

**GRADE 12**

**Calculus – First Principles**

**WEBSITE NOTES**

**TOPIC:**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- First principles.
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**Example 1**

To differentiate from first principles (definition) use the formula below

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Determine  $f'(x)$  from first principles if  $f(x) = -3x^2$

$$f(x+h) = -3(x+h)^2$$

$$= -3(x^2 + 2xh + h^2)$$

$$= -3x^2 - 6xh - 3h^2 \text{ to get } f(x+h) \text{ we replace } x \text{ with } x+h \text{ and get}$$

$$-3(x+h)^2$$

Expand the brackets and Make sure you multiply the  $-3$  with each term in the brackets

Substituting into  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  the definition of the derivative gives

$$f'(x) = \lim_{h \rightarrow 0} \frac{-3x^2 - 6xh - 3h^2 - (-3x^2)}{h} \quad f(x) = -3x^2 \text{ so}$$

$$= \lim_{h \rightarrow 0} \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h}$$

Take out a common factor of  $h$  so you can cancel it with the  $h$  in the denominator.

As  $h$  goes to 0,  $-6x - 3h$  goes to  $-6x$ .

$$= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$$

$$= \lim_{h \rightarrow 0} (-6x - 3h)$$

$$= -6x$$

**You can always expect a question to determine the derivative ( $f'(x)$ ) using first principles**

**Example 2 (Try Yourself)**

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|--|-------------|
| 1. Determine $f'(x)$ from first principles if $f(x) = 5x^2 - 4x + 2$ | (6)         |
| 2. Determine $f'(x)$ from first principles if $f(x) = \frac{2}{x}$   | (6)         |
|  | <b>[12]</b> |