GRADE 11
Trigonometry
WEBSITE NOTES 2

TOPIC: Trig functions and revision grade 10 trigonometry

- Basic graphs defined by $y = a \sin x$, $y = a \cos x$ and $y = \tan x$ for $\theta \in [-360^0 ; 360^0]$
- Investigate the effect of k and p on the graphs of the functions defined by:
  $y = \sin (kx)$, $y = \cos (kx)$, $y = \tan (kx)$
- $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$)

GENERAL EQUATIONS OF TRIG FUNCTIONS

$y = a \sin b(x + p) + q$

a	Amplitude
b	Compress the graph of f(x) horizontally by a factor of b. For Trig graphs it will decrease the period.
p	Shifts the graph left or right by p units (if p is positive then it will shift left)
q	Shifts the graph up or down by q units

- To work out your critical values (values where the graph cuts the x-axis **– the intervals)**

$$Period = \frac{360°}{b}$$

$$Intervals = \frac{Period}{\mathbf{4}}$$

$y = a \cos b(x + p) + q$

a	Amplitude
b	Compress the graph of f(x) horizontally by a factor of b. For Trig graphs it will decrease the period.
p	Shifts the graph left or right by p units (if p is positive then it will shift left)
q	Shifts the graph up or down by q units

- To work out your critical values (values where the graph cuts the x-axis **– the intervals)**

$$Period = \frac{360°}{b}$$

$$Intervals = \frac{Period}{\mathbf{4}}$$

$y = a \tan b(x + p) + q$

a	The value of a affects the y-value of each point. Each y-value is multiplied by a.
b	Compress the graph of f(x) horizontally by a factor of b. For Trig graphs it will decrease the period.
p	Shifts the graph left or right by p units (if p is positive then it will shift left)
q	Shifts the graph up or down by q units

- To work out your critical values (values where the graph cuts the x-axis – **the intervals)**

$$Period = \frac{180°}{b}$$

$$Intervals = \frac{Period}{\mathbf{4}}$$

Revision of Trig Functions

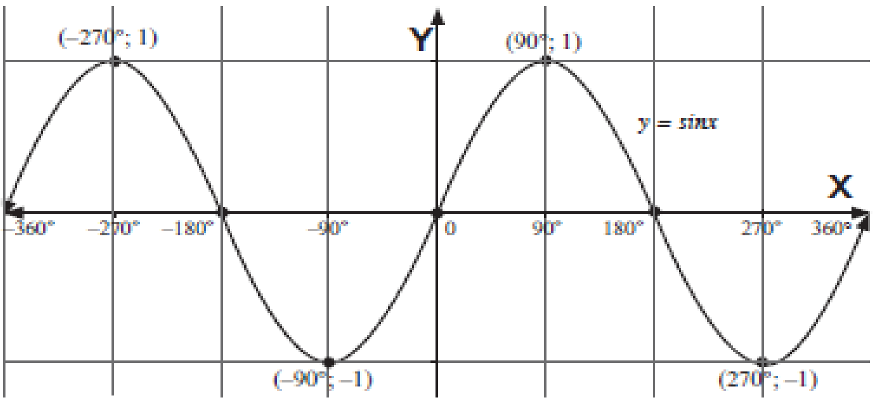
Example 1

Sketch the graph of $y = \sin x$ for x

- We can make use of a table or a calculator to determine the critical points on the graph.
- The endpoints of the domain must be included i.e. $x = -360°$ and $x = 360°$
- All intercepts with the x and y axis must be indicated as well as all minimum and maximum points (turning points)

Solution

x	–360°	–270°	–180°	–90°	0°	90°	180°	270°	360°
y	0	1	0	–1	0	1	0	–1	0



Domain: all the possible x values on the graph
Range: all the possible y-values on the graph
Amplitude: the maximum distance from the equilibrium position(in the above graph the equilibrium position is the x-axis.
Period: number of degrees to complete a wave or a cycle.

Example 2

Use the graph $y = \sin x$ above to answer these questions:

1. What are the maximum and minimum values of $y = \sin x$? (2)
2. Write down the domain and the range of $y = \sin x$. (4)
3. Write down the x-intercepts of $y = \sin x$. (2)
4. What is the amplitude of the graph of $y = \sin x$? (1)
5. What is the period of the graph of $y = \sin x$? (1)
[10]

Solutions

	$y = \sin x$		
1	Maximum Values	1 ✓ , at $x = -270°$ and $90°$	
	Minimum Values	-1 ✓, at $x = -90°$ and $270°$	(2)
2	Domain	$x \in [-360°;360°]$, $x \in \mathbb{R}$✓✓	
	Range	$[-1;1]$ $y \in \mathbb{R}$✓✓	(4)
3	x-intercepts	$-360°$, $-180°$, $0°$, $180°$ and $360°$.✓✓	(2)
4	Amplitude	1✓	(1)
5	Period	$360°$✓	(1)

