples:

- 5 + 6 = 6 + 5 … = 11
- -5 + 6 = 6 + (-5) … = 6 – 5 = 1

Note! Use brackets to keep the negative sign

❑ ASSOCIATIVE PROPERTY

Grouping integers to make addition or subtraction easier, does not change the answer!

$$\boldsymbol{a - b + c - d = (a + c) - (b + d)}$$

Example:

- $7 - 8 + 3 - 2 \quad = (7 + 3) - (8 + 2)$

$$= 10 - (10)$$

$$= 10 - 10$$

$$= 0$$

Addition Properties

Exercise

Calculate:

1. $5 + (+9)$
2. $5 - (+9)$
3. $5 + (-9)$
4. $5 - (-9)$
5. $\frac{8}{-2}$
6. -3×-5
7. $-6 \div -2$
8. $7(-2)$
9. $(-4)(-6)$
10. $\frac{4}{-16}$

Exercise

11. What is the additive inverse of:

    11.1 16

    11.2 -9

12. Calculate:

    12.1 $-3 + 8 - 7 + 1$

    12.2 $12 - 9 - 11 + 21$

❑ SQUARES & CUBES

Squaring

- Multiplying a number by itself (twice)
- Examples:

i) $5^2 \quad = 5 \times 5 \quad = 25$

ii) $(-7)^2 \quad = -7 \times -7 \quad = 49$

iii) $-4^2 \quad = -(4 \times 4) \quad = -16$

Cubing

- Multiplying a number by itself three times
- Examples:

i) $2^3 \quad = 2 \times 2 \times 2 \quad = 8$

ii) $(-3)^3 \quad = -3 \times -3 \times -3 \quad = -27$

iii) $-5^3 \quad = -(5 \times 5 \times 5) \quad = -125$

❑ SQUARE ROOTS

Square-Rooting

- What number multiplied by itself gives us the question?
- Examples:

i) $\sqrt{16} = 4$

ii) $\sqrt{81} = 9$

iii) $\sqrt{-25} = undefined$

e

Understanding Square Roots

$+5 \times +5 = 25$ and $-5 \times -5 = 25$

Can never get -25!

❑ CUBE ROOTS

Cube-Rooting

- What number multiplied by itself three times gives us the question?
- Examples:

i) $\sqrt[3]{125} \quad = 5$

ii) $\sqrt[3]{27} \quad = 3$

iii) $\sqrt[3]{-8} \quad = -2$

Understanding Cube Roots

$$-2 \times -2 \times -2 = -8$$

Can get -8!

Exercise

Calculate:

1. $\sqrt{36}$
2. $\sqrt[3]{8}$
3. 4^3
4. 8^2
5. $(-9)^2$
6. $\sqrt{-64}$
7. -5^2
8. -3^3
9. $(-2)^3$
10. $\sqrt[3]{-27}$

COMMON FRACTIONS

❑ COMMON FRACTIONS

* A number that is written as $\frac{x}{y}$, where x (numerator) and y (denominator) are whole numbers and $y \neq 0$.
* Common fractions are divided into:

i) Proper fractions

- numerator < denominator e.g. $\frac{2}{3}$

ii) Improper fractions

- denominator > numerator e.g. $\frac{3}{2}$

iii) Mixed fractions

- whole number + fraction e.g. $1\frac{2}{3}$

❑ MULTIPLYING FRACTIONS

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

[Multiplying Fractions]

Examples:

1. $\frac{2}{3} \times \frac{4}{5} = \frac{2\times4}{3\times5} = \frac{8}{15}$

2. $\frac{2}{7} \times \frac{5}{6} = \frac{2\times5}{7\times6} = \frac{10}{32}$

$= \frac{5}{16}$

Simplify the fraction by dividing by the highest common factor … = 2

$\frac{5}{16} = \frac{10}{32}$

equivalent fractions

3. $\frac{x^2}{8y} \times \frac{y^4}{z^3} \times \frac{6z^5}{x^3} = \frac{6x^2y^4z^5}{8yz^3x^3}$

e.g. More x's at bottom by 1; More y's at top by 3 etc.

