GRADE 12
Calculus – First Principles
WEBSITE NOTES

TOPIC:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

- First principles.

Example 1

To differentiate from first principles (definition) use the formula below

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Determine $f'(x)$ from first principles if $f(x) = -3x^2$

$f(x+h) = -3(x+h)^2$

$= -3(x^2 + 2xh + h^2)$

$= -3x^2 - 6xh - 3h^2$ to get $f(x+h)$ we replace x with $x + h$ and get $-3(x+h)^2$

Expand the brackets and Make sure you multiply the –3 with each term in the brackets

Substituting into $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ the definition of the derivative gives

$$f'(x) = \lim_{h\to 0} \frac{-3x^2 - 6xh - 3h^2 - (-3x^2)}{h}$$

$f(x) = -3x^2$ $f(x) = -3x^2$ so

$$= \lim_{h\to 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h}$$

Take out a common factor of h so you can cancel it with the h in the denominator.

As h goes to 0, $-6x - 3h$ goes to $-6x$.

$$= \lim_{h\to 0} \frac{h(-6x - 3h)}{h}$$

$$= \lim_{h\to 0} (-6x - 3h)$$

$$= -6x$$

<u>You can always expect a question to determine the derivative (f'(x)) using first principles</u>

<u>Example 2 (Try Yourself)</u>

1.	Determine $f'(x)$ from first principles if $f(x) = 5x^2 - 4x + 2$	(6)
2.	Determine $f'(x)$ from first principles if $f(x) = \frac{2}{x}$	(6)
		[12]

<u>Answers</u>

1. $f(x+h) = 5(x+h)^2 - 4(x+h) + 2$

$= 5(x^2 + 2xh + h^2) - 4x - 4h + 2$

$= 5x^2 + 10xh + 5h^2 - 4x - 4h + 2$ ✓

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 4x - 4h + 2 - (5x^2 - 4x + 2)}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2 - 4h}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{h(10x + 5h - 4)}{h} \checkmark$$

$$= \lim_{h \to 0} (10x + 5h - 4) \checkmark$$

$$= 10x - 4 \checkmark$$

(6)

2. $f(x+h) = \frac{2}{x+h}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \checkmark \checkmark$$

$$= \lim_{h \to 0} \frac{\frac{2x}{x(x+h)} - \frac{2(x+h)}{x(x+h)}}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{\frac{2x - 2x - 2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{x(x+h)} \times \frac{1}{h} \checkmark$$

$$= \lim_{h \to 0} \frac{-2}{x(x+h)} \checkmark$$

$$\simeq \frac{-2}{x(x)} = \frac{-2}{x^2} \checkmark$$

(6)

[12]