GRADE 12
Calculus 2– Differential Rules
WEBSITE NOTES

TOPIC:

- Rules of differentiation.

NOTE: The notation we use for the derivative of $y = f(x)$ is

$f'(x)$ or y' or $\frac{dy}{dx}$ or $D_x[f(x)]$.

When we find the derivative of a function, we say we **differentiate** the function.

NOTE!!!

If the question asks you to find the derivative but does not say USING FIRST PRINCIPLES, then you use the DIFFERENTIATION RULES as below.

Rules

1. If $f(x) = b$ then $f'(x) = 0$
   where b is a constant

Example 1

If $h(x) = 12$, then $h'(x) = 0$

2. If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

Example 2

If $k(x) = x^5$, then $k'(x) = 5x^4$

3. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Example 3

If $f(x) = x^5 + x^4$, then $\frac{d}{dx}f(x) = 5x^4 + 4x^3$

NOTE!!!

$\frac{d}{dx}f(x)$

Is the same as saying – the derivative of the function f(x) or f '(x)

4. $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$

Example 4

If $f(x) = 3x^5$ then

$\frac{d}{dx}f(x) = 3 \times \frac{d}{dx}f(x)\ (x^5) = 3 \times 5x^4 = 15x^4$

Sometimes you may need to use distributive law in order to get the function in standard form. It must be in a standard form before you differentiate.

Multiply out first using FOIL

Example 5

Determine $f'(x)$ if $f(x) = (3x + 2)(x - 5)$

Solution

$f(x) = 3x^2 - 13x - 10$

$\therefore f'(x) = 6x - 13$

Sometimes you may need to change roots into exponents before doing differentiation.

Example 6

$\sqrt{x} = x^{\frac{1}{2}}$

so $\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}}$

- $\frac{dy}{dx}$ *as the derivative of* y *with respect to* x
- $\frac{d}{dx}\sqrt{x}$ *as the derivative of* $\sqrt{x}$ *with respect to* x
- $\frac{d}{dx}f(x)$ *as the derivative of* $f(x)$ *with respect to* x

REMEMBER

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$f'(x)$ or y' or $\frac{dy}{dx}$ or $D_x[f(x)]$.

When we find the derivative of a function, we say we **differentiate** the function.

Example 7 (Try Yourself)

You need to find the Derivative of each question using the rules and not first principles because it does not say first principles.

Remember STANDARD FORM and ROOTS into EXPONENTS first

a) Evaluate $D_x[(x^3 - 3)^2]$

b) Find $f'(x)$ if $f(x) = \sqrt[3]{x}$

c) Find $\frac{d}{dx}\sqrt[3]{x^5}$

d) Differentiate $f(x)$ if $f(x) = \sqrt{x^4}$

e) Find $f'(x)$ if $f(x) = \sqrt{16x^3}$

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Example 8 (Try Yourself)

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REMEMBER THE DERIVATIVE IS THE GRADIENT AT A POINT.

TO WORK OUT A TANGENT OF A GRAPH YOU NEED TO WORK THE GRADIENT OUT FIRST.

QUESTION 8

8.1 If $f(x) = \frac{4}{x}$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 5x^2 + 5x + 2$ (2)

8.2.2 $D_x\left[\sqrt[3]{x^2} - \frac{1}{2}x\right]$ (3)

8.3 Given: $p(x) = x^3 + 2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of p to have a negative gradient. (3)

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