Method 1: "Where are there more and by how many"

$$= \frac{3y^3z^2}{4x}$$

$$= \frac{6x^{2-3}y^{4-1}z^{5-3}}{8}$$

Method 2: Use exponent laws

$$= \frac{6x^{-1}y^3z^2}{8}$$

Subtract exponents when dividing

$$= \frac{3y^3z^2}{4x}$$

Only change negative exponents to positive exponents when asked! But look – same answer!

❑ DIVIDING FRACTIONS

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c} = \frac{ad}{bc}$$

Reciprocal

Examples:

1. $\frac{2}{7} \div \frac{5}{6} = \frac{2}{7} \times \frac{6}{5} = \frac{2\times 6}{7\times 5} = \frac{12}{35}$

2. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{2\times 5}{3\times 4} = \frac{10}{12} = \frac{5}{6}$

Simplify: Divide by HCF = 2

$\frac{10}{12} = \frac{5}{6}$ equivalent fractions

Dividing Fractions

3. $\frac{x^2}{8y} \div \frac{y^4}{z^3} \div \frac{6z^5}{x^3} = \frac{x^2}{8y} \times \frac{z^3}{y4} \times \frac{x^3}{6z^5}$

$= \frac{x^2 \times z^3 \times x^3}{8y \times y^4 \times 6z^5}$

$= \frac{x^5 z^3}{48y^5 z^5}$

$= \frac{x^5}{48y^5 z^2}$

$= \frac{x^5 z^{-2}}{48y^5}$

$= \frac{x^5}{48y^5 z^2}$

Simplify by adding exponents of same bases

TWO reciprocals!

Method 1: "Where are there more and by how many"

Method 2: Use exponent laws

Exercise

Evaluate:

1. $\frac{12}{9} \times -\frac{3}{5}$

2. $5\frac{3}{4} \div \frac{3}{2}$

3. $\frac{6x^3}{7y^2} \times \frac{14y^4}{2x^7}$

4. $\frac{a^2b^3}{5c^4} \div \frac{a^4b^4}{15c^4}$

5. $\frac{25x^6}{4y} \times \frac{2y^3}{15x^8} \div \frac{3x}{6y^9}$

ADDING & SUBTRACTING FRACTIONS

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \quad \text{or} \quad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$$

Examples:

1. $\frac{2}{3}+\frac{4}{3}=\frac{2+4}{3}=\frac{6}{3}=\frac{2}{1}$

2. $\frac{2}{6}-\frac{5}{6}=\frac{2-5}{6}=\frac{-3}{6}=\frac{-1}{2}$

Note!

$$\frac{-1}{2}=\frac{1}{-2}=-\frac{1}{2}$$

❑ ADDING & SUBTRACTING FRACTIONS

$$\frac{a}{e} + \frac{b}{f} = \frac{af + be}{ef}$$

<u>Step 1:</u>

Find the Lowest Common Denominator (LCD)

<u>Step 2:</u>

Find the numerator by doing the following:

$$\frac{LCD}{denominator} \times numerator$$

3. $\dfrac{3}{4} + \dfrac{2}{5}$

$= \dfrac{5(3)+4(2)}{20}$

$= \dfrac{15+8}{20}$

$= \dfrac{23}{20}$

Step 1:

LCD = 20!

