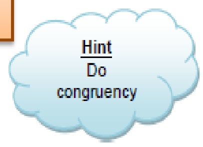
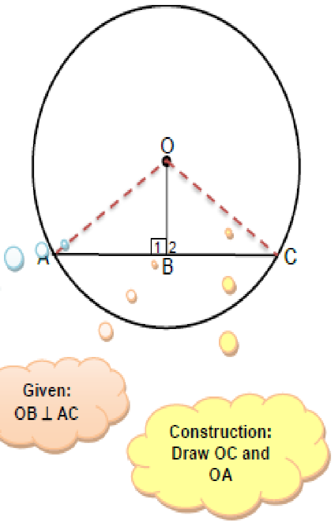
Euclidean Geometry
Gr 11
Theorems

Theorem 1

The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

line from centre ⊥ to chord





GIVEN: OB ⊥ AC

To Prove: AB = BC

Construction: Draw OA and OC

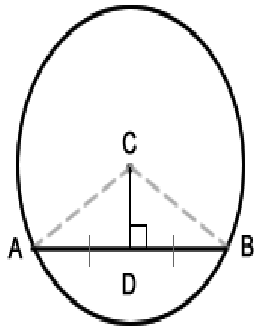
Proof:

In Δ OAB and Δ OCB

(i) OA = OC [radii]
(ii) OB = OB [common]
(iii) $\hat{B}_1 = \hat{B}_2 = 90°$ [given; OB⊥ AC]

∴ Δ OAB ≡ ΔOCB [hyp; 90˚∠;side]
AB = BC [from congruency]

Theorem 2



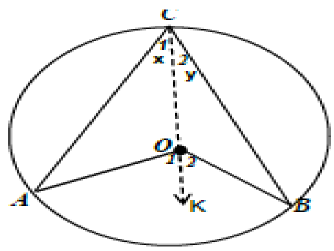
The perpendicular bisector of a chord contains the centre of the circle

Statement	Reason
In ΔCAD and ΔCBD,	
AD = DB	Given
∠CDA = ∠CDB	CD ⊥to AB
CD = CD	Reflexive proerty
ΔCDA and ΔCDB	SAS
CA = CB	Congruent triangles
Q is center of circle	The center of the circle is the only point within the circle that has points on the circumference equal distance from it.

Theorem 3

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre).

∠ at centre = 2 ×∠ at circumference



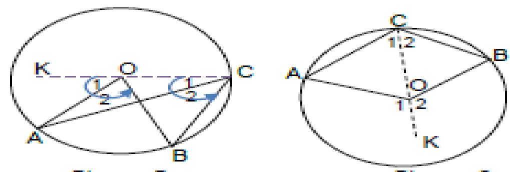


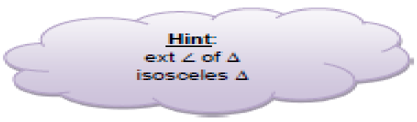
Diagram B Diagram C

Given: Circle with centre O and arc AB subtended $A\hat{O}B$ at the centre and $A\hat{C}B$ on the circumference.

To Prove: $B\hat{O}C = 2B\hat{A}C$

Construction: Draw AO extended.

Proof:



In diagram A, B and C:

Let $\hat{A}_1 = x$ and $\hat{A}_2 = y$

OA = OB [radii]

$\hat{A}_1 = \hat{B} = x$ [∠s opp equal sides]

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ [ext ∠ of Δ]

$\therefore \hat{O}_1 = 2x$

$\therefore \hat{O}_1 = 2\hat{A}_1$

Similarly $\hat{O}_2 = 2y$

$\therefore \hat{O}_2 = 2\hat{A}_2$

In diagrammes A and C:

$\therefore \hat{O}_1 + \hat{O}_2 = 2x + 2y$

$= 2(x + y)$

$A\hat{O}B = 2A\hat{C}B$

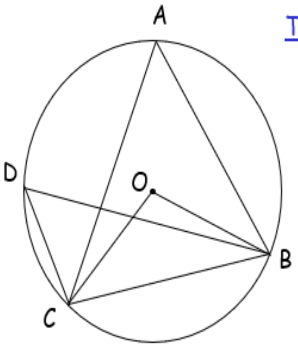
In diagram B :

$\hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

$A\hat{O}B = 2A\hat{C}B$

Theorem 4

To Prove that angles subtended by an arc or chord in the same segment are equal.



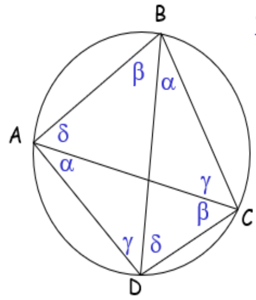
To prove that angle CAB = angle BDC

- With centre of circle O draw lines OB and OC.
- Angle COB = 2 x angle CAB (Theorem 1).
- Angle COB = 2 x angle BDC (Theorem 1).
- 2 x angle CAB = 2 x angle BDC
- Angle CAB = angle BDC

[illegible]

Theorem 5

To prove that the opposite angles of a cyclic quadrilateral are supplementary (Sum to 180°).



To prove that angles A + C and B + D = 180^0

- Draw straight lines AC and BD
- Chord DC subtends equal angles α (same segment)
- Chord AD subtends equal angles β (same segment)
- Chord AB subtends equal angles γ (same segment)
- Chord BC subtends equal angles δ (same segment)
- $2(\alpha + \beta + \gamma + \delta) = 360^\circ$ (Angle sum quadrilateral)
- $\alpha + \beta + \gamma + \delta = 180^\circ$

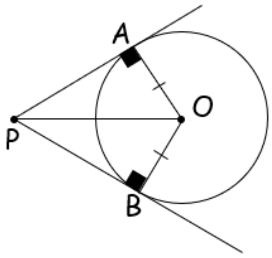
Angles A + C and B + D = 180^0 QED

Theorem 5

α	β	γ	δ
alpha	beta	gamma	delta

Theorem 6

To prove that the two tangents drawn from a point outside a circle are of equal length.

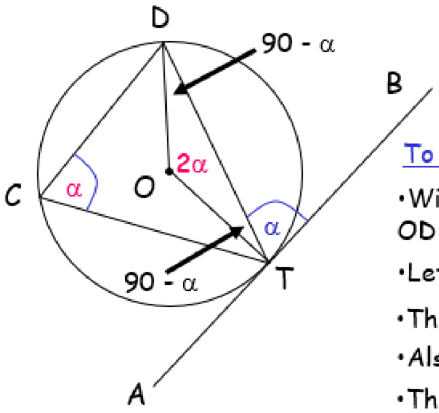


To prove that AP = BP.

- With centre of circle at O, draw straight lines OA and OB.
- OA = OB (radii of the same circle)
- Angle PAO = PBO = 90° (tangent radius).
- Draw straight line OP.
- In triangles OBP and OAP, OA = OB and OF is common to both.
- Triangles OBP and OAP are congruent (RHS
- Therefore AP = BP. QED

Theorem 7

To prove that the angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.



To prove that angle BTD = angle TCD

- With centre of circle O, draw straight lines OD and OT.
- Let angle DTB be denoted by α.
- Then angle DTO = 90 - α (Theorem 4 tan/rad)
- Also angle TDO = 90 - α (Isos triangle)
- Therefore angle TOD = 180 –(90 - α + 90 - α) = 2α (angle sum triangle)

- Angle TCD = α (Theorem 1 angle at the centre)
- Angle BTD = angle TCD QED