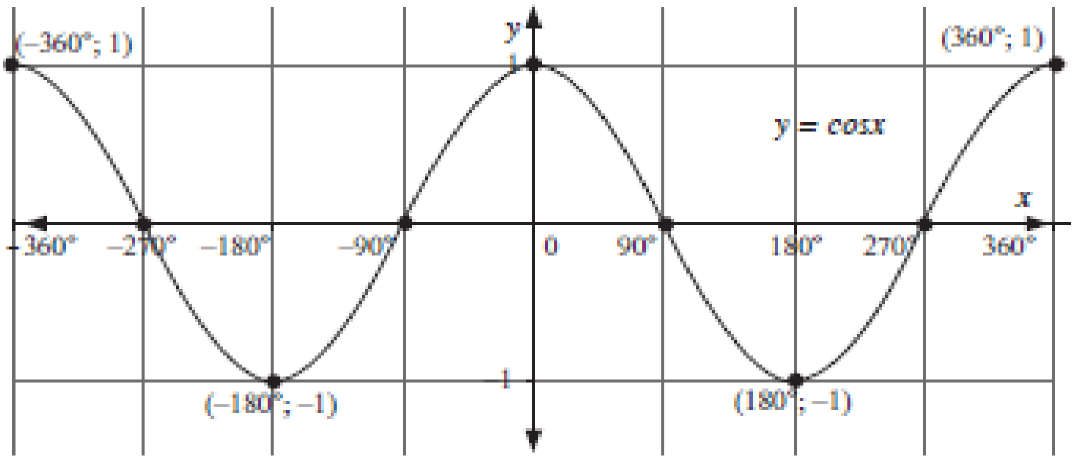
[10]

Example 3

Sketch the graph of $y = \cos x$ for $x \in [-360°;360°]$

- We can make use of a table or a calculator to determine the critical points on the graph.
- The endpoints of the domain must be included i.e. $x = -360°$ and $x = 360°$
- All intercepts with the x and y axis must be indicated as well as all minimum and maximum points (turning points)

x	$-360°$	$-270°$	$-180°$	$-90°$	$0°$	$90°$	$180°$	$270°$	$360°$
y	1	0	-1	0	1	0	-1	0	1



Example 4

For y = cos x

$y = \cos x$	
Maximum Values	1, at $x = 0°$ and 360°
Minimum Values	–1, at $x = –180°$ and 180°
x-intercepts	–270°, –90°, 90° and 270°.
Amplitude	1
Period	360°
Domain	$x \in [-360°;360°], x \in \mathbb{R}$
Range	$[-1; 1]\ y \in \mathbb{R}$

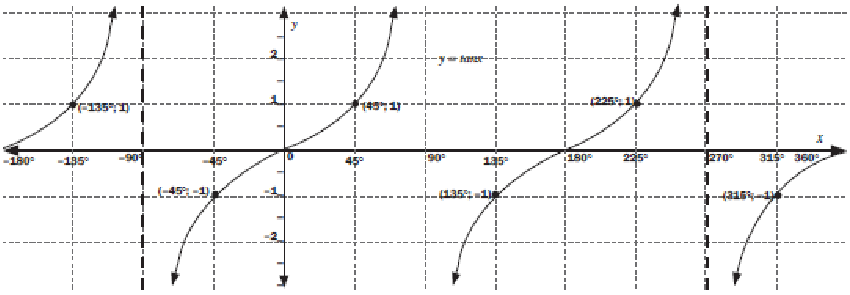
Example 5

Sketch the graph of $y = \tan x$ for $x \in [-180°;180°]$

- All intercepts with the x and y axis must be indicated.
- The endpoints of the domain must be included i.e. $x = -180°$and $x = 360°$
- The equations of the asymptotes must be written on the graph.

Answer

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	1	undefined	–1	0	1	undefined	–1	0	1	undefined	–1	0



	$y = \tan x$	
1	Asymptotes	$x = –90°$, $x = 90°$ and $x = 270°$
2	x-intercepts	–180°, 0°, 180° and 360°.
3	Period	180°
4	Domain	$x \in [-180°;360°], x \in \mathbb{R}$
5	Range	$(-\infty; \infty),\ y \in \mathbb{R}$

Example 6 (Try yourself)

1. Given $f(x) = 2\cos x$ and $g(x) = \sin(x + 30°)$

a) Sketch the graphs of f and g on the same set of axes for $x \in [-150°; 180°]$

Clearly show all intercepts with the axes and the coordinates of turning points. (7)

Use your graph to answer the following questions:

b) Write down the period of f. (1)

c) For which values of x is $f(x) = g(x)$? (2)

d) For which values of x is $f(x) > 0$? (2)

e) For which values of x is $g(x)$ increasing? (2)

f) Determine one value of x for which $f(x) - g(x) = 1,5$. (1)

g) If the curve of f is moved down one unit, write down the new equation of f. (2)

h) If the curve of g is moved 45° to the left, write down the new equation of g. (2)