Step 2:

$\dfrac{LCD}{denominator} \times numerator$

i.e. $\dfrac{20}{4} \times 3$ and $\dfrac{20}{5} \times 2$

4. $\dfrac{3}{2xy^2} - \dfrac{4}{7x^3y}$

$$= \frac{7x^2(3) - 2y(4)}{14x^3y^2}$$

$$= \frac{21x^2 - 8y}{14}$$

Step 1:
LCD number = 14
LCD variables = x^3y^2
[One of each variable and the highest power of each]

Unlike terms, so we can't simplify further

Step 2:

$$\frac{LCD}{denominator} \times numerator$$

i.e. $\dfrac{14x^3y^2}{2xy^2} \times 3$ and $\dfrac{14x^3y^2}{7x^3y} \times 4$

Exercise

Evaluate:

1. $4\frac{2}{3}+\frac{1}{5}$
2. $\frac{1}{2}-\frac{2}{7}$
3. $\frac{8y}{x}-\frac{4x}{y}$
4. $\frac{6}{xy^3}+\frac{1}{x^2y}$
5. $\frac{1}{2y}+\frac{3}{10y}-\frac{5}{4y}$

❑ SQUARES, SQUARE ROOTS, CUBES & CUBE ROOTS IN FRACTIONS

* Simplify fraction as far as possible before squaring, cubing or rooting

* Apply operation to numerator *and* denominator

Examples:

1. $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

Remember your exponent laws!

2. $\left(-\frac{3}{2}\right)^3 = \frac{(-3)^3}{2^3} = -\frac{27}{8}$

Beware of the negative!

3. $\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

Changed mixed fraction into improper fraction first!

4. $\sqrt[3]{-\frac{1}{8}} = \frac{\sqrt[3]{-1}}{\sqrt[3]{8}} = -\frac{1}{2}$

5. $\left(\frac{2}{3} - \frac{5}{4}\right)^2 = \left(\frac{4(2) - 3(5)}{12}\right)^2$

$= \left(\frac{8 - 15}{12}\right)^2$

$= \left(-\frac{7}{12}\right)^2$

$= \frac{49}{144}$

First subtract the fractions before squaring them!

<u>Square Roots & Fractions</u>

Exercise

Evaluate:

1. $\left(\frac{1}{5}-2\right)^2$

2. $\left(3\frac{1}{2}+\frac{2}{3}\right)^3$

3. $\sqrt{6\frac{1}{4}}$

4. $\sqrt[3]{3\frac{3}{8}}$

DECIMAL FRACTIONS

CONVERTING A FRACTION TO A DECIMAL

Multiply both the numerator and denominator by a number, in order to get the denominator to a power of 10.

Note that $\frac{2}{2} = 1$, so we haven't changed the question!

Examples:

1. $\frac{1}{5} = \frac{1}{5} \times \frac{2}{2} = \frac{2}{10} = 0{,}2$
2. $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 0{,}75$
3. $\frac{7}{8} = \frac{7}{8} \times \frac{125}{125} = \frac{875}{1000} = 0{,}875$

Converting between Fractions and Decimals

Exercise

Convert the following to decimals:

1. $\frac{2}{5}$
2. $\frac{3}{8}$
3. $\frac{27}{25}$
4. 19%
5. 202%

❑ ADDING & SUBTRACTING DECIMALS

Write down the numbers in a column, making sure that the decimals are all lined up (so fill in zero's (0) if need be!)

Example:

0,34 + 0,5 + 0,123:

$$
\begin{array}{r}
0{,}340 \\
+\ 0{,}500 \\
\underline{+\ 0{,}123} \\
=\ 0{,}963
\end{array}
$$

Note how the zero's are added as place holders!

Adding & Subtracting Decimals

❑ ADDING & SUBTRACTING DECIMALS

Use the associative property of integers … i.e.

$$\boldsymbol{a - b + c - d = (a + c) - (b + d)}$$

Example:

9,72 – 3,1 + 1,8 – 2,505

= (9,72 + 1,8) – (3,1 + 2,505)

$$\begin{array}{lr} & 9{,}72 \\ + & 1{,}80 \\ \hline = & 11{,}52 \end{array} \qquad \begin{array}{lr} & 3{,}100 \\ + & 2{,}505 \\ \hline = & 5{,}605 \end{array} \qquad \begin{array}{lr} & 11{,}520 \\ - & 5{,}605 \\ \hline = & 5{,}915 \end{array}$$

Add each bracket separately and then subtract!

Exercise

Simplify without the use of a calculator:

1. $3{,}425 + 5{,}89 + 7{,}1$
2. $8{,}99 - 1{,}4 - 3{,}764$
3. $7{,}2 - 3{,}66 + 2{,}105$
4. $3{,}89 + 5{,}6 - 6{,}022$
5. $29 + 6{,}3 - 7{,}884 - 1{,}26$

❑ MULTIPLYING DECIMALS

* First convert the decimals to fractions
* Then multiply the numerators and denominators
* Lastly, convert back into a decimal

Multiplying & Dividing Decimal Fractions

Examples:

1. $0{,}5 \times 0{,}3 = \frac{5}{10} \times \frac{3}{10} = \frac{15}{100} = 0{,}15$
2. $0{,}09 \times 0{,}2 = \frac{9}{100} \times \frac{2}{10} = \frac{18}{1000} = 0{,}018$
3. $4{,}1 \times 2{,}07 = \frac{41}{10} \times \frac{207}{100} = \frac{8487}{1000} = 8{,}487$

❑ DIVIDING DECIMALS

* First convert the decimals to fractions
* Then divide by the method of "tip & times"
* Then multiply the numerators and denominators & simplify
* Lastly, convert back into a decimal

Examples:

1. $4{,}2 \div 1{,}2 =$

$$\frac{42}{10} \div \frac{12}{10}$$

$$= \frac{42}{10} \times \frac{10}{12}$$

$$= \frac{420}{120}$$

$$= \frac{7}{2}$$

$= 3{,}5$

Examples:

2\. $5{,}5 \div 1{,}1 =$

$$\frac{55}{10} \div \frac{11}{10}$$

$$= \frac{55}{10} \times \frac{10}{11}$$

$$= \frac{550}{110}$$

$$= \frac{5}{1}$$

$$= 5{,}0$$

3\. $26{,}88 \div 6{,}4$

$$= \frac{2688}{100} \div \frac{64}{10}$$

$$= \frac{2688}{100} \times \frac{10}{64}$$

$$= \frac{26880}{6400}$$

$$= \frac{21}{5}$$

$$= 4{,}2$$

Exercise

Simplify without a calculator:

1. $6{,}67 \times 2{,}3$
2. $13{,}8 \div 6$
3. $3{,}5 \times 8{,}07$
4. $15{,}87 \div 6{,}9$
5. $12{,}6 \times 4{,}02 \div 1{,}1$

❑ SQUARES, SQUARE ROOTS, CUBES & CUBE ROOTS IN DECIMALS

* First convert the decimal to a fraction

* Then square, cube or root the fraction

* Remember to apply the exponent laws to the numerator & denominator

* Lastly, convert back into a decimal

Examples:

1. $(0,7)^2 =$

$$\left(\frac{7}{10}\right)^2$$

$$= \frac{7^2}{10^2}$$

$$= \frac{49}{100} = 0,49$$

Examples:

2. $(0{,}2)^3 = \left(\frac{2}{10}\right)^3$

$= \frac{2^3}{10^3}$

$= \frac{8}{1000}$

$= 0{,}008$

3. $\sqrt{0{,}81} = \sqrt{\frac{81}{100}}$

$= \frac{\sqrt{81}}{\sqrt{100}}$

$= \frac{9}{10}$

$= 0{,}9$

4. $\sqrt[3]{0{,}125} = \sqrt[3]{\frac{125}{1000}}$

$= \frac{\sqrt[3]{125}}{\sqrt[3]{1000}}$

$= \frac{5}{10}$

$= 0{,}5$

Exercise

Simplify:

1. $(\,0{,}12)^2$
2. $(0{,}003)^3$
3. $\sqrt{0{,}121}$
4. $\sqrt[3]{0{,}